1. Let $U = \mathbb{C} \setminus [-i, i]$ be the complex plane less the closed interval joining $-i$ to $i$. When $\gamma$ is a closed contour in $U$ prove that
$$\int_{\gamma} \frac{d\zeta}{\zeta^2 + 1} = 0.$$

2. Let $V = \mathbb{C} \setminus ((-i\infty, -i] \cup [i, i\infty))$ be the complex plane less the closed rays from $-i$ to $-i\infty$ and from $i$ to $i\infty$ along the imaginary axis. When $\gamma$ is a closed contour in $V$ prove that
$$\int_{\gamma} \frac{d\zeta}{\zeta^2 + 1} = 0.$$

3. Let $f$ be holomorphic in $\mathbb{D}$ and there satisfy the inequality $|f(z)| \leq 1 - |z|$. Use the Cauchy integral formula to prove that $f$ is identically zero.

4. Let $\gamma$ be a closed contour in $\mathbb{C}$ that avoids the point $a$. Prove that
$$\frac{1}{2\pi i} \int_{\gamma} \frac{1}{\zeta - a} d\zeta \in \mathbb{Z}.$$ 

*Suggestion:* Let $\gamma$ be parametrized over $[0, 1]$ and consider the integral from 0 to $t$ as a function of $t$. 

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1