The test will be closed-book. The test will consist of 2-3 problems. Here is a collection of sample problems.

(1) Find the equilibria and determine their stability using the linearization method:

\[ x' = x(2 - x), \]

also sketch the phase line for this differential equation. Argue that all positive solutions \( x(t) > 0 \) approach the limiting value 2 as \( t \to \infty \), and that the convergence is exponential.

(2) Consider the following model of microbial growth:

\[ s' = D(s_0 - s) - p(s)x, \quad x' = p(s)x - Dx. \]

Here, \( s(t) \) is the concentration of a growth limiting resource, and \( x(t) \) is the microbial concentration. Let \( p(s) = \frac{ms}{k+s} \). Interpret the meaning of this function and the remaining parameters of the model. Find the equilibria and determine their stability using linearization (note: there may be one or two equilibria, depending on the parameters). Show that the total biomass \( s + x \) approaches \( s_0 \) in the limit \( t \to \infty \).

(3) Sketch the bifurcation diagram of the equation

\[ x' = x^2(1 - x^2) + \mu, \]

treating \( \mu \) as the bifurcation parameter. Determine the bifurcation points/values. What type of bifurcations are these?

(4) Let \( A \) be a \( 2 \times 2 \) matrix. Classify the eigenvalues of \( A \) in terms of the trace and the determinant of \( A \). Derive the necessary and sufficient conditions for \( A \) to have both eigenvalues with negative real parts.

(5) Sketch the phase portrait of the \( SI \) epidemic model

\[ S' = -bSI, \quad I' = bSI - dI. \]