Differential Equations
Test 3 sample

Answer FOUR questions and show all work.

1. Define the Laplace transform $F$ of $f$. Write down $F(s)$ when $f(t)$ is $t^n$, $e^{at}$, $e^{at} \cos bt$, $e^{at} \sin bt$ and calculate one of these from the definition.

2. State how the Laplace transforms $G$ and $F$ are related when $g(t) = tf(t)$. Calculate the Laplace transforms of $t \cos bt$ and $t \sin bt$.

3. Calculate the inverse Laplace transform $f$ of $F$ in each case:

   (i) $F(s) = (s + 1)/(s - 1)^3$; (ii) $F(s) = s^2/(s^4 - 1)$.

4. Use the Laplace transform to solve the initial value problem

   $y'' + y = t$; $y(0) = 0$, $y'(0) = 1$.

5. Define the convolution product $f \ast g$ and use it to calculate $h(t)$ when $H(s) = 1/[s^3(s^2 + 1)]$. 

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