

1 **Supporting information: A metabolic perspective on**
2 **competition and body size reductions with warming**

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1 Appendix: s and T dependence of parameters

We here provide details for the parameterizations described in the section "Phytoplankton: Parameterization" of the main text. Results for the dependence of μ_{max} and scaled assimilation affinity on cell size were from table 1 of Edwards *et al.* (2012), using their allometries common to freshwater and marine phytoplankton under N-limited growth. Confidence intervals provided by Edwards *et al.* (2012) were converted to standard errors by dividing half their length by 1.959964. The standard error of the scaling exponent of H_{gro} was computed as the square root of the sum of the squares of the standard errors of the scalings exponents of μ_{max} and scaled assimilation affinity, since the scaling exponent of H_{gro} was a difference of the two other exponents. This effectively assumes errors in the exponents are independent.

Results for $E_{\mu_{max}}$ from Bissinger *et al.* (2008) are from the first row of their table 1, converted from exponential dependencies to Arrhenius dependencies. Their confidence intervals were converted to the standard error for $E_{\mu_{max}}$ presented in the main text by converting an exponential dependence using each confidence interval endpoint to an Arrhenius dependence using the exponential prefactor 0.81 they provide (see their equation 2). The resulting confidence range for $E_{\mu_{max}}$ was converted to a standard error by dividing half its length by 1.959964.

Data on the viscosity of sea water at a range of temperatures were downloaded from the Chemical Hazards Response Information System of the U.S. Coastguard (www.chrismanual.com/Intro/prop.htm; downloaded data are reproduced in table S1). Temperatures were converted to °K and viscosities to $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$. A linear model was fitted with response variable \log_e viscosity and predictor $\frac{1}{kT}$. The slope was 0.1781 (SE 0.001) and the intercept was -13.8738 (standard error 0.0405). The linear model explained 99.98% of the variation in \log_e viscosity.

2 Appendix: a specific T dependence of H_{gro}

Equation 9 of the main text establishes a power law dependence of H_{gro} on cell size, s . Although not needed for the results of the main text, we here provide theoretical and empirical arguments supporting a particular T dependence of H_{gro} . By the equation for H_{gro} in equations 6 in the main text, it suffices to understand the T dependencies of H_{up} , Q_{min} , and V_{max} ; the T dependence of μ_{max} is in the main text.

We begin with theoretical arguments for H_{up} and V_{max} , largely taken from Aksnes & Egge (1991). Nutrients are actively transported into a cell by ion uptake sites on the cell surface. Aksnes & Egge (1991) mechanistically modeled this process using the same reasoning behind the Holling type II functional response used in predator-prey models (Holling, 1959). They obtained a formula for uptake rate, as limited by encounters between ions and uptake sites, and by the time it takes for an uptake site to process an ion once encountered. Converting their formulas to our notation provides

$$V_{max} = \frac{n}{h} \quad (1)$$

52 and

$$H_{up} = \frac{1}{hAv}, \quad (2)$$

53 where n is the number of uptake sites occurring on the cell surface (sites \cdot cell $^{-1}$), h is the
54 time required for an uptake channel to process an ion, i.e., the “handling time” (days), A
55 is the capture area of an uptake site (μm^2), and v is a “mass transfer coefficient” which is
56 an average equivalent relative velocity between ions and uptake sites ($\mu\text{m} \cdot \text{day}^{-1}$). We can
57 model the s and T dependence of V_{max} and H_{up} by modeling the s and T dependence of n ,
58 h , A and v . This is the strategy used by Litchman *et al.* (2007) and Verdy *et al.* (2009),
59 though they did not include temperature dependence.

60 We assume A is independent of both s and T . We assume n is proportional to cell surface
61 area and independent of T , so

$$n \propto s^{2/3}. \quad (3)$$

62 The handling time, h is the time required for the biochemical reactions of ion uptake, and
63 as such seems likely to have inverse Arrhenius temperature dependence. We assume h does
64 not depend systematically on s , so

$$h \propto \exp\left(\frac{E_h}{kT}\right), \quad (4)$$

65 with $E_h > 0$. Aksnes & Egge (1991) used the approximation

$$v = \frac{D}{r}, \quad (5)$$

66 where D is the coefficient of molecular diffusion for the nutrient ($\mu\text{m}^2 \cdot \text{day}^{-1}$), r is the
67 equivalent spherical radius of the cell (μm), and we assume nutrient transport to the cell
68 is largely via diffusion as opposed to cell motion through the medium. The Einstein-Stokes
69 equation gives

$$D \propto \frac{T}{\eta}, \quad (6)$$

70 where η is the dynamic viscosity of the medium. Aksnes & Egge (1991) realized that tem-
71 perature influenced D as described in this proportionality, and then in turn influenced v and
72 H_{up} as described in equations 5 and 2, however, they said “as the temperature-dependent
73 viscosity also influences diffusion, the overall influence of temperature on molecular diffusion
74 is not straightforward.” But the influence of T on viscosity is known:

$$\eta \propto \exp\left(\frac{E_\eta}{kT}\right), \quad (7)$$

75 where $E_\eta = 0.1781$ (Appendix S1). Combining equations 5, 6 and 7, we have

$$v \propto s^{-1/3} T \exp\left(\frac{-E_\eta}{kT}\right), \quad (8)$$

76 so that dependencies on s and T of n , h , A and v have been modeled.

77 Substituting the expressions for n , h , and v (equations 3, 4, and 8) into equations 1 and
 78 2, we have

$$V_{max} \propto s^{2/3} \exp\left(\frac{-E_h}{kT}\right) \propto s^{2/3} \exp\left(\frac{E_h(T - T_0)}{kTT_0}\right) \quad (9)$$

79 and

$$H_{up} \propto \frac{s^{1/3} \exp\left(\frac{-E_{H_{up}}}{kT}\right)}{T} \propto \frac{s^{1/3} \exp\left(\frac{E_{H_{up}}(T - T_0)}{kTT_0}\right)}{T} \quad (10)$$

80 where

$$E_{H_{up}} = E_h - E_\eta. \quad (11)$$

81 The cell-size scaling exponents predicted here (2/3 for V_{max} and 1/3 for H_{up}) are similar to
 82 the values obtained empirically by Edwards *et al.* (2012) (see their table 1, “common slope”
 83 column); they got scaling exponent for V_{max} for N uptake equal to 0.82 (SE 0.077) and scaling
 84 exponent for H_{up} for N uptake equal to 0.33 (SE 0.041). See also Litchman *et al.* (2007), who
 85 obtained similar scaling exponents. The agreement of theory with empirical results on cell-
 86 size scaling exponents lends credence to theoretical predictions for temperature dependence.

87 We tested the theoretically predicted functional form for the temperature dependence
 88 of H_{up} and obtained an estimate for $E_{H_{up}}$ via a metastudy. We found in the literat-
 89 ure 79 measurements of H_{up} for 16 different species or groups at temperatures ranging
 90 from 1°C to 29°C; all data and references are in table S3. Cell-size measurements gener-
 91 ally were not available. We fitted a linear mixed-effects model (Pinheiro & Bates, 2000),
 92 $y_{kl} = E_{H_{up}}x_{kl} + b + \delta_k + \epsilon_{kl}$, where k indexes unique species-study combinations in the data, l
 93 indexes measurements (at different temperatures) taken within a species-study combination, y_{kl}
 94 represents $\log_e(H_{up}) + \log_e(T)$ for the kl -th data point, and x_{kl} represents $(T_{kl} - T_0)/(kT_{kl}T_0)$
 95 where T_{kl} is the temperature (°K) for the kl -th data point and $T_0 = 293.15^\circ\text{K}$. The random
 96 factor δ_k , which takes an independent value for each species-study combination, is normally
 97 distributed with a variance parameter determined in fitting. The δ_k account for differences
 98 among species and methodological differences among studies. The ϵ_{kl} term is a normally
 99 distributed residual error term. The main result of this fitting was an estimate $E_{H_{up}} = 0.277$
 100 (SE 0.084), but we also obtained estimates for the δ_k and plotted $y_{kl} - \delta_k$ against $E_{H_{up}}x_{kl} + b$,
 101 using the estimated values of the parameters $E_{H_{up}}$ and b , to demonstrate that linearity of the
 102 fixed effect was supported (figure S1A). The package lme4 in the R programming language
 103 was used to fit the model. The activation energy for V_{max} can be obtained from equation 11,
 104 i.e., $E_{V_{max}} = E_h = E_{H_{up}} + E_\eta = 0.456$. The standard error on this estimate is practically the
 105 same as that for $E_{H_{up}}$ because the standard error for E_η is very small (Appendix S1).

106 To assess the T dependence of Q_{min} , we carried out another metastudy. We found in
 107 the literature 41 measurements of Q_{min} for 7 different species or groups at temperatures
 108 ranging from 3°C to 25°C; all data and references are in table S4. Cell-size measurements
 109 generally were not available. We fitted a linear mixed-effects model (Pinheiro & Bates, 2000),
 110 $y_{kl} = E_{Q_{min}}x_{kl} + b + \delta_k + \epsilon_{kl}$, where k indexes unique species-study combinations in the data, l
 111 indexes measurements (at different temperatures) taken within a species-study combination,
 112 y_{kl} represents $\log_e(Q_{min})$ for the kl -th data point, and x_{kl} represents $(T_{kl} - T_0)/(kT_{kl}T_0)$.
 113 The random factor δ_k and the residual error ϵ_{kl} are conceptually the same as in the mixed-
 114 effects model for H_{up} described above. The main result of this fitting was an estimate

115 $E_{Q_{min}} = -0.230$ (SE 0.056), but we also obtained estimates for the δ_k and plotted $y_{kl} - \delta_k$
 116 against $E_{Q_{min}} x_{kl} + b$, using the estimated values of the parameters $E_{Q_{min}}$ and b , to demonstrate
 117 that linearity of the fixed effect was supported (figure S1B). Thus Q_{min} has the form

$$Q_{min} \propto s^{e_{Q_{min}}} \exp\left(\frac{E_{Q_{min}}(T - T_0)}{kTT_0}\right), \quad (12)$$

118 with $E_{Q_{min}} = -0.230$, where the power-law cell-size dependence used here is supported by
 119 the results of Edwards *et al.* (2012), with their value $e_{Q_{min}} = 0.84$ (SE 0.046).

120 Combining the dependencies for H_{up} , Q_{min} , V_{max} , and μ_{max} gives

$$H_{gro} \propto \frac{s^{e_{H_{up}} + e_{Q_{min}} + e_{\mu_{max}} - e_{V_{max}}} \exp\left(\frac{(E_{H_{up}} + E_{Q_{min}} + E_{\mu_{max}} - E_{V_{max}})(T - T_0)}{kTT_0}\right)}{T}, \quad (13)$$

121 where the value $e_{H_{gro}} = e_{H_{up}} + e_{Q_{min}} + e_{\mu_{max}} - e_{V_{max}} = \frac{1}{3} + 0.84 - 0.28 - \frac{2}{3} = 0.227$ is close to
 122 the value derived from the regressions of Edwards *et al.* (2012) (table 1 of the main text).

123 3 Appendix: derivative of G with respect to T

124 The invasion fitness, G , of an invader of cell size s_{in} into a resident of cell size s_{re} was derived
 125 in the main text (main text equation 11). The derivative of G with respect to T was needed
 126 to establish the effects of temperature on the relative competitive abilities of differently sized
 127 plankton cells. The derivative is computed here:

$$\frac{\partial G}{\partial T} = \frac{\frac{\partial \mu_{max}}{\partial T}(s_{in}, T) g_m(s_{re}) s_{re}^{e_{H_{gro}}}}{D} \quad (14)$$

$$- \frac{\mu_{max}(s_{in}, T) g_m(s_{re}) s_{re}^{e_{H_{gro}}} s_{in}^{e_{H_{gro}}} \frac{\partial \mu_{max}}{\partial T}(s_{re}, T)}{D^2} \quad (15)$$

128 where

$$D = g_m(s_{re}) s_{re}^{e_{H_{gro}}} + s_{in}^{e_{H_{gro}}} (\mu_{max}(s_{re}, T) - g_m(s_{re})). \quad (16)$$

129 Substituting $\frac{\partial \mu_{max}}{\partial T} = \frac{E_{\mu_{max}}}{kT^2} \mu_{max}$ and simplifying gives;

$$\frac{\partial G}{\partial T} = \frac{\mu_{max}(s_{in}, T) \frac{E_{\mu_{max}}}{kT^2} g_m(s_{re})^2 s_{re}^{e_{H_{gro}}} (s_{re}^{e_{H_{gro}}} - s_{in}^{e_{H_{gro}}})}{D^2}. \quad (17)$$

130 The denominator, D^2 , is strictly positive because $m = g_m$ is positive and $\mu_{max}(s_{re}) - g_m(s_{re}) >$
 131 0 is a condition for the resident to have become established in the first place. Therefore the
 132 sign of $\frac{\partial G}{\partial T}$ is the same as the sign of the numerator of equation 17, which is positive for
 133 $s_{re} > s_{in}$ and negative for $s_{re} < s_{in}$ since $e_{H_{gro}} > 0$.

4 Appendix: R^* for the case $m = k_m s^{e_m}$

In section "Phytoplankton: Model analysis and results" of the main text, the case in which mortality is $m = g_m(s) = k_m s^{e_m}$ is considered, and it is stated that an evolutionarily stable size (ESS) exists under certain conditions. We here demonstrate this and compute the ESS. Equation 12 of the main text is minimized with changes in s exactly when

$$\tilde{R}^* = \frac{s^{e_{H_{gro}} + e_m}}{f_{\mu_{max}}(T) s^{e_{\mu_{max}}} - k_m s^{e_m}} \quad (18)$$

is minimized, where we use the shorthand $f_{\mu_{max}} = k_{\mu_{max}} \exp\left(\frac{E_{\mu_{max}}(T-T_0)}{kTT_0}\right)$. Taking derivatives and simplifying gives

$$\frac{\partial \tilde{R}^*}{\partial s} = \frac{s^{e_{H_{gro}} + e_m - 1} ((e_m + e_{H_{gro}} - e_{\mu_{max}}) f_{\mu_{max}}(T) s^{e_{\mu_{max}}} - e_{H_{gro}} k_m s^{e_m})}{(f_{\mu_{max}}(T) s^{e_{\mu_{max}}} - k_m s^{e_m})^2}. \quad (19)$$

Cell size s is viable only if the denominator of this expression is greater than zero. The sign of equation 19 is the same as the sign of its numerator, which in turn is the same as the sign of

$$(e_m + e_{H_{gro}} - e_{\mu_{max}}) \frac{f_{\mu_{max}}(T) s^{e_{\mu_{max}}}}{k_m s^{e_m}} - e_{H_{gro}}, \quad (20)$$

since $k_m s^{e_m} > 0$ and $s^{e_{H_{gro}} + e_m - 1} > 0$.

If $e_{\mu_{max}} < e_m$ then equation 20 is always positive, because then $e_m + e_{H_{gro}} - e_{\mu_{max}} > e_{H_{gro}}$, and we already know the quotient in equation 20 is greater than 1 for viable cell sizes, s . So in that case, $\partial R^*/\partial s > 0$ for all viable s and we have runaway selection to small sizes. There is also a maximum viable size

$$s_{max} = \left(\frac{f_{\mu_{max}}(T)}{k_m} \right)^{\frac{1}{e_m - e_{\mu_{max}}}} \quad (21)$$

if $e_{\mu_{max}} < e_m$.

If $e_m < e_{\mu_{max}} - e_{H_{gro}}$, then $e_m + e_{H_{gro}} - e_{\mu_{max}} < 0$, and equation 20 is always negative because $e_{H_{gro}} > 0$. So in that case, $\partial R^*/\partial s < 0$ for all viable s , and we have runaway selection to large sizes. There is a minimum viable size

$$s_{min} = \left(\frac{k_m}{f_{\mu_{max}}(T)} \right)^{\frac{1}{e_{\mu_{max}} - e_m}} \quad (22)$$

if $e_m < e_{\mu_{max}}$.

If $e_{\mu_{max}} - e_{H_{gro}} < e_m < e_{\mu_{max}}$, then equation 20 has a root,

$$s_{ESS} = \left(\frac{e_{H_{gro}} k_m}{(e_m + e_{H_{gro}} - e_{\mu_{max}}) f_{\mu_{max}}(T)} \right)^{\frac{1}{e_{\mu_{max}} - e_m}}. \quad (23)$$

Because equation 20 is a monotonically increasing function of s for $e_{\mu_{max}} - e_{H_{gro}} < e_m < e_{\mu_{max}}$, it is negative for $s < s_{ESS}$ and positive for $s > s_{ESS}$, so s_{ESS} is a local and global minimum

157 of \tilde{R}^* and therefore of R^* .

158 5 Appendix: The equilibrium assumption

159 The use of R^* and G made in the main text in analyzing the phytoplankton model depends
160 on the assumption that equilibria are stable. This seems likely because model nutrients are
161 supplied at a constant rate. Nevertheless, we tested the assumption numerically using a
162 Monte Carlo method, for a wide range of parameters encompassing biologically reasonable
163 values. Edwards *et al.* (2012) provide allometries of the form $y = ks^e$ for $y = Q_{min}, \mu_{max},$
164 $H_{up},$ and $V_{max},$ giving estimates and confidence intervals for $\log_{10}(k)$ and e in all cases. Con-
165 fidence intervals were converted to standard errors by dividing half their length by 1.959964.
166 Montagnes & Franklin (2001) provide an allometry for $Q_{max},$ with estimates and standard
167 errors for $\log_{10}(k)$ and $e.$ See table S2 for all values taken from these references. On each
168 Monte Carlo run, the parameters $\log_{10}(k)$ and e for each allometry were drawn from nor-
169 mal distributions with mean equal to the estimated value from Edwards *et al.* (2012) or
170 Montagnes & Franklin (2001) and standard deviation equal to twice the standard error re-
171 ported in those references. Twice the standard error was used to add a buffer of additional
172 uncertainty. Cell sizes $\log_{10}(s)$ were drawn randomly and uniformly between 0 and 8 and the
173 allometries resulting from the randomly generated values of the k and e for each allometry
174 were used to generate values for $Q_{min}, \mu_{max}, H_{up}, V_{max},$ and $Q_{max}.$ When Q_{max} was less
175 than Q_{min} the values were swapped. Input nutrient concentration $R_0 = 40$ was used follow-
176 ing Litchman *et al.* (2009). Nutrient supply rates, $d,$ were taken as a uniformly distributed
177 random variable between 0.05 and 0.95 divided by a uniformly distributed random variable
178 between 1 and 50. The first variable represents replacement fractions in mixing events and
179 the second represents mixing-event periods; both ranges are from Litchman *et al.* (2009).
180 Mortality, $m,$ was a uniform random variable between 1×10^{-4} and 0.02. The parameter
181 μ_{∞} was established by solving the μ_{max} equation in equations 6 in the main text for $\mu_{\infty}.$
182 For each of 10,000 runs, equilibria were computed following the equations of Verdy *et al.*
183 (2009). When equilibria existed (9,464 runs), the system Jacobian was computed at the
184 equilibrium. In all cases, the real parts of the eigenvalues of the Jacobian were negative,
185 indicating stability. Temperature was not accounted for in this analysis because data are
186 not available on the temperature dependence of all model parameters; nevertheless it seems
187 unlikely model equilibria are ever unstable for biologically reasonable parameters given the
188 wide uncertainties incorporated above and the result that every simulated equilibrium was
189 stable. We find it plausible to expect that local stability is a mathematical necessity for any
190 equilibrium, but we do not know of a proof of this.

191 6 Appendix: Non-equilibrium dynamics

192 The R^* and invasion-fitness approach as described here is valid only when the equilibrium
193 is stable, but in contrast to the phytoplankton model (Appendix S5), an equilibrium of the
194 Rosenzweig-MacArthur model need not be stable. A Hopf bifurcation occurs at a parameter
195 boundary given by Vasseur & McCann (2005); past the boundary, the model has a stable
196 limit cycle. Invasion fitness into a resident exhibiting limit-cycle oscillations can still be

197 computed by averaging the growth rate of the putative invader over the limit-cycle resource
198 density driven by the resident (see, e.g., Klausmeier, 2008; Kooi & Troost, 2006; Litchman
199 *et al.*, 2009). A non-equilibrium approach will require all model parameters to be written as
200 functions of s and T instead of just those appearing in expressions for R^* and G , but this
201 should be possible for the Rosenzweig-MacArthur model if α and t_h can be described. Savage
202 *et al.* (2004) published theory and data on body mass and temperature dependence of r and
203 K , and Vasseur & McCann (2005) cover the body mass and temperature dependence of r as
204 well as m and μ_{max} . An important direction for future work will be elucidating the thermal
205 dimension of non-equilibrial competitive interactions.

206 References

- 207 Aksnes, D. & Egge, J. (1991) A theoretical model for nutrient uptake in phytoplankton. *Marine*
208 *Ecology Progress Series*, **70**, 65–72.
- 209 Bissinger, J., Montagnes, D., Sharples, J. & Atkinson, D. (2008) Predicting marine phytoplankton
210 maximum growth rates from temperature: Improving the Eppley curve using quantile regression.
211 *Limnology and Oceanography*, **53**, 487–493.
- 212 Edwards, K., Thomas, M., Klausmeier, C. & Litchman, E. (2012) Allometric scaling and taxonomic
213 variation in nutrient utilization traits and maximum growth rate of phytoplankton. *Limnology*
214 *and Oceanography*, **57**, 554–566.
- 215 Holling, C. (1959) The components of predation as revealed by a study of small-mammal predation
216 of the European pine sawfly. *Canadian Entomologist*, **91**, 293–320.
- 217 Klausmeier, C. (2008) Floquet theory: a useful tool for understanding nonequilibrium dynamics.
218 *Theoretical Ecology*, **1**, 153–163.
- 219 Kooi, B. & Troost, T. (2006) Advantage of storage in a fluctuating environment. *Theoretical Popu-*
220 *lation Biology*, **70**, 527–541.
- 221 Litchman, E., Kalusmeier, C., Schofield, O. & Falkowski, P. (2007) The role of functional traits and
222 trade-offs in structuring phytoplankton communities: scaling from cellular to ecosystem level.
223 *Ecology Letters*, **10**, 1170–1181.
- 224 Litchman, E., Kalusmeier, C. & Yoshiyama, K. (2009) Contrasting size evolution in marine and
225 freshwater diatoms. *Proceedings of the National Academy of Sciences*, **106**, 2665–2670.
- 226 Montagnes, D. & Franklin, D. (2001) Effect of temperature on diatom volume, growth rate, and
227 carbon and nitrogen content: reconsidering some paradigms. *Limnology and Oceanography*, **46**,
228 2008–2018.
- 229 Pinheiro, J. & Bates, D. (2000) *Mixed-Effects Models in S and S-Plus*. Springer, New York.
- 230 Savage, V., J.M., G., Brown, J., West, G. & Charnov, E. (2004) Effects of body size and temperature
231 on population growth. *The American Naturalist*, **163**, 429–441.
- 232 Vasseur, D. & McCann, K. (2005) A mechanistic approach for modelling temperature-dependent
233 consumer-resource dynamics. *American Naturalist*, **166**, 184–198.

234 Verdy, A., Follows, M. & Flierl, G. (2009) Optimal phytoplankton cell size in an allometric model.
235 *Marine Ecology Progress Series*, **379**, 1–12.

Tables

Table 1 Viscosity of sea water at a range of temperatures. Data were taken from the Chemical Hazards Response Information System of the U.S. Coast Guard (www.chrismanual.com/Intro/prop.htm), and are for “standard” sea water containing 35g salts per kg of solution.

Temp. (°F)	Visc. (Centipoise)	Temp. (°K)	Visc. ($\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$)
30	1.88	272.04	0.00188
40	1.61	277.59	0.00161
50	1.40	283.15	0.00140
60	1.21	288.71	0.00121
70	1.06	294.26	0.00106
80	0.92	299.82	0.00092
90	0.82	305.37	0.00082
100	0.73	310.93	0.00073

Table 2 Parameter estimates used in Monte Carlo simulations (see Appendix S5). Values from Edwards *et al.* (2012) are from their table 1, using values fitted to pooled freshwater and marine data. Values from Montagnes & Franklin (2001) are from their table 3.

Parameter	Estimate	SE	Source
$e_{Q_{min}}$	0.84	0.046	Edwards <i>et al.</i> (2012)
$\log_{10}(k_{Q_{min}})$	-9.0	0.128	Edwards <i>et al.</i> (2012)
$e_{\mu_{max}}$	-0.28	0.018	Edwards <i>et al.</i> (2012)
$\log_{10}(k_{\mu_{max}})$	0.65	0.059	Edwards <i>et al.</i> (2012)
$e_{H_{up}}$	0.33	0.041	Edwards <i>et al.</i> (2012)
$\log_{10}(k_{H_{up}})$	-0.61	0.138	Edwards <i>et al.</i> (2012)
$e_{V_{max}}$	0.82	0.077	Edwards <i>et al.</i> (2012)
$\log_{10}(k_{V_{max}})$	-8.0	0.281	Edwards <i>et al.</i> (2012)
$e_{Q_{max}}$	0.809	0.011	Montagnes & Franklin (2001)
$\log_{10}(k_{Q_{max}})$	-8.03	0.039	Montagnes & Franklin (2001)

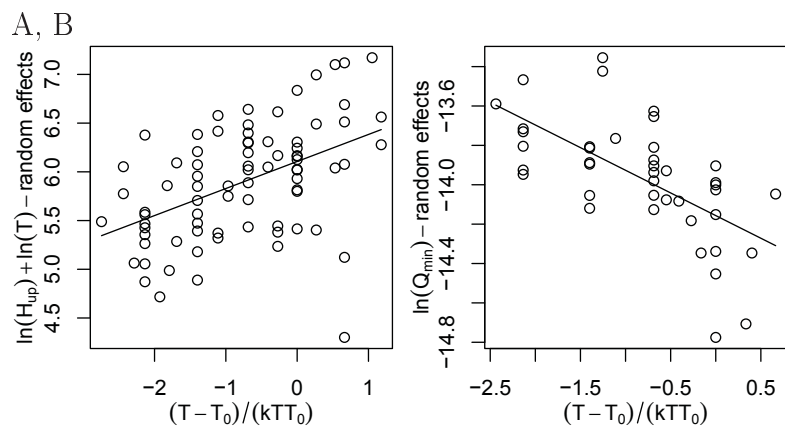


Fig. 1 Plots of metastudy data, corrected for random effects; lines have fixed-effect slopes and intercepts (see Appendix S2).

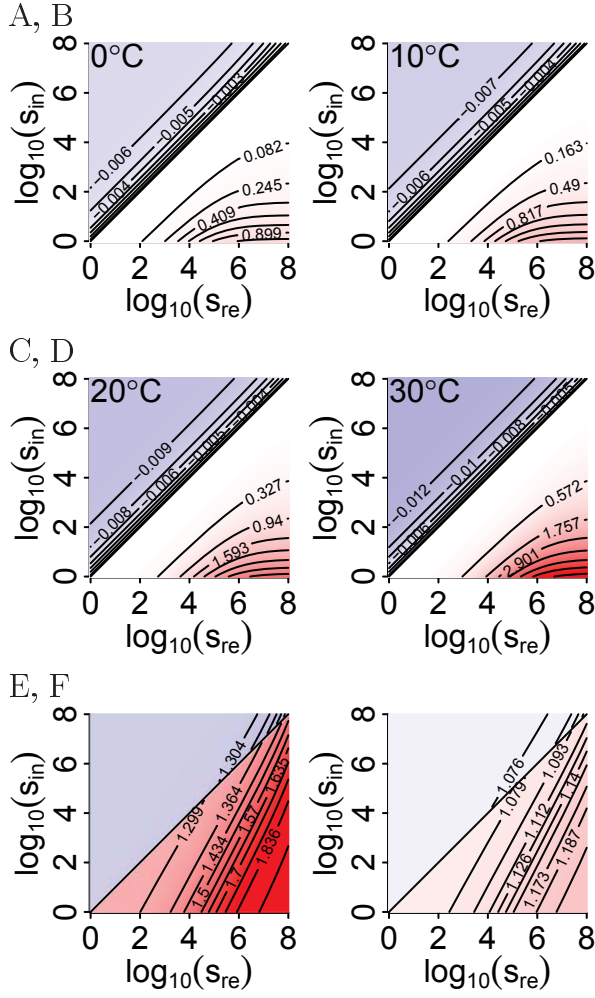


Fig. 2 (A-D) Pairwise invasibility plots at different temperatures for $m = 0.01 \exp(E_m(T - T_0)/(kTT_0))$ with $E_m = 0.1781$. These plots are analogous to figure 4A-D of the main text, but for a different model of m . (E-F) Ratios of pairwise invasibilities at different temperatures for the same m . These plots are analogous to figure 4E-F of the main text. Plots show $G(s_{in}, s_{re}, T_1)/G(s_{in}, s_{re}, T_2)$ for $T_1 = 20^\circ\text{C}$ and $T_2 = 10^\circ\text{C}$ (E) and for $T_1 = 23^\circ\text{C}$ and $T_2 = 20^\circ\text{C}$ (F). The same result of accentuated invasion fitness of smaller invaders at warmer temperatures was also true for $k_m = 0.0001, 0.001$, and 0.02 .