1. Consider the first-order initial value problem

\[ f' = 1 + f^2, \quad f(0) = 0 \]

for \( f \) on some open interval \( I \) about 0. Define

\[ g = \frac{1 - f^2}{1 + f^2} \]

and write \( h(t) = g(t/2) \) when \( t \in 2I \).

(i) Determine a second-order initial value problem satisfied by \( h \).

(ii) Use this to identify \((h, \text{ hence}) f \) and the maximal interval \( I \).

[Duplication formulas for trigonometric functions may be assumed.]

2. When \(|s| < 1\) write

\[ \ell(s) = s - s^2/2 + s^3/3 - s^4/4 + \ldots. \]

(i) By (termwise) differentiation, show that \( \ell(s) = \log(1 + s) \).

(ii) Prove that

\[ |\log(1 + s) - s| \leq \frac{1}{2} \frac{s^2}{1 - |s|}. \]

(iii) Let \( t \in \mathbb{R} \): prove that \( \lim_{n \to \infty} (1 + \frac{t}{n})^n = e^t \).

[Take \( s = \frac{t}{n} \) with \( |t| < n \).]