1. Write the following scalar equations in matrix normal form:
   (i) $z^2 u'' + zu' + (z^2 - m^2)u = 0$ (Bessel equation)
   (ii) $u'' - zu = 0$ (Airy equation)
   (iii) $u''' - 3u'' + zu' - z^2 u = \cos z$

2. Let $v_1(t), v_2(t)$ be the vector functions
   
   $v_1(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$
   $v_2(t) = \begin{pmatrix} t |t| \\ 2|t| \end{pmatrix}$

   (i) Show that $W(v_1(t), v_2(t)) = 0$. (ii) Show that $v_1(t)$ and $v_2(t)$ are linearly independent on $(-\infty, \infty)$. (iii) Show that they are linearly dependent on $(-\infty, 0)$ and $(0, \infty)$.

3. Find a fundamental solution matrix for the system
   
   $v' = \begin{pmatrix} -10 & -15 & -6 \\ 8 & 13 & 6 \\ -5 & -9 & -5 \end{pmatrix} v$

   Hint: the eigenvalues are $\pm 1$ and $-2$.

4. Find a fundamental solution matrix for the system
   
   $v' = \begin{pmatrix} -11 & -9 & -14 & -17 \\ 4 & 3 & 5 & 7 \\ 1 & 0 & 1 & 2 \\ 4 & 4 & 5 & 6 \end{pmatrix} v$

   Hint: eigenvalues with multiplicities are $2, -1, -1, -1$.

5. Find a real fundamental solution matrix for the system
   
   $v' = \begin{pmatrix} 9 & 10 & 0 \\ -8 & -11 & -3 \\ 6 & 10 & 5 \end{pmatrix} v$

   (eigenvalues are $-1$ and $2 \pm i$).

6. If $A$ is the $3 \times 3$ matrix in problem 3, find all solutions of

   $v' = Av + te^{zt} \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

   using the method of undetermined coefficients.
7. If $A$ is the $3 \times 3$ matrix in problem 3, find all solutions of

$$v' = Av + e^{-t} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

using the method of variation of parameters.

8. Again with $A$ the $3 \times 3$ matrix in problem 3, find the matrix exponential $e^{At}$. If $X(t)$ is the fundamental solution matrix you found in problem 3, find an invertible matrix $C$ such that $e^{At} = X(t)C$.

9. Find a solution of the initial value problem

$$v' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} v + \begin{pmatrix} \sin t \\ -\cos t \end{pmatrix} \quad v(0) = 0$$

using the method of Laplace transforms (see Ch. 9 Group Projects B, page 564).