1. Let $X$ be complete and let $f : X \to X$ be such that $g = f \circ f \circ f$ is a (strict) contraction. Must $f$ have any fixed points? If so, how many?

2. Let $f : X \to Y$ be a continuous bijection, with $Y$ compact. Must $f^{-1}$ be continuous? Proof or counterexample.

3. Let $f : X \to Y$ map Cauchy sequences to Cauchy sequences. Must $f$ be uniformly continuous? Proof or counterexample.

4. Let $Z = \{(x, y) : x \in \mathbb{Q} \text{ or } y \in \mathbb{Q}\} \subseteq \mathbb{R}^2$. Let $f : Z \to \mathbb{R}$ be continuous, with $f(1,1)f(-1,-1) < 0$. Must there exist $p \in Z$ such that $f(p) = 0$? Proof or counterexample.