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Controlling transient chaos to prevent species extinction

Luke Shulenburger [a], Ying-Cheng Lai [b], Tolga Yalçınkaya [c], Robert D. Holt [d]

[a] Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[b] Departments of Physics and Astronomy and of Mathematics, University of Kansas, Lawrence, KS 66045, USA
[c] Advanced Research Division, Sprint, Overland Park, KS 66011, USA
[d] Department of Systematics and Ecology, University of Kansas, Lawrence, KS 66045, USA

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Abstract

Recent work has suggested that species extinctions in ecological systems can occur as a consequence of deterministic transient chaos even in the absence of external disturbances. We argue and present a practical method to demonstrate that species extinctions due to transient chaos can be effectively prevented by applying small, ecologically feasible perturbations to the populations at appropriate but rare times. This may be of significant importance to the challenging environmental problems of species preservations. © 1999 Elsevier Science B.V. All rights reserved.

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Extinction of species has been one of the biggest mysteries in nature [1]. A common belief about local extinction is that they are typically caused by external environmental factors such as sudden changes in climate. For a species of very small population size, small random changes in the population (known as 'demographic stochasticity') can also lead to its extinction. Clearly, the question of how species extinction occur is extremely complex, as each species typically lives in an environment that involves interaction with many other species (e.g., through competition for common food sources, predator-prey interactions, etc.) as well as physical factors such as the weather and disturbances (e.g., landslides). From a mathematical point of view, a dynamical model for the population size of a species is complex, involving temporal and spatial variations, external driving, and random perturbations. Such a system should, in general, be modeled by nonlinear partial differential equations with random and/or regular external driving forces. A difficulty associated with this approach is that the analysis and numerical solution of stochastic and/or driven nonlinear partial differential equations present an extremely challenging problem in mathematics – a general method for tackling such equations still remains unknown.

Nonetheless, in certain situations the problem of species extinction can become simpler. Recently, it

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was suggested by McCann and Yodzis [2] that deterministic chaos in very simple but plausible ecosystem models, mathematically described by coupled ordinary differential equations, can provide a hint as to how local species extinction can arise without the necessity to consider temporal or spatial variations and external factors. The key observation is that the population dynamics of a large class of ecosystems can be effectively modeled by deterministic chaotic systems [3], and that the behavior of transient chaos is often typical in nonlinear dynamical systems [4,5]. In the case of *transient chaos*, the dynamical variables of the system behave chaotically for a period of time and then switch their behavior suddenly, say, to a fixed value or to some regular patterns (including zero population density). For an ecosystem that exhibits transient chaos, the implication is that the population size of some species can behave chaotically for a (long) period of time and then decreases to zero in a relatively short period of time. It was shown by McCann and Yodzis [2] that such a transient chaotic behavior, which is responsible for species extinction, can indeed occur in a simple three-species food chain model which incorporates biologically reasonable assumptions about species interactions [6].

The aim of this Letter is to demonstrate that if species extinction is caused by transient chaos, then it is possible for human being to intervene externally by applying perturbations so as to effectively prevent species from becoming extinct. We devise a simple scheme to control transient chaos by applying occasional feedback perturbations to populations of the species. As such, the natural dynamics of the species population is hardly influenced, and yet, the population, though still exhibiting chaotic behavior, will never become zero. We describe the principle of controlling transient chaos and illustrate our idea by utilizing a representative example, the food-chain model studied by McCann and Yodzis [2]. *The implication is that in a realistic ecological environment, an appropriate amount of artificially imposed change to population sizes or some small disturbance to the environment, only very rarely applied, can prevent species extinction over long time scales.* Potentially, our idea can be of importance to the significant and growing environmental problem of species preservation.

We consider the following model of a simple three-species food chain: a resource species, a prey (consumer), and a predator [2]:

$$\frac{dR}{dt} = R\left(1 - \frac{R}{K}\right) - \frac{x_C y_C CR}{R + R_0},$$

$$\frac{dC}{dt} = x_C C\left(\frac{y_C R}{R + R_0} - 1\right) - \frac{x_P y_P PC}{C + C_0},$$

$$\frac{dP}{dt} = x_P P\left(-1 + \frac{y_P C}{C + C_0}\right), \qquad (1)$$

where R, C, and P are the population densities of the resource, consumer, and predator, respectively, K is the resource carrying capacity, x_C, y_C, x_P, y_P, R_0, and C_0 are parameters which are positive [1]. The resource population, growing alone, equilibrates its carrying capacity K. The resource population and intermediate consumer, without the top consumer, either settle to a stable equilibrium, or to a stable limit cycle, a kind of 'biological oscillator.' The oscillations are generated by the saturating feeding response, which permits the resource to periodically 'escape' control by the consumer. With the top consumer, there are in a sense two coupled oscillators in the food chain. It is well-known that coupled oscillators can lead to complex dynamics [7]. This provides an intuitive insight into why the model can give rise to chaotic dynamics.

Realistic values for parameters can be derived from bioenergetics. We fix $x_C = 0.4$, $y_C = 2.009$, $x_P = 0.08$, and $y_P = 2.876$ so that both the consumer and the predator can be either invertebrate or vertebrate ectotherms (e.g., fish), with a reasonable predator to prey (consumer to resource) body mass ratio [2]. We also fix $R_0 = 0.16129$ and $C_0 = 0.5$. Although the above parameter choices are rather arbitrary, they are ecologically meaningful [2]. The resource carrying capacity, K, however, can be differ-

[1] The biological assumptions of the model are as follows: (1) The life histories of each species involve continuous growth and overlapping generations, with no age structure (this permits the use of differential equations); (2) The resource population (R) grows logistically; (3) Each consumer species (immediate consumer C, top consumer P) without food dies off exponentially; (4) Each consumer's feeding rate, [e.g., $x_C y_C R/(R + R_0)$], saturates at high food levels.

ent in different environments. Thus, we vary K over some reasonable range to assess different dynamical behaviors of the system.

To understand how species extinction can occur in the model Eq. (1), it is insightful to look into the dynamics of the predator population from the perspective of chaos. It can be shown from a bifurcation analysis that chaotic attractors can occur for $0.9 < K < K_c \approx 0.99976$ via a period-doubling cascade [2]. None of the populations corresponding to trajectories on the chaotic attractor is extinct because the attractor is located in a phase-space region away from the origin $[(R,C,P) = (0,0,0)]$. In this parameter range, however, there is also a limit-cycle attractor, located in the plane of $P = 0$, which coexists with the chaotic attractor. Trajectories on this attractor thus correspond to the situation where the top predator population is extinct. Therefore, for a fixed $K \lesssim K_c$, depending on the choice of the initial condition, the system either asymptotes to the chaotic attractor or to the limit cycle with $P = 0$. For $K \lesssim K_c$, there is still a finite distance from the tip of the chaotic attractor to the basin boundary. Thus, for any initial condition chosen in the basin of the chaotic attractor, the population of the top predator $P(t)$ behaves chaotically in time but never decreases to zero because the attractor lives in a region where $P(t) \neq 0$. In this case, the predator never becomes extinct.

As the carrying capacity K increases passing through the critical value K_c, the predator will eventually become extinct for almost all initial conditions. This is quite counter-intuitive, but it can be easily understood from the dynamics. At $K = K_c$, the tip of the chaotic attractor touches the basin boundary, a dynamical event called the *crisis* [4]. Roughly, this crisis creates ‘holes’ on the basin boundary from which trajectories in the blank region can now leak through the holes and enter the basin of the limit-cycle attractor at $P = 0$, resulting in an ultimate extinction of the top predator population. For $K \gtrsim K_c$, a typical trajectory spends a lot of time near the original chaotic attractor, before it exits one of the holes and asymptotes to the limit-cycle attractor. This is the phenomenon of transient chaos. Dynamically, after the crisis, the original chaotic attractor is converted into a non-attracting chaotic set, called a *chaotic saddle* [4]. Thus, we see that a species extinction can indeed occur as a result of a non-attracting chaotic saddle in the phase space which physically leads to transient chaos.

How can the extinction of the predator population be prevented? One way is to decrease the resource carrying capacity K so that the sustained chaotic motion on the attractor is restored. But ecologically, it may not be easy to adjust the carrying capacity of an environment and if this can be done, it may take some time to do so after detecting that the predator population is in danger. Thus, it may occur that the predator will already have become extinct before the carrying capacity is changed. Here we propose an alternative approach to restore sustained chaotic motions without the need to vary the carrying capacity of the environment but instead, by making use of the idea of converting transient chaos into sustained chaos via small feedback control [8–11]. The key observation [9] is that one can, in principle, identify the ‘dangerous’ exit regions surrounding the collision points between the chaotic attractor and the basin boundary by monitoring the populations of R, C, and P. If it is determined that the populations are close to a dangerous region, small but deliberately chosen perturbations to the populations are applied to guarantee that no immediate exit from the hole occurs. We conceive that it may be practically feasible to introduce a small population change to R, C, and P. These perturbations are not necessary, as long as the dynamical trajectory stays in the original basin of the chaotic attractor and avoids the dangerous region. By targeting a set of points in the exit region for which the trajectory maps back to the region of recurrent chaotic motion, one can compute the required perturbations. Usually, the perturbations need to be applied only rarely. This technique may be of practical use: by applying small but occasional adjustments to the population at appropriate times estimated from time series, species extinction can be prevented. From an ecological point of view, it may be more feasible to make tiny adjustments to the local populations than to change the carrying capacity of the environment.

A potential problem, when designing the control algorithm based on the map derived from a Poincaré surface of section, is that a substantial fraction of trajectories will escape and asymptote to the limit cycle at $P = 0$ without even being controlled. The reason is that it usually takes a long time for a trajectory to return to the surface of section. In the case of transient chaos, a trajectory may then never pierce through the surface of section before exiting the region in which the originally sustained chaotic motion occurs. We thus propose the following approach to maintain sustained chaotic motion for almost all transient chaotic trajectories. First, we identify, in the three-dimensional phase space (R,C,P), a critical two-dimensional plane defined by $P = P_{crit} =$ constant, which separates the region in which recurrent chaotic motions occur and the region in which the dynamics is such that the population $P(t)$ go directly to zero. This plane needs not be the basin boundary, nor is it a Poincaré surface of section. The criteria for choosing this critical plane are: (1) ecologically – it is chosen with respect to the population that can become extinct; and (2) dynamically – it should be sufficiently close to the originally recurrent chaotic region. *The plane $P = P_{crit}$, thus represents a critical level of the endangered population at which human intervention must be introduced to prevent the extinction of the species P.* The concept of a 'threshold population size' may provide a useful rule-of-thumb for manipulating the dynamics, similar ideas have been used elsewhere in conservation theory [12]. The fact that the critical plane is chosen close to the recurrent chaotic region indicates that arbitrarily close to but above the critical plane, there exists an infinite number of points in the phase space, trajectories starting from which can resume recurrent chaotic motions for at least a finite amount of time. To illustrate this, we consider the case where $K = 1.02$. Fig. 1 shows the recurrent time, or the lifetime span, for trajectories resulting from a grid of 500×500 points chosen from a two-dimensional region in the (R,P) plane at $C = 0.5$, where the lifetime is defined to be the time that the trajectory spends in the phase space region with $P(t) > P_{crit}$. We choose, through a simple search procedure, a critical plane at $P_{crit} = 0.57$. In Fig. 1, brighter spots indicate longer lifetime. For example, the yellow and red spots represent points with greater lifetimes than the blue spots. It can be seen that the distribution of the lifetime is highly nonuniform, due to the multifractal structure of the natural measure of the chaotic set.

The setting of a critical plane and the fact that there exist an infinite number of 'hot' spots with long chaotic recurrent times immediately above the plane provide us with a simple but feasible way to design the intervention or control. Say the population $P(t)$ falls slightly below the critical level at time t. Let (R_-,C_-,P_-) be the values of the state variables at this time, where $P_- \lesssim P_{crit}$, and let (R_+,C_+,P_+) be the values of the state variables a little before t, where $P_+ \gtrsim P_{crit}$. At time t, arbitrarily small random adjustments $[\delta R(t), \delta C(t), \delta P(t)]$ are made to *all* the populations so that the trajectory falls into a point, in the phase space, within a small ball centered at (R_+,C_+,P_+). With a nonzero probability, the trajectory will be close to one of the hot spots contained in the small ball so that a finite time of recurrent chaotic motion can occur. Note that it is useless to kick the trajectory back directly to the point (R_+,C_+,P_+), as this point maps to (R_-,C_-,P_-) immediately. In so far as the trajectory executes a recurrent chaotic motion for $P > P_{crit}$, no external perturbations are necessary. As such, we find that the small perturbations to the populations are needed only very rarely. Fig. 2(a) shows a controlled population $P(t)$ for $K = 1.02$, which indicates a sustained sizable population of the predator through a long time. Fig. 2(b) shows

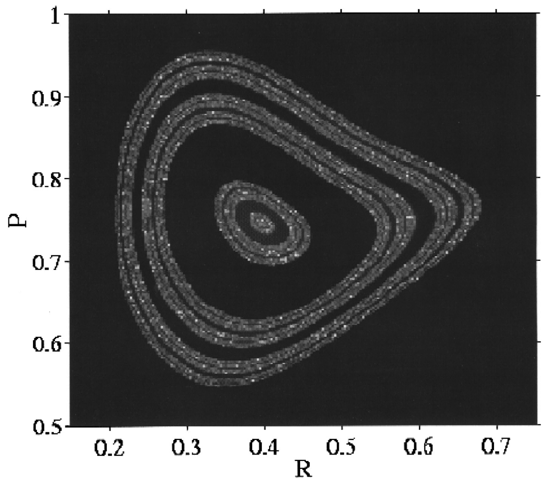

Fig. 1. The lifetime plot for trajectories resulting from a grid of 500×500 points chosen from a two-dimensional region in the (R,P) plane at $C = 0.5$, where the lifetime is defined to be the time that the trajectory spends in the phase space region with $P(t) > P_{crit} = 0.57$. The brightness represent the lifetimes. In particular, brighter spots indicate longer lifetime.

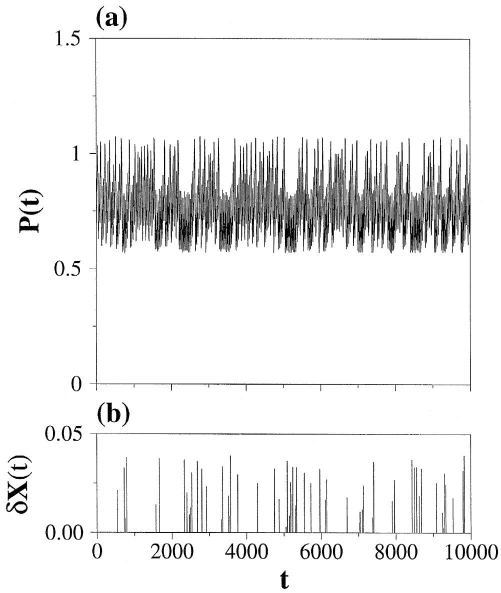

Fig. 2. (a) A controlled population $P(t)$ for $K=1.02$, which indicates a sustained sizable population of the predator through a long time. (b) The magnitude of the applied perturbations $\Delta X(t)$ versus time.

the magnitude of the applied perturbations $\Delta X(t) \equiv \sqrt{[\delta R(t)]^2 + [\delta C(t)]^2 + [\delta P(t)]^2}$ versus time. It can be seen that the required perturbations $[\delta R(t), \delta C(t), \delta P(t)]$ are indeed small [$\Delta X(t) <$ 0.04, compared with the size of the population which is about one] and rare [only about 100 perturbations are applied in a time interval of (0,10000)]. Numerical computations reveal that the chaotic population $P(t)$ can be maintained practically indefinitely through the use of occasional and small adjustments to all the populations, for almost all initial conditions chosen in the original basin of the chaotic attractor, as exemplified in Fig. 2(b). Our approach can thus prevent species extinction effectively.

An issue of practical interest is how often small adjustments need to be applied so that finite species populations can be maintained. To address this question, we observe that the time intervals for successive adjustments of the populations are in fact the recurrent times that the trajectory stays in the region where $P > P_{\text{crit}}$. The recurrent time can be considered as random due to the nature of the chaotic saddle in the phase space. It is known that for trajectories on the chaotic saddle, the probability distribution for the recurrent times decays exponentially [4]. Specifically, say we distribute N_0 (large) initial conditions in the vicinity of the chaotic saddle. All these initial conditions lead to trajectories that eventually asymptote to $P=0$. Let $N(t)$ be the trajectories that still satisfy $P > P_{\text{crit}}$ at time t. Then one typically has [4], $N(t) \sim \exp(-t/\langle\tau\rangle)$, where $\langle\tau\rangle$ is the average lifetime of a typical chaotic trajectory, which is also the average time interval for applying the control. For $K=1.02$, we find $\langle\tau\rangle \approx 209$, which means that roughly, 50 adjustments to the populations need to be made in a time interval of length of 10000. This agrees qualitatively with the result in Fig. 2(b).

An ecosystem can in principle be highly complicated. A practical problem is whether the deterministic chaotic model is a good one in a given situation. The model we utilize in this paper, however, has incorporated within itself biologically and ecologically reasonable assumptions and, hence, it is believed that the model captures the essential dynamics involved in an environment where three species interact in a fashion of a food chain [2]. Even then, the neglected degrees of freedom would show up as small corrections and there is always random noise present in any environment. It thus becomes important to assess the influence of random noise on our control strategy. Here we wish to point out that the simplicity embedded in our control method makes it evident that control is robust against the influence of small noise. The reason is that in our algorithm, we have made a deliberate effort to avoid the need to utilize detailed and more accurate information about the dynamics, such as the Jacobian matrices and the stable and unstable eigenvalues associated with target points which are used commonly in the practice of chaos control. As such, we believe that if deterministic chaos is the main culprit for the extinction of a species for a particular system, it is then possible to use the principle outlined in this Letter to effectively prevent this extinction from occurring even in noisy environment, regardless of the details of the system dynamics. This may be of significant value to important environmental problems such as species preservation.

While we claim that our control technique is robust against small noise, an ecological system can be under the influence of large noise, can be quite nonstationary, and may even involve interaction with other ecosystems. Thus, it is important to discuss the applicability of our technique in realistic ecological environment. Due to its simplicity, the implementation of our control technique consists of the following three straightforward phases: (1) obvservation, (2) computation, and (3) control. In the observation phase, time evolution of the species populations is recorded. This phase can be saved if there are already data available, which can be the case, say, where certain endangered species needs to be saved but the history of its population is more or less known. In the phase of computation, data obtained from the observations or history are utilized to reconstruct the dynamics in the phase space, which can be done by using the well-studied technique of delay-coordinate embedding [13–15]. As a result of this computation, dynamical invariant sets such as the one shown in Fig. 1 which is responsible for transient chaos can be obtained and, hence, the critical region (such as the plane $P = P_{\rm crit}$ in our numerical example) in the phase space can be determined in which control is to be activated. Finally, perturbations can be applied to species populations to keep the dynamics inside the phase-space region where the chaotic invariant set lives, as demonstrated in our numerical example. Apparently, all the above three phases can be realized even in noisy and nonstationary environment.

We remark that the procedure we have presented in this Letter applies generally to controlling transient chaos in deterministic flows [16,8–11]. Our algorithm overcomes the difficulties caused by the more standard use of the discrete-map-type of controlling procedure based on a Poincaré surface of section so that almost all transient chaotic trajectories can be controlled. To our knowledge, this represents a successful attempt to attack the problem of controlling transient chaos in general deterministic flows, which remains less explored despite the large body of existing works on controlling chaos. Sustaining transient chaotic motion has become an interesting area of recent investigation due to their potential relevance to problems such as biological health [17]. Our work may thus help to provide broadly useful insights into this rapidly growing area of research.

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