Write final answers on this sheet. Turn in all relevant work on separate sheets. Good luck!

(1) Find the Laplace transform \( \mathcal{L}\{f(t)\}(s) \) for the period 2 function with
\[
f(t) = \begin{cases} 
2t & \text{if } 0 \leq t < 1 \\
4 - 2t & \text{if } 1 < t < 2
\end{cases}
\]

(2) Show that the gamma function \( \Gamma \) has the property that \( \Gamma(n + 1) = n\Gamma(n) \) for any positive integer \( n \).

(3) Find \( \mathcal{L}^{-1} \) for the following functions
   (a) \( \frac{3s + 8}{s^2 - 8s + 25} \)
   (b) \( \frac{e^{-3s}}{s^4} \)
   (c) \( \frac{s + 3}{(s - 1)^2(s^2 + 4)} \)

(4) Solve the IVP
\[
y'' - 4y' = \begin{cases} 
3 & \text{if } 0 \leq t < 2 \\
0 & \text{if } t > 2
\end{cases}; y(0) = 1, y'(0) = 0
\]

(5) A mass of 3 kg is attached to a spring with stiffness \( k = 50 \text{N/m} \). The spring is released from rest 1 meter to the left of the spring’s equilibrium position. Five seconds later, the mass is struck, giving it an impulse of 10 Ns. Find a differential equation for the position \( y(t) \) of the mass at time \( t \) and use Laplace transforms to solve for \( y(t) \).

(6) Find the transfer and impulse response function for the ODE
\[
y'' + 4y = g(t).
\]
Use the impulse response function to find a formula for the solution to the above ODE with initial conditions
\[
y(0) = 1, y'(0) = 1.
\]