MAP 4484 Review for Midterm 2

The test will be closed-book. The test will consist of 2-3 problems. Here is a collection of sample problems.

(1) Find the equilibria and determine their stability using the linearization method:

$$x' = x(2 - x),$$

also sketch the phase line for this differential equation. Argue that all positive solutions $x(t) > 0$ approach the limiting value 2 as $t \to \infty$, and that the convergence is exponential.

(2) Consider the following model of microbial growth:

$$s' = D(s_0 - s) - p(s)x, \quad x' = p(s)x - Dx.$$

Here, $s(t)$ is the concentration of a growth limiting resource, and $x(t)$ is the microbial concentration. Let $p(s) = \frac{ms}{k+s}$. Interpret the meaning of this function and the remaining parameters of the model. Find the equilibria and determine their stability using linearization (note: there may be one or two equilibria, depending on the parameters). Show that the total biomass $s + x$ approaches s_0 in the limit $t \to \infty$.

(3) Sketch the bifurcation diagram of the equation

$$x' = x^2(1 - x^2) + \mu,$$

treating μ as the bifurcation parameter. Determine the bifurcation points/values. What type of bifurcations are these?

(4) Let A be a 2×2 matrix. Classify the eigenvalues of A in terms of the trace and the determinant of A. Derive the necessary and sufficient conditions for A to have both eigenvalues with negative real parts.

(5) Sketch the phase portrait of the SI epidemic model

$$S' = -bSI, \quad I' = bSI - dI.$$