# Quiz 1 Solutions

(1) $\int_0^{\pi} x^2 \cos(x)$

| $u$ | | $dv$ |
|---|---|---|
| $x^2$ | $+$ | $\cos x$ |
| $2x$ | $-$ | $\sin x$ |
| $2$ | $+$ | $-\cos x$ |
| $0$ | | $-\sin x$ |

$\int x^2 \cos(x)$

$= x^2 \sin x + 2x\cos x - 2\sin x \Big|_0^{\pi}$

$= [\pi^2 \sin\pi + 2\pi\cos\pi - 2\sin\pi] - [0^2 \sin 0 + 2(0)\cos 0 - 2\sin 0]$

$= -2\pi$

(2) $\int x \ln x \, dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$

$= \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$

$= \frac{x^2}{2} \ln x - \frac{1}{2}\int x \, dx$

$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$

# Quiz 2 Solutions

(1) $\int \tan^{2025} x \sec^4 x \, dx$

Save a copy of $\sec^2 x$ for $du$, turn rest of secs $\to$ tans, $u = \tan x$

$\int \tan^{2025} x \cdot \sec^2 x \cdot \sec^2 x \, dx$

$= \int \tan^{2025} x \, (\tan^2 x + 1) \cdot \sec^2 x \, dx$

Let $u = \tan x$

$du = \sec^2 x \, dx$

$= \int u^{2025}(u^2+1)\,du = \int (u^{2027} + u^{2025})\,du$

$= \frac{u^{2028}}{2028} + \frac{u^{2026}}{2026} + C = \frac{\tan^{2028} x}{2028} + \frac{\tan^{2026} x}{2026} + C$

(2) $\int \sin^4 x \, dx$

$= \int \sin^2 x \cdot \sin^2 x \, dx$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$= \int \left[\frac{1}{2}(1 - \cos 2x)\right]^2 dx$

$= \frac{1}{4}\int (1 - \cos 2x)^2 \, dx$

$= \frac{1}{4}\int (1 - 2\cos 2x + \cos^2(2x))\,dx$

$\cos^2(2x) = \frac{1}{2}(1 + \cos 4x)$

$= \frac{1}{4}\int (1 - 2\cos 2x + \frac{1}{2}(1 + \cos 4x))\,dx$

$= \frac{1}{4}\int (3/2 - 2\cos 2x + 1/2 \cos 4x)\,dx$

$= \frac{1}{4}[3x/2 - \sin 2x + \frac{1}{8}\sin 4x] + C$

$= 3x/8 - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$

# Quiz 3 Solutions

(1) $\displaystyle\int \frac{1}{\sqrt{x^2+9}}\,dx$ $\qquad x = 3\tan\theta$

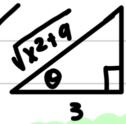
$\qquad dx = 3\sec^2\theta\, d\theta$

$$\int \frac{3\sec^2\theta\, d\theta}{\sqrt{(3\tan\theta)^2+3^2}} = \int \frac{3\sec^2\theta\, d\theta}{\sqrt{9(\tan^2\theta+1)}} = \int \frac{3\sec^2\theta\, d\theta}{3\sqrt{\sec^2\theta}} = \int \frac{\sec^2\theta}{\sec\theta}\, d\theta = \int \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$x = 3\tan\theta$

$\frac{O}{A} = \frac{x}{3} = \tan\theta$




$$= \ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C$$

(2) $\displaystyle\int \frac{1}{x^2-x-2}\,dx$

Note $x^2 - x - 2 = (x-2)(x+1)$

$$\frac{1}{x^2-x-2} = \frac{A}{x-2} + \frac{B}{x+1}$$

Cover up

A: $\frac{1}{2+1} = \frac{1}{3}$

B: $\frac{1}{-1-2} = -\frac{1}{3}$

$$\int \frac{1}{x^2-x-2}\,dx = \int \left(\frac{\frac{1}{3}}{x-2} + \frac{-\frac{1}{3}}{x+1}\right) dx$$

$$= \frac{1}{3}\ln|x-2| - \frac{1}{3}\ln|x+1| + C$$

# Quiz 4 Solutions

(1) $\displaystyle\int_1^{\infty} xe^{-x^2}\,dx$

$\displaystyle = \lim_{t\to\infty} \int_1^t xe^{-x^2}\,dx$

First, calculate the indefinite integral

$\displaystyle\int xe^{-x^2}\,dx$ $\qquad$ Let $u = -x^2$

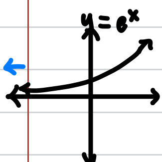
$\displaystyle = \int -\frac{1}{2}e^u\,du$ $\qquad du = -2x\,dx$

$\displaystyle = -\frac{1}{2}e^u + C$ $\qquad \Rightarrow -\frac{1}{2}du = x\,dx$

$\displaystyle = -\frac{1}{2}e^{-x^2} + C$

now return to our limit, plug in

$$\int_1^{\infty} xe^{-x^2}\,dx = \lim_{t\to\infty}\left[-\tfrac{1}{2}e^{-x^2}\right]_1^t$$




$$= \lim_{t\to\infty}\left[\cancel{-\frac{1}{2}e^{-t^2}}^{\,0} + \frac{1}{2}e^{-1}\right]$$

$$= \frac{1}{2}e^{-1} = \frac{1}{2e}$$

as $t\to\infty$,

$e^{-t^2} \to 0$

(2) $\int \frac{x+4}{x^2+2x+10}\,dx$

$u = x^2+2x+10$
$du = (2x+2)\,dx$

$\frac{1}{2}\int \frac{2x+8}{x^2+2x+10}\,dx$

$= \frac{1}{2}\int \frac{(2x+2)+6}{x^2+2x+10}\,dx$

$= \frac{1}{2}\int \left(\underbrace{\frac{2x+2}{x^2+2x+10}} + \frac{6}{x^2+2x+10}\right)dx$

$\int \frac{2x+2}{x^2+2x+10}\,dx$

$u = x^2+2x+10$
$du = 2x+2\;dx$

$= \int \frac{du}{u} = \ln|u| + C$

$= \ln|x^2+2x+10| + C$

$\int \frac{6}{x^2+2x+10}\,dx$

$= 6\int \frac{1}{x^2+2x+10}\,dx$

complete □:

$\frac{2}{2} = (1)^2 = 1$

so,

$x^2+2x+10 =$
$(x+1)^2 - 1 + 10$
$= (x+1)^2 + 9$

$= 6\int \frac{1}{(x+1)^2+3^2}\,dx$

$= 6 \cdot \frac{1}{3}\arctan\left(\frac{x+1}{3}\right) + C$

$= 2\arctan\left(\frac{x+1}{3}\right) + C$

$= \frac{1}{2}\left(\ln|x^2+2x+10| + 2\arctan\left(\frac{x+1}{3}\right)\right) + C$

$= \frac{1}{2}\ln|x^2+2x+10| + \arctan\left(\frac{x+1}{3}\right) + C$

can drop the abs. value bars, since this quadrant always positive