1. Let $S$ be a finite set and $M = \mathcal{P}(S)$ its power set. Define $d(A, B) = |A \Delta B|$ when $A, B \in M$. Prove that $d$ is a metric on $M$. *Suggestion:* First prove that $A \Delta B \subseteq (A \Delta C) \cup (C \Delta B)$.

2. Let $d_0$ be a metric on the set $M$. Prove that $d = d_0/(1 + d_0)$ is a metric on $M$. *Suggestion:* First inspect the function from $[0, \infty)$ to $[0, 1)$ given by the rule $t \mapsto t/(1 + t)$.

3. Let $d$ be derived from $d_0$ as above. Prove that the metric spaces $(M, d)$ and $(M, d_0)$ have precisely the same open sets. *Suggestion:* First relate their open balls.

4. Let $(M, d)$ be a bounded metric space and $S$ a set. Decide (carefully, of course) whether $D$ defined by the rule

$$D(f, g) = \sup\{d(f(s), g(s)) : s \in S\}$$

is a metric on the set $\text{Map}(S, M)$ comprising all functions from $S$ to $M$. 