

Instructions: Answer the questions in the spaces provided. Show all of your work.

Q 1. (6 points) Find the equation the plane containing the points $\overset{P}{(2, -1, 3)}$, $\overset{Q}{(1, 2, -2)}$, and $\overset{R}{(5, -1, 2)}$.

need a normal vector η and pt (x_0, y_0, z_0)

To find normal vector, take 2 vectors in plane and cross!

$$\vec{PQ} = \langle 1-2, 2-(-1), -2-3 \rangle = \langle -1, 3, -5 \rangle$$

$$\vec{PR} = \langle 5-2, -1-(-1), 2-3 \rangle = \langle 3, 0, -1 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -1 & 3 & -5 \\ 3 & 0 & -1 \end{vmatrix} = i(-3) - j(1-15) + k(-9) = -3i - 16j - 9k$$

$$\hookrightarrow \langle -3, -16, -9 \rangle$$

$$\text{or } n = \langle 3, 16, 9 \rangle$$

NOW

Pick any pt in plane! I choose P!

We get

$$3(x-2) + 16(y+1) + 9(z-3) = 0$$

$$3x - 6 + 16y + 16 + 9z - 27 = 0$$

$$3x + 16y + 9z - 17 = 0$$

Q 2. (4 points) Suppose that $\vec{u} \cdot \vec{v} = 18$ and $\vec{u} \times \vec{v} = \langle 8, -1, -4 \rangle$ and θ is the angle between $\vec{u}$ and $\vec{v}$. Find $\tan \theta$.

We have 2 relevant formulas!

$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta \quad \cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$$

$$|\vec{u} \times \vec{v}| = |\vec{u}| |\vec{v}| \sin \theta \quad \sin \theta = \frac{|\vec{u} \times \vec{v}|}{|\vec{u}| |\vec{v}|}$$

$$\text{Recall } \tan \theta = \frac{\sin \theta}{\cos \theta}, \text{ so } \tan \theta = \frac{\frac{|\vec{u} \times \vec{v}|}{|\vec{u}| |\vec{v}|}}{\frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}} = \frac{|\vec{u} \times \vec{v}|}{\vec{u} \cdot \vec{v}}$$

$\vec{u} \cdot \vec{v} = 18$ from the question

$\vec{u} \times \vec{v} = \langle 8, -1, -4 \rangle$, but we need magnitude

$$\begin{aligned} \hookrightarrow |\vec{u} \times \vec{v}| &= \sqrt{8^2 + (-1)^2 + (-4)^2} \\ &= \sqrt{64 + 1 + 16} \\ &= \sqrt{81} \\ &= 9 \end{aligned}$$

$$\text{So } \tan \theta = \frac{|\vec{u} \times \vec{v}|}{\vec{u} \cdot \vec{v}} = \frac{9}{18} = \frac{1}{2}$$