1. Let $(X, d)$ be a metric space. Prove that the function $d : X \times X \to \mathbb{R}$ is continuous when $X \times X$ is given the ‘max’ metric. [Argue from the ‘$\varepsilon, \delta$’ definition of continuity.]

2. Let the metric space $X$ be compact and $f : X \to X$ continuous. Assume that $f$ ‘almost’ has a fixed point, in the sense

$$(\forall \varepsilon > 0)(\exists x \in X) d(x, f(x)) < \varepsilon.$$

Prove that $f$ actually has a fixed point. Do this in two ways:
(i) argue sequentially, taking $\varepsilon$ to be $1/n$ for each positive integer $n$ in turn;
(ii) compose $F : X \to X \times X : x \mapsto (x, f(x))$ and $d : X \times X \to \mathbb{R}$. 

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1