MAP 4484 Review for Midterm 1

The test will be closed-book. The test will consist of 2-3 problems. Below are the representative problems.

(1) Find the fixed point of the given map and determine their stability for various values of a and b:

$$x_{n+1} = \frac{ax_n}{b + x_n}, \quad x_n \geq 0, \quad a, b > 0.$$

(2) Consider the piece-wise linear function f defined as

$$f(x) = \begin{cases} 3x, & 0 \leq x \leq 1/3, \\ 2 - 3x, & 1/3 < x < 2/3, \\ 3x - 2, & 2/3 \leq x \leq 1. \end{cases}$$

Find the fixed points and determine their stability. Does there exist an orbit of period 2? If yes, how many? Which orbits of period 2 are stable?

(3) Find the explicit solution of the following difference equation:

$$x_{n+2} = 3x_{n+1} - 2x_n, \quad n \geq 0, \quad x_0 = 1, \; x_1 = 3.$$

(4) Consider the following competition model:

$$\begin{cases} x_{n+1} = \frac{4x_n}{1+x_n+3y_n}, \\ y_{n+1} = \frac{4y_n}{1+3x_n+y_n}. \end{cases}$$

Show that there exists a unique coexistence fixed point (x^*, y^*). Find it. Evaluate the Jacobian matrix at (x^*, y^*). Is this coexistence point stable or unstable?