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Topological Data Analysis Worksheet 2

Exercise 1. Consider the following filtered simplicial complex.

$$X_0 \;\hookrightarrow\; X_1 \;\hookrightarrow\; X_2 \;\hookrightarrow\; X_3 \;\hookrightarrow\; X_4 \;\hookrightarrow\; X_5 \;\hookrightarrow\; X_6 \;\hookrightarrow\; X_7 \;\hookrightarrow\; X_8 \;\hookrightarrow\; X_9$$
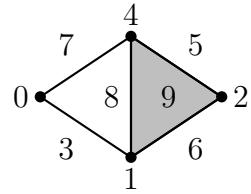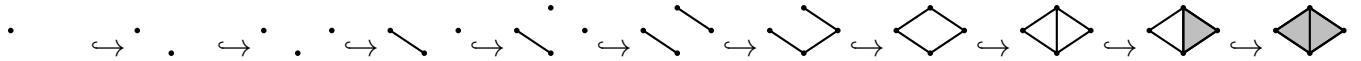

(a) Write down boundary matrices for ∂_1 and ∂_2, ordering the simplices using the order they appear in the filtration.

(b) Apply the persistence algorithm to reduce these matrices using column operations.

(c) Give the birth–death pairs for H_0 and H_1.

(d) Plot the corresponding barcode and persistence diagram.

Exercise 2. Consider the following filtered simplicial complex.
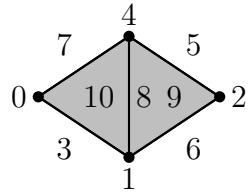
$$X_0 \;\hookrightarrow\; X_1 \;\hookrightarrow\; X_2 \;\hookrightarrow\; X_3 \;\hookrightarrow\; X_4 \;\hookrightarrow\; X_5 \;\hookrightarrow\; X_6 \;\hookrightarrow\; X_7 \;\hookrightarrow\; X_8 \;\hookrightarrow\; X_9 \;\hookrightarrow\; X_{10}$$

(a) Compute the rank functions for reduced homology $\tilde H_0$ and $\tilde H_1$.
(b) Apply Möbius inversion to obtain the corresponding persistence diagrams.
(c) Compare these persistence diagrams to the ones obtained in the previous exercise.

Exercise 3. Consider the previous filtered simplicial complex and its rank function.

(a) Compute the graded functions (rank $\geq k$) for $k = 1, 2$ for $\tilde{H}_0$ and $\tilde{H}_1$.

(b) Apply Möbius inversion to obtain the graded persistence diagrams for $k = 1, 2$ for $\tilde{H}_0$ and $\tilde{H}_1$.

(c) Verify that the sum of the graded rank functions is the corresponding rank function.

(d) Verify that the sum of the graded persistence diagrams is the corresponding persistence diagram.

$\tilde{H}_0, 1$	0	1	2	3	4	5	6	7	8	9
0										
1	—									
2	—	—								
3	—	—	—							
4	—	—	—	—						
5	—	—	—	—	—					
6	—	—	—	—	—	—				
7	—	—	—	—	—	—	—			
8	—	—	—	—	—	—	—	—		
9	—	—	—	—	—	—	—	—	—	

$\tilde{H}_0, 2$	0	1	2	3	4	5	6	7	8	9
0										
1	—									
2	—	—								
3	—	—	—							
4	—	—	—	—						
5	—	—	—	—	—					
6	—	—	—	—	—	—				
7	—	—	—	—	—	—	—			
8	—	—	—	—	—	—	—	—		
9	—	—	—	—	—	—	—	—	—	

$\tilde{H}_1, 1$	0	1	2	3	4	5	6	7	8	9
0										
1	—									
2	—	—								
3	—	—	—							
4	—	—	—	—						
5	—	—	—	—	—					
6	—	—	—	—	—	—				
7	—	—	—	—	—	—	—			
8	—	—	—	—	—	—	—	—		
9	—	—	—	—	—	—	—	—	—	

$\tilde{H}_1, 2$	0	1	2	3	4	5	6	7	8	9
0										
1	—									
2	—	—								
3	—	—	—							
4	—	—	—	—						
5	—	—	—	—	—					
6	—	—	—	—	—	—				
7	—	—	—	—	—	—	—			
8	—	—	—	—	—	—	—	—		
9	—	—	—	—	—	—	—	—	—	