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Supplementary Materials for

Ecosystem context and historical contingency in apex predator recoveries

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The PDF file includes:

- fig. S1. Table of equilibrium solutions for three-species Lotka-Volterra model of IGP for the basal resource (*R*), the mesopredator (*N*), and the apex predator (*P*).
- fig. S2. Response of the equilibrium densities of the apex predator (*P*) and mesopredator (*N*) to increases in the resource's carrying capacity (*K*) for stable equilibria in which all three species coexist (RNP), only the resource and mesopredator coexist (RN), and only the resource and the apex predator coexist (RP).
- fig. S3. Stable (solid line) and unstable (dashed line) equilibrium densities for the apex predator (*P*) and mesopredator (*N*) across a range of the apex predator's prey preferences.

Full Model Equilibrium	Resource (R)	Mesopredator (N)	Apex Predator (P)
Trivial	0	0	0
Unfeasible	0	$\frac{d}{ae\omega}$	$-\frac{\delta}{a\omega}$
Resource only	K	0	0
Resource & Mesopredator	$\frac{\delta}{e\alpha}$	$\frac{r(-\delta+e\alpha K)}{e\alpha^2 K}$	0
Resource & Apex predator	$\frac{d}{ae-ae\omega}$	0	$-\frac{r(d+aeK(-1+\omega))}{a^2eK(-1+\omega)^2}$
3-spp. coexistence	$\frac{K(d\alpha+ae(\delta(-1+\omega)-r\omega))}{ae((-1+e)\alpha K(-1+\omega)-r\omega)}$	$\frac{aeK(-1+\omega)(\delta(-1+\omega)-r\omega)+d(e\alpha K(-1+\omega)-r\omega)}{ae\omega((-1+e)\alpha K(-1+\omega)-r\omega)}$	$\frac{d\alpha^2K+a(r\delta\omega+\alpha K(\delta(-1+\omega)-er\omega))}{a^2\omega((-1+e)\alpha K(-1+\omega)-r\omega)}$

Food Chain Equilibrium $\omega=1$	Resource (R)	Mesopredator (N)	Apex Predator (P)
Trivial	0	0	0
Unfeasible	0	$\frac{d}{ae\varphi}$	$-\frac{d\delta}{a}$
Resource only	K	0	0
Resource & Mesopredator	$\frac{\delta}{e\alpha}$	$\frac{r(-d\delta+e\alpha K)}{e\alpha^2 K}$	0
3-spp. coexistence	$K-\frac{d\alpha K}{aer\varphi}$	$\frac{d}{ae\varphi}$	$\frac{aer\alpha K\varphi-d(\alpha^2K+ar\delta\varphi)}{a^2r\varphi}$

Competition Equilibrium $\omega=0$	Resource (R)	Mesopredator (N)	Apex Predator (P)
Trivial	0	0	0
Resource only	K	0	0
Resource & Mesopredator	$\frac{d\delta}{e\alpha}$	$\frac{r(-d\delta+e\alpha K)}{e\alpha^2 K}$	0
Resource & Top predator	$K-\frac{d\alpha K}{aer\varphi}$	$\frac{d}{ae\varphi}$	$\frac{aer\alpha K\varphi-d(\alpha^2K+ar\delta\varphi)}{a^2r\varphi}$

fig. S1. Table of equilibrium solutions for three-species Lotka-Volterra model of IGP for the basal resource (*R*), the mesopredator (*N*), and the apex Predator (*P*). Also shown are equilibriums for a food chain module ($\omega = 1$) and competition mode ($\omega = 0$).

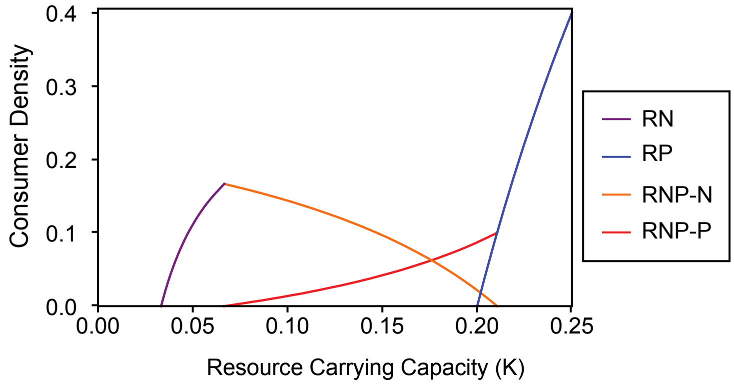

fig. S2. Response of the equilibrium densities of the apex predator (*P*) and mesopredator (*N*) to increases in the resource's carrying capacity (*K*) for stable equilibria in which all three species coexist (RNP), only the resource and mesopredator coexist (RN), and only the resource and the apex predator coexist (RP). RNP-N and RNP-P are the respective densities of the mesopredator and apex predator when all three species coexist. Parameter values reflect baseline values used in Figs. 2 to 4 of the main text.

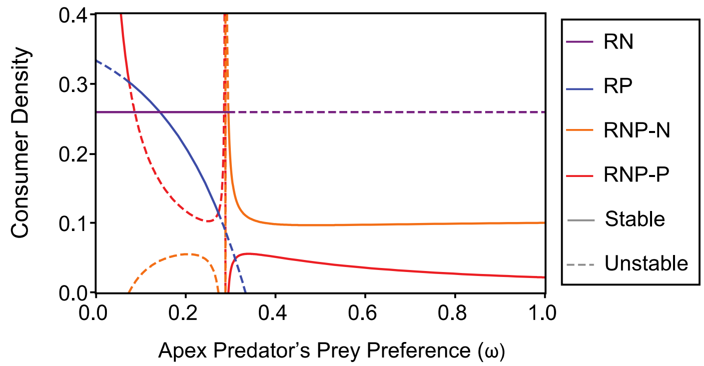

fig. S3. Stable (solid line) and unstable (dashed line) equilibrium densities for the apex predator (*P*) and mesopredator (*N*) across a range of the apex predator's prey preferences. Like fig. S2, RNP-N and RNP-P are the respective densities of the mesopredator and apex predator when all three species coexist.

At the extremes of omega value range for exploitative completion (omega = 0), the model reduces to Lotka's competition model (*117*) under slow-fast dynamics assumption with two species exploit a single resource. Similarly, when omega = 1 the model simplifies to a linear trophic chain, where it has been demonstrated that increase in productivity of the basal resource differentially leads to an increase of the density of the apex predator (*116*).