1. The holomorphic function $f : \mathbb{D} \to \mathbb{D}$ has a zero of order $N$ at 0. Prove:
   (i) if $z \in \mathbb{D}$ then $|f(z)| \leq |z|^N$; (ii) if equality holds here for some $z \neq 0$ then there exists $\alpha \in \partial \mathbb{D}$ such that $f(w) = \alpha w^N$ for each $w \in \mathbb{D}$.

2. Let $U \subseteq \mathbb{C}$ be a connected open set and $f : U \to \mathbb{C}$ a holomorphic function. Assume that for each $z \in U$ there exists $n = n_z \in \mathbb{N}$ such that $f^{(n)}(z) = 0$. Prove that $f$ is actually a polynomial.

3. Let $U$ be the square $\{ z \in \mathbb{C} : |\text{Re} z| < 1 \text{ and } |\text{Im} z| < 1 \}$; let the complex-valued function $f$ be holomorphic on $U$ and continuous on its closure $\overline{U}$. Deduce as much as possible about $f$ from the hypothesis that $f(z) = 0$ whenever $z \in \overline{U}$ has $\text{Re} z = 1$. 