Name: Solutions

Score: _____

1. Evaluate

$$\int_1^4 \frac{\sqrt{x}+3x}{x^{\frac{5}{2}}}\,dx$$

* Remember: no product/quotient rule, so split over addition/subtraction!

$\int_1^4 \frac{\sqrt{x}+3x}{x^{5/2}}\,dx$

$= \int_1^4 \left(\frac{\sqrt{x}}{x^{5/2}} + \frac{3x}{x^{5/2}}\right)dx = \int_1^4 \left(\frac{x^{1/2}}{x^{5/2}} + \frac{3x^{2/2}}{x^{5/2}}\right)dx = \int_1^4 \left(x^{-2} + 3x^{-3/2}\right)dx$

$= -x^{-1} + \frac{3x^{-1/2}}{(-1/2)}\Big|_1^4$

$= -\frac{1}{x} - \frac{6}{\sqrt{x}}\Big|_1^4$

$= \left[-\frac{1}{4} - \frac{6}{\sqrt{4}}\right] - \left[-1 - \frac{6}{\sqrt{1}}\right]$ ($-1-6=-7$)

$= -\frac{1}{4} - 3 + 7$

$= 15/4$

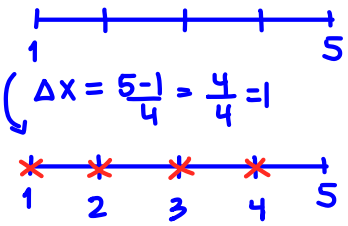
* This was L30 content (my bad), so I graded very kindly and only on the steps regarding earlier lectures, although by the time you took this quiz, you definitely covered the L30 material.

2. Approximate the area under the curve

$$f(x) = x^2 + x$$

over the interval $[1, 5]$ by using a Riemann Sum with **four** subintervals of equal width, using the **left** endpoints of each subinterval.

$f(x) = x^2+x$



$\Delta x = \frac{5-1}{4} = \frac{4}{4} = 1$

$f(1) = 1^2+1 = 2$
$f(2) = 2^2+2 = 6$
$f(3) = 3^2+3 = 12$
$f(4) = 4^2+4 = 20$

$\Delta x\,(f(x_0) + \dots f(x_3)) \Rightarrow 1\,(f(1)+f(2)+f(3)+f(4))$

$= 1(2+6+12+20)$

$= 40$

[alternate solution below]

$L_n = \sum_{i=0}^{n-1} \Delta x\, f(x_i) = \sum_{i=0}^{3} \Delta x f(x_i)$

$= \Delta x \sum_{i=0}^{3} f(x_i)$

$\Delta x = \frac{5-1}{4} = \frac{4}{4} = 1$

$= 1 \sum_{i=0}^{3} f(x_i)$

$= \sum_{i=0}^{3} (1+i)^2 + (1+i)$

$x_i = a + \Delta x \cdot i$

$= 1 + 1 \cdot i$

$= 1 + i$

$f(x) = x^2 + x$, so

$f(x_i) = x_i^2 + x_i$

$= (1+i)^2 + (1+i)$

$i=0: (1+0)^2 + (1+0) = 1+1 = 2$

$i=1: (1+1)^2 + (1+1) = 2^2 + 2 = 6$

$i=2: (1+2)^2 + (1+2) = 12$

$i=3: (1+3)^2 + (1+3) = 4^2 + 4 = 20$

$= 2 + 6 + 12 + 20$

$= 40$

This way is more formal, as you can see, so I personally prefer the 1st method I showed.