Modern Analysis 1
Homework 04

The connectedness of each line segment (bounded or unbounded, with or without endpoints) in the Euclidean plane may be quoted without proof.

1. Prove that in the Euclidean plane each figure $H$ is connected, whether the verticals are bounded or unbounded, with or without endpoints.

2. Prove that if $h \in \mathbb{R}$ then the following subset of the Euclidean plane is connected:

$$\{(x, y) \in \mathbb{R}^2 : (x = \pm 1) \text{ or } (|x| < 1 \text{ and } y \geq h)\};$$

a sketch would be appropriate and should be helpful.

3. Let $C_1 \supseteq C_2 \supseteq \cdots$ be a sequence of connected closed subsets of a metric space $X$ with nonempty intersection $\bigcap_{n=1}^{\infty} C_n$. Must this intersection be connected? Proof or counterexample.