

1 **Supporting information: A metabolic perspective on**  
2 **competition and body size reductions with warming**

3 **Daniel C Reuman<sup>1,2,\*</sup>, Robert D Holt<sup>3</sup>,**  
4 **Gabriel Yvon-Durocher<sup>4,5</sup>**

- 5 1. *Division of Ecology and Evolution, Imperial College London, Silwood Park Campus, Ascot,*  
6 *SL5 7PY, U.K.*  
7 2. *Laboratory of Populations, Rockefeller University, 1230 York Ave, New York, 10065, USA.*  
8 3. *Department of Zoology, 223 Bartram Hall, PO Box 118525, University of Florida, Gaines-*  
9 *ville, FL 32611-8525, USA.*  
10 4. *School of Biological and Chemical Sciences, Queen Mary University of London, London, E1*  
11 *4NS, U.K.*  
12 5. *Environment and Sustainability Institute, University of Exeter, Penryn, Cornwall, TR10 9EZ,*  
13 *U.K.*

14 \*Correspondence author(s). [d.reuman@imperial.ac.uk](mailto:d.reuman@imperial.ac.uk)

# 1 Appendix: $s$ and $T$ dependence of parameters

We here provide details for the parameterizations described in the section "Phytoplankton: Parameterization" of the main text. Results for the dependence of  $\mu_{max}$  and scaled assimilation affinity on cell size were from table 1 of Edwards *et al.* (2012), using their allometries common to freshwater and marine phytoplankton under N-limited growth. Confidence intervals provided by Edwards *et al.* (2012) were converted to standard errors by dividing half their length by 1.959964. The standard error of the scaling exponent of  $H_{gro}$  was computed as the square root of the sum of the squares of the standard errors of the scalings exponents of  $\mu_{max}$  and scaled assimilation affinity, since the scaling exponent of  $H_{gro}$  was a difference of the two other exponents. This effectively assumes errors in the exponents are independent.

Results for  $E_{\mu_{max}}$  from Bissinger *et al.* (2008) are from the first row of their table 1, converted from exponential dependencies to Arrhenius dependencies. Their confidence intervals were converted to the standard error for  $E_{\mu_{max}}$  presented in the main text by converting an exponential dependence using each confidence interval endpoint to an Arrhenius dependence using the exponential prefactor 0.81 they provide (see their equation 2). The resulting confidence range for  $E_{\mu_{max}}$  was converted to a standard error by dividing half its length by 1.959964.

Data on the viscosity of sea water at a range of temperatures were downloaded from the Chemical Hazards Response Information System of the U.S. Coastguard ([www.chrismanual.com/Intro/prop.htm](http://www.chrismanual.com/Intro/prop.htm); downloaded data are reproduced in table S1). Temperatures were converted to °K and viscosities to  $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ . A linear model was fitted with response variable  $\log_e$  viscosity and predictor  $\frac{1}{kT}$ . The slope was 0.1781 (SE 0.001) and the intercept was  $-13.8738$  (standard error 0.0405). The linear model explained 99.98% of the variation in  $\log_e$  viscosity.

## 2 Appendix: a specific $T$ dependence of $H_{gro}$

Equation 9 of the main text establishes a power law dependence of  $H_{gro}$  on cell size,  $s$ . Although not needed for the results of the main text, we here provide theoretical and empirical arguments supporting a particular  $T$  dependence of  $H_{gro}$ . By the equation for  $H_{gro}$  in equations 6 in the main text, it suffices to understand the  $T$  dependencies of  $H_{up}$ ,  $Q_{min}$ , and  $V_{max}$ ; the  $T$  dependence of  $\mu_{max}$  is in the main text.

We begin with theoretical arguments for  $H_{up}$  and  $V_{max}$ , largely taken from Aksnes & Egge (1991). Nutrients are actively transported into a cell by ion uptake sites on the cell surface. Aksnes & Egge (1991) mechanistically modeled this process using the same reasoning behind the Holling type II functional response used in predator-prey models (Holling, 1959). They obtained a formula for uptake rate, as limited by encounters between ions and uptake sites, and by the time it takes for an uptake site to process an ion once encountered. Converting their formulas to our notation provides

$$V_{max} = \frac{n}{h} \quad (1)$$

52 and

$$H_{up} = \frac{1}{hAv}, \quad (2)$$

53 where  $n$  is the number of uptake sites occurring on the cell surface (sites  $\cdot$  cell $^{-1}$ ),  $h$  is the  
54 time required for an uptake channel to process an ion, i.e., the “handling time” (days),  $A$   
55 is the capture area of an uptake site ( $\mu\text{m}^2$ ), and  $v$  is a “mass transfer coefficient” which is  
56 an average equivalent relative velocity between ions and uptake sites ( $\mu\text{m} \cdot \text{day}^{-1}$ ). We can  
57 model the  $s$  and  $T$  dependence of  $V_{max}$  and  $H_{up}$  by modeling the  $s$  and  $T$  dependence of  $n$ ,  
58  $h$ ,  $A$  and  $v$ . This is the strategy used by Litchman *et al.* (2007) and Verdy *et al.* (2009),  
59 though they did not include temperature dependence.

60 We assume  $A$  is independent of both  $s$  and  $T$ . We assume  $n$  is proportional to cell surface  
61 area and independent of  $T$ , so

$$n \propto s^{2/3}. \quad (3)$$

62 The handling time,  $h$  is the time required for the biochemical reactions of ion uptake, and  
63 as such seems likely to have inverse Arrhenius temperature dependence. We assume  $h$  does  
64 not depend systematically on  $s$ , so

$$h \propto \exp\left(\frac{E_h}{kT}\right), \quad (4)$$

65 with  $E_h > 0$ . Aksnes & Egge (1991) used the approximation

$$v = \frac{D}{r}, \quad (5)$$

66 where  $D$  is the coefficient of molecular diffusion for the nutrient ( $\mu\text{m}^2 \cdot \text{day}^{-1}$ ),  $r$  is the  
67 equivalent spherical radius of the cell ( $\mu\text{m}$ ), and we assume nutrient transport to the cell  
68 is largely via diffusion as opposed to cell motion through the medium. The Einstein-Stokes  
69 equation gives

$$D \propto \frac{T}{\eta}, \quad (6)$$

70 where  $\eta$  is the dynamic viscosity of the medium. Aksnes & Egge (1991) realized that tem-  
71 perature influenced  $D$  as described in this proportionality, and then in turn influenced  $v$  and  
72  $H_{up}$  as described in equations 5 and 2, however, they said “as the temperature-dependent  
73 viscosity also influences diffusion, the overall influence of temperature on molecular diffusion  
74 is not straightforward.” But the influence of  $T$  on viscosity is known:

$$\eta \propto \exp\left(\frac{E_\eta}{kT}\right), \quad (7)$$

75 where  $E_\eta = 0.1781$  (Appendix S1). Combining equations 5, 6 and 7, we have

$$v \propto s^{-1/3} T \exp\left(\frac{-E_\eta}{kT}\right), \quad (8)$$

76 so that dependencies on  $s$  and  $T$  of  $n$ ,  $h$ ,  $A$  and  $v$  have been modeled.

77 Substituting the expressions for  $n$ ,  $h$ , and  $v$  (equations 3, 4, and 8) into equations 1 and  
 78 2, we have

$$V_{max} \propto s^{2/3} \exp\left(\frac{-E_h}{kT}\right) \propto s^{2/3} \exp\left(\frac{E_h(T - T_0)}{kTT_0}\right) \quad (9)$$

79 and

$$H_{up} \propto \frac{s^{1/3} \exp\left(\frac{-E_{H_{up}}}{kT}\right)}{T} \propto \frac{s^{1/3} \exp\left(\frac{E_{H_{up}}(T - T_0)}{kTT_0}\right)}{T} \quad (10)$$

80 where

$$E_{H_{up}} = E_h - E_\eta. \quad (11)$$

81 The cell-size scaling exponents predicted here (2/3 for  $V_{max}$  and 1/3 for  $H_{up}$ ) are similar to  
 82 the values obtained empirically by Edwards *et al.* (2012) (see their table 1, “common slope”  
 83 column); they got scaling exponent for  $V_{max}$  for N uptake equal to 0.82 (SE 0.077) and scaling  
 84 exponent for  $H_{up}$  for N uptake equal to 0.33 (SE 0.041). See also Litchman *et al.* (2007), who  
 85 obtained similar scaling exponents. The agreement of theory with empirical results on cell-  
 86 size scaling exponents lends credence to theoretical predictions for temperature dependence.

87 We tested the theoretically predicted functional form for the temperature dependence  
 88 of  $H_{up}$  and obtained an estimate for  $E_{H_{up}}$  via a metastudy. We found in the literat-  
 89 ure 79 measurements of  $H_{up}$  for 16 different species or groups at temperatures ranging  
 90 from 1°C to 29°C; all data and references are in table S3. Cell-size measurements gener-  
 91 ally were not available. We fitted a linear mixed-effects model (Pinheiro & Bates, 2000),  
 92  $y_{kl} = E_{H_{up}}x_{kl} + b + \delta_k + \epsilon_{kl}$ , where  $k$  indexes unique species-study combinations in the data,  $l$  in-  
 93 dexes measurements (at different temperatures) taken within a species-study combination,  $y_{kl}$   
 94 represents  $\log_e(H_{up}) + \log_e(T)$  for the  $kl$ -th data point, and  $x_{kl}$  represents  $(T_{kl} - T_0)/(kT_{kl}T_0)$   
 95 where  $T_{kl}$  is the temperature (°K) for the  $kl$ -th data point and  $T_0 = 293.15^\circ\text{K}$ . The random  
 96 factor  $\delta_k$ , which takes an independent value for each species-study combination, is normally  
 97 distributed with a variance parameter determined in fitting. The  $\delta_k$  account for differences  
 98 among species and methodological differences among studies. The  $\epsilon_{kl}$  term is a normally  
 99 distributed residual error term. The main result of this fitting was an estimate  $E_{H_{up}} = 0.277$   
 100 (SE 0.084), but we also obtained estimates for the  $\delta_k$  and plotted  $y_{kl} - \delta_k$  against  $E_{H_{up}}x_{kl} + b$ ,  
 101 using the estimated values of the parameters  $E_{H_{up}}$  and  $b$ , to demonstrate that linearity of the  
 102 fixed effect was supported (figure S1A). The package lme4 in the R programming language  
 103 was used to fit the model. The activation energy for  $V_{max}$  can be obtained from equation 11,  
 104 i.e.,  $E_{V_{max}} = E_h = E_{H_{up}} + E_\eta = 0.456$ . The standard error on this estimate is practically the  
 105 same as that for  $E_{H_{up}}$  because the standard error for  $E_\eta$  is very small (Appendix S1).

106 To assess the  $T$  dependence of  $Q_{min}$ , we carried out another metastudy. We found in  
 107 the literature 41 measurements of  $Q_{min}$  for 7 different species or groups at temperatures  
 108 ranging from 3°C to 25°C; all data and references are in table S4. Cell-size measurements  
 109 generally were not available. We fitted a linear mixed-effects model (Pinheiro & Bates, 2000),  
 110  $y_{kl} = E_{Q_{min}}x_{kl} + b + \delta_k + \epsilon_{kl}$ , where  $k$  indexes unique species-study combinations in the data,  $l$   
 111 indexes measurements (at different temperatures) taken within a species-study combination,  
 112  $y_{kl}$  represents  $\log_e(Q_{min})$  for the  $kl$ -th data point, and  $x_{kl}$  represents  $(T_{kl} - T_0)/(kT_{kl}T_0)$ .  
 113 The random factor  $\delta_k$  and the residual error  $\epsilon_{kl}$  are conceptually the same as in the mixed-  
 114 effects model for  $H_{up}$  described above. The main result of this fitting was an estimate

115  $E_{Q_{min}} = -0.230$  (SE 0.056), but we also obtained estimates for the  $\delta_k$  and plotted  $y_{kl} - \delta_k$   
 116 against  $E_{Q_{min}} x_{kl} + b$ , using the estimated values of the parameters  $E_{Q_{min}}$  and  $b$ , to demonstrate  
 117 that linearity of the fixed effect was supported (figure S1B). Thus  $Q_{min}$  has the form

$$Q_{min} \propto s^{e_{Q_{min}}} \exp\left(\frac{E_{Q_{min}}(T - T_0)}{kTT_0}\right), \quad (12)$$

118 with  $E_{Q_{min}} = -0.230$ , where the power-law cell-size dependence used here is supported by  
 119 the results of Edwards *et al.* (2012), with their value  $e_{Q_{min}} = 0.84$  (SE 0.046).

120 Combining the dependencies for  $H_{up}$ ,  $Q_{min}$ ,  $V_{max}$ , and  $\mu_{max}$  gives

$$H_{gro} \propto \frac{s^{e_{H_{up}} + e_{Q_{min}} + e_{\mu_{max}} - e_{V_{max}}} \exp\left(\frac{(E_{H_{up}} + E_{Q_{min}} + E_{\mu_{max}} - E_{V_{max}})(T - T_0)}{kTT_0}\right)}{T}, \quad (13)$$

121 where the value  $e_{H_{gro}} = e_{H_{up}} + e_{Q_{min}} + e_{\mu_{max}} - e_{V_{max}} = \frac{1}{3} + 0.84 - 0.28 - \frac{2}{3} = 0.227$  is close to  
 122 the value derived from the regressions of Edwards *et al.* (2012) (table 1 of the main text).

### 123 3 Appendix: derivative of $G$ with respect to $T$

124 The invasion fitness,  $G$ , of an invader of cell size  $s_{in}$  into a resident of cell size  $s_{re}$  was derived  
 125 in the main text (main text equation 11). The derivative of  $G$  with respect to  $T$  was needed  
 126 to establish the effects of temperature on the relative competitive abilities of differently sized  
 127 plankton cells. The derivative is computed here:

$$\frac{\partial G}{\partial T} = \frac{\frac{\partial \mu_{max}}{\partial T}(s_{in}, T) g_m(s_{re}) s_{re}^{e_{H_{gro}}}}{D} \quad (14)$$

$$- \frac{\mu_{max}(s_{in}, T) g_m(s_{re}) s_{re}^{e_{H_{gro}}} s_{in}^{e_{H_{gro}}} \frac{\partial \mu_{max}}{\partial T}(s_{re}, T)}{D^2} \quad (15)$$

128 where

$$D = g_m(s_{re}) s_{re}^{e_{H_{gro}}} + s_{in}^{e_{H_{gro}}} (\mu_{max}(s_{re}, T) - g_m(s_{re})). \quad (16)$$

129 Substituting  $\frac{\partial \mu_{max}}{\partial T} = \frac{E_{\mu_{max}}}{kT^2} \mu_{max}$  and simplifying gives;

$$\frac{\partial G}{\partial T} = \frac{\mu_{max}(s_{in}, T) \frac{E_{\mu_{max}}}{kT^2} g_m(s_{re})^2 s_{re}^{e_{H_{gro}}} (s_{re}^{e_{H_{gro}}} - s_{in}^{e_{H_{gro}}})}{D^2}. \quad (17)$$

130 The denominator,  $D^2$ , is strictly positive because  $m = g_m$  is positive and  $\mu_{max}(s_{re}) - g_m(s_{re}) >$   
 131  $0$  is a condition for the resident to have become established in the first place. Therefore the  
 132 sign of  $\frac{\partial G}{\partial T}$  is the same as the sign of the numerator of equation 17, which is positive for  
 133  $s_{re} > s_{in}$  and negative for  $s_{re} < s_{in}$  since  $e_{H_{gro}} > 0$ .

## 4 Appendix: $R^*$ for the case $m = k_m s^{e_m}$

In section "Phytoplankton: Model analysis and results" of the main text, the case in which mortality is  $m = g_m(s) = k_m s^{e_m}$  is considered, and it is stated that an evolutionarily stable size (ESS) exists under certain conditions. We here demonstrate this and compute the ESS. Equation 12 of the main text is minimized with changes in  $s$  exactly when

$$\tilde{R}^* = \frac{s^{e_{H_{gro}} + e_m}}{f_{\mu_{max}}(T) s^{e_{\mu_{max}}} - k_m s^{e_m}} \quad (18)$$

is minimized, where we use the shorthand  $f_{\mu_{max}} = k_{\mu_{max}} \exp\left(\frac{E_{\mu_{max}}(T-T_0)}{kTT_0}\right)$ . Taking derivatives and simplifying gives

$$\frac{\partial \tilde{R}^*}{\partial s} = \frac{s^{e_{H_{gro}} + e_m - 1} ((e_m + e_{H_{gro}} - e_{\mu_{max}}) f_{\mu_{max}}(T) s^{e_{\mu_{max}}} - e_{H_{gro}} k_m s^{e_m})}{(f_{\mu_{max}}(T) s^{e_{\mu_{max}}} - k_m s^{e_m})^2}. \quad (19)$$

Cell size  $s$  is viable only if the denominator of this expression is greater than zero. The sign of equation 19 is the same as the sign of its numerator, which in turn is the same as the sign of

$$(e_m + e_{H_{gro}} - e_{\mu_{max}}) \frac{f_{\mu_{max}}(T) s^{e_{\mu_{max}}}}{k_m s^{e_m}} - e_{H_{gro}}, \quad (20)$$

since  $k_m s^{e_m} > 0$  and  $s^{e_{H_{gro}} + e_m - 1} > 0$ .

If  $e_{\mu_{max}} < e_m$  then equation 20 is always positive, because then  $e_m + e_{H_{gro}} - e_{\mu_{max}} > e_{H_{gro}}$ , and we already know the quotient in equation 20 is greater than 1 for viable cell sizes,  $s$ . So in that case,  $\partial R^*/\partial s > 0$  for all viable  $s$  and we have runaway selection to small sizes. There is also a maximum viable size

$$s_{max} = \left( \frac{f_{\mu_{max}}(T)}{k_m} \right)^{\frac{1}{e_m - e_{\mu_{max}}}} \quad (21)$$

if  $e_{\mu_{max}} < e_m$ .

If  $e_m < e_{\mu_{max}} - e_{H_{gro}}$ , then  $e_m + e_{H_{gro}} - e_{\mu_{max}} < 0$ , and equation 20 is always negative because  $e_{H_{gro}} > 0$ . So in that case,  $\partial R^*/\partial s < 0$  for all viable  $s$ , and we have runaway selection to large sizes. There is a minimum viable size

$$s_{min} = \left( \frac{k_m}{f_{\mu_{max}}(T)} \right)^{\frac{1}{e_{\mu_{max}} - e_m}} \quad (22)$$

if  $e_m < e_{\mu_{max}}$ .

If  $e_{\mu_{max}} - e_{H_{gro}} < e_m < e_{\mu_{max}}$ , then equation 20 has a root,

$$s_{ESS} = \left( \frac{e_{H_{gro}} k_m}{(e_m + e_{H_{gro}} - e_{\mu_{max}}) f_{\mu_{max}}(T)} \right)^{\frac{1}{e_{\mu_{max}} - e_m}}. \quad (23)$$

Because equation 20 is a monotonically increasing function of  $s$  for  $e_{\mu_{max}} - e_{H_{gro}} < e_m < e_{\mu_{max}}$ , it is negative for  $s < s_{ESS}$  and positive for  $s > s_{ESS}$ , so  $s_{ESS}$  is a local and global minimum

157 of  $\tilde{R}^*$  and therefore of  $R^*$ .

## 158 5 Appendix: The equilibrium assumption

159 The use of  $R^*$  and  $G$  made in the main text in analyzing the phytoplankton model depends  
160 on the assumption that equilibria are stable. This seems likely because model nutrients are  
161 supplied at a constant rate. Nevertheless, we tested the assumption numerically using a  
162 Monte Carlo method, for a wide range of parameters encompassing biologically reasonable  
163 values. Edwards *et al.* (2012) provide allometries of the form  $y = ks^e$  for  $y = Q_{min}, \mu_{max},$   
164  $H_{up},$  and  $V_{max},$  giving estimates and confidence intervals for  $\log_{10}(k)$  and  $e$  in all cases. Con-  
165 fidence intervals were converted to standard errors by dividing half their length by 1.959964.  
166 Montagnes & Franklin (2001) provide an allometry for  $Q_{max},$  with estimates and standard  
167 errors for  $\log_{10}(k)$  and  $e.$  See table S2 for all values taken from these references. On each  
168 Monte Carlo run, the parameters  $\log_{10}(k)$  and  $e$  for each allometry were drawn from nor-  
169 mal distributions with mean equal to the estimated value from Edwards *et al.* (2012) or  
170 Montagnes & Franklin (2001) and standard deviation equal to twice the standard error re-  
171 ported in those references. Twice the standard error was used to add a buffer of additional  
172 uncertainty. Cell sizes  $\log_{10}(s)$  were drawn randomly and uniformly between 0 and 8 and the  
173 allometries resulting from the randomly generated values of the  $k$  and  $e$  for each allometry  
174 were used to generate values for  $Q_{min}, \mu_{max}, H_{up}, V_{max},$  and  $Q_{max}.$  When  $Q_{max}$  was less  
175 than  $Q_{min}$  the values were swapped. Input nutrient concentration  $R_0 = 40$  was used follow-  
176 ing Litchman *et al.* (2009). Nutrient supply rates,  $d,$  were taken as a uniformly distributed  
177 random variable between 0.05 and 0.95 divided by a uniformly distributed random variable  
178 between 1 and 50. The first variable represents replacement fractions in mixing events and  
179 the second represents mixing-event periods; both ranges are from Litchman *et al.* (2009).  
180 Mortality,  $m,$  was a uniform random variable between  $1 \times 10^{-4}$  and 0.02. The parameter  
181  $\mu_{\infty}$  was established by solving the  $\mu_{max}$  equation in equations 6 in the main text for  $\mu_{\infty}.$   
182 For each of 10,000 runs, equilibria were computed following the equations of Verdy *et al.*  
183 (2009). When equilibria existed (9,464 runs), the system Jacobian was computed at the  
184 equilibrium. In all cases, the real parts of the eigenvalues of the Jacobian were negative,  
185 indicating stability. Temperature was not accounted for in this analysis because data are  
186 not available on the temperature dependence of all model parameters; nevertheless it seems  
187 unlikely model equilibria are ever unstable for biologically reasonable parameters given the  
188 wide uncertainties incorporated above and the result that every simulated equilibrium was  
189 stable. We find it plausible to expect that local stability is a mathematical necessity for any  
190 equilibrium, but we do not know of a proof of this.

## 191 6 Appendix: Non-equilibrium dynamics

192 The  $R^*$  and invasion-fitness approach as described here is valid only when the equilibrium  
193 is stable, but in contrast to the phytoplankton model (Appendix S5), an equilibrium of the  
194 Rosenzweig-MacArthur model need not be stable. A Hopf bifurcation occurs at a parameter  
195 boundary given by Vasseur & McCann (2005); past the boundary, the model has a stable  
196 limit cycle. Invasion fitness into a resident exhibiting limit-cycle oscillations can still be

197 computed by averaging the growth rate of the putative invader over the limit-cycle resource  
198 density driven by the resident (see, e.g., Klausmeier, 2008; Kooi & Troost, 2006; Litchman  
199 *et al.*, 2009). A non-equilibrium approach will require all model parameters to be written as  
200 functions of  $s$  and  $T$  instead of just those appearing in expressions for  $R^*$  and  $G$ , but this  
201 should be possible for the Rosenzweig-MacArthur model if  $\alpha$  and  $t_h$  can be described. Savage  
202 *et al.* (2004) published theory and data on body mass and temperature dependence of  $r$  and  
203  $K$ , and Vasseur & McCann (2005) cover the body mass and temperature dependence of  $r$  as  
204 well as  $m$  and  $\mu_{max}$ . An important direction for future work will be elucidating the thermal  
205 dimension of non-equilibrial competitive interactions.

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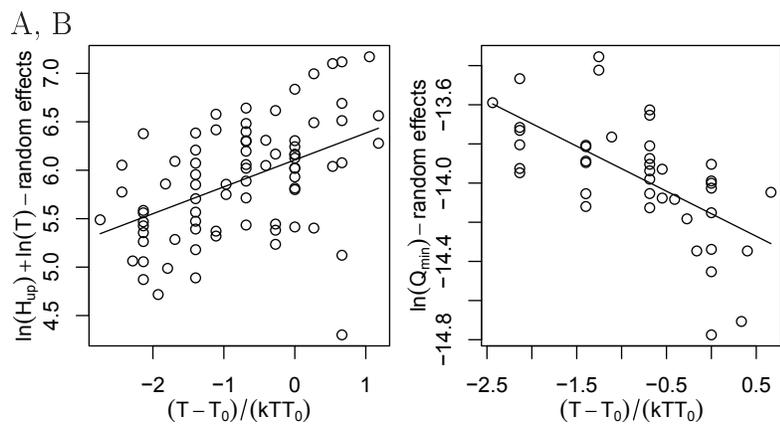
## Tables

**Table 1** Viscosity of sea water at a range of temperatures. Data were taken from the Chemical Hazards Response Information System of the U.S. Coast Guard ([www.chrismanual.com/Intro/prop.htm](http://www.chrismanual.com/Intro/prop.htm)), and are for “standard” sea water containing 35g salts per kg of solution.

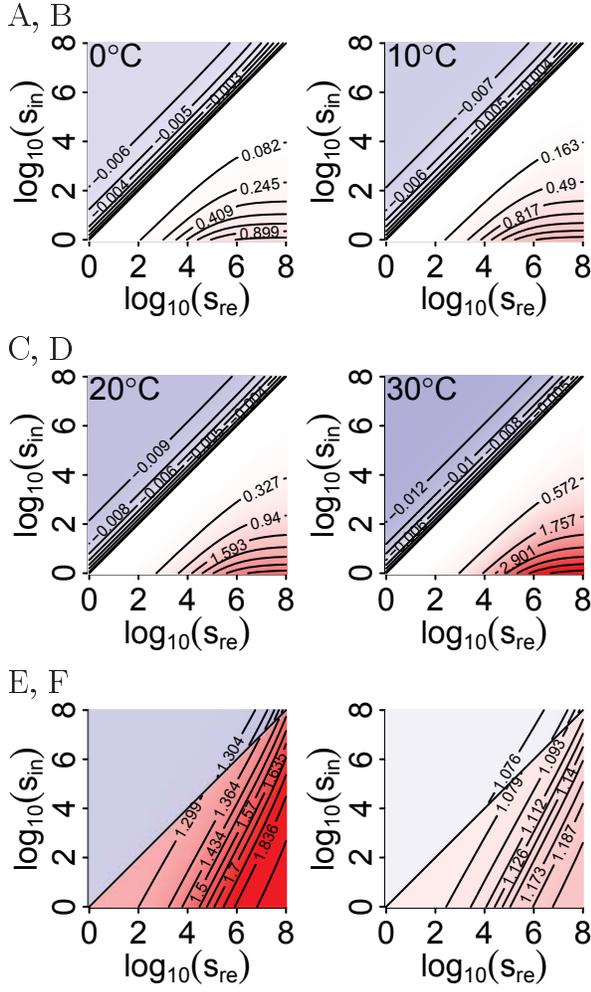
Temp. (°F)	Visc. (Centipoise)	Temp. (°K)	Visc. ( $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ )
30	1.88	272.04	0.00188
40	1.61	277.59	0.00161
50	1.40	283.15	0.00140
60	1.21	288.71	0.00121
70	1.06	294.26	0.00106
80	0.92	299.82	0.00092
90	0.82	305.37	0.00082
100	0.73	310.93	0.00073

**Table 2** Parameter estimates used in Monte Carlo simulations (see Appendix S5). Values from Edwards *et al.* (2012) are from their table 1, using values fitted to pooled freshwater and marine data. Values from Montagnes & Franklin (2001) are from their table 3.

Parameter	Estimate	SE	Source
$e_{Q_{min}}$	0.84	0.046	Edwards <i>et al.</i> (2012)
$\log_{10}(k_{Q_{min}})$	-9.0	0.128	Edwards <i>et al.</i> (2012)
$e_{\mu_{max}}$	-0.28	0.018	Edwards <i>et al.</i> (2012)
$\log_{10}(k_{\mu_{max}})$	0.65	0.059	Edwards <i>et al.</i> (2012)
$e_{H_{up}}$	0.33	0.041	Edwards <i>et al.</i> (2012)
$\log_{10}(k_{H_{up}})$	-0.61	0.138	Edwards <i>et al.</i> (2012)
$e_{V_{max}}$	0.82	0.077	Edwards <i>et al.</i> (2012)
$\log_{10}(k_{V_{max}})$	-8.0	0.281	Edwards <i>et al.</i> (2012)
$e_{Q_{max}}$	0.809	0.011	Montagnes & Franklin (2001)
$\log_{10}(k_{Q_{max}})$	-8.03	0.039	Montagnes & Franklin (2001)



**Fig. 1** Plots of metastudy data, corrected for random effects; lines have fixed-effect slopes and intercepts (see Appendix S2).



**Fig. 2** (A-D) Pairwise invasibility plots at different temperatures for  $m = 0.01 \exp(E_m(T - T_0)/(kTT_0))$  with  $E_m = 0.1781$ . These plots are analogous to figure 4A-D of the main text, but for a different model of  $m$ . (E-F) Ratios of pairwise invasibilities at different temperatures for the same  $m$ . These plots are analogous to figure 4E-F of the main text. Plots show  $G(s_{in}, s_{re}, T_1)/G(s_{in}, s_{re}, T_2)$  for  $T_1 = 20^\circ\text{C}$  and  $T_2 = 10^\circ\text{C}$  (E) and for  $T_1 = 23^\circ\text{C}$  and  $T_2 = 20^\circ\text{C}$  (F). The same result of accentuated invasion fitness of smaller invaders at warmer temperatures was also true for  $k_m = 0.0001, 0.001$ , and  $0.02$ .