Final Exam Review Guide

1. Final Exam Format

The final exam will consist of two parts, each comprising roughly 50% of the exam. The first part will consist of short questions that should take no longer than 5 minutes each to complete. Determining whether an equation is exact, finding integration factors, finding homogeneous solutions to Cauchy-Euler/constant coefficient equations, taking $\mathcal{L}, \mathcal{L}^{-1}$ for simple expressions, and finding the radius of convergence of a power series solution are examples of this type of problem. The second part will consist of short answer problems similar in format to what you have seen on the four in-class exams. These problems may combine techniques from multiple sections. I will definitely get creative on this part and ask some unconvential questions.

2. Review Guide

Below is a list of topics as well as the place in the book where they appear. I have provided relevant exercises next to a topic if that topic was not properly covered by previous homework problems. Otherwise, cross reference that section with the exercise list on the course webpage for practice problems. Exercise numbers offset in brackets indicate that they are all the same type of problem. Rather than working all from a bracketed section, you should just work one or two to ensure that you understand the procedure. I also strongly recommend reviewing the problems on the previous in-class exams. Good luck!

(1) Chatper 1: Introduction
   (a) Classification of DEs [**1.1 p.4**]
      - Ordinary vs. Partial
      - Linear DEs, order of a DE
   (b) Directon Fields [**1.3**]
      - Sketching direction field/solutions of IVPs
      - Qualitative features of solutions (bounded? limit at ∞?) [**1.2 Ex. 1**]

(2) Chapter 2: First Order ODEs
   (a) Existence and uniqueness for first order equations [**1.2 Thm. 1**]
   (b) Solutions to separable, linear, exact equations [**2.2-2.4**]
   (c) Find integrating factors for linear and exact equations[**2.2 Ex.1, 2.5 Thm. 3**]
   (d) Reduction of Bernouli equations to linear equations.[**2.6**]
   (e) General changes of variables.[**2.6**]

(3) Chapter 4: Second Order ODEs
   (a) Linear Independence of solutions and the relation between the Wronskian and linear independence [**4.2 Def. 1, Lem. 1**]
   (b) General solutions for homogeneous constant coefficient equations[**4.2,4.3 (2nd order)**
   (c) Finding a particular solution to a nonhomogeneous equation: Method of Undetermined Coefficients and Variation of Parameters [**4.5 p.186, 4.6 p.191**]
   (d) The Superposition Principle [**4.5 Thm. 3**]
   (e) General solutions for nonhomogeneous constant coefficient equations: $y = y_p + y_h$ [**4.5 p.184**]

- (f) Variable Coefficient equations [**4.7**]
  - General solutions for Cauchy-Euler equations [**4.7 pp.195-196**]
  - Superposition, Variation of Parameters, $y_h = c_1 y_1 + c_2 y_2$ still hold, but finding y_1, y_2 is hard. [**4.7 pp.197-198**]
  - Reduction of Order: Given y_1, find y_2 [**4.7 Thm. 8**]
- (g) Existence and Uniqueness [**4.7 Thm. 5**]
- (h) Free Mechanical Vibration [**4.9**]
  - Classify as under/over/critically damped
  - Amplitude, (quasi)period, (quasi)frequency

(4) Chapter 7: Laplace Transforms
- (a) Definition of the Laplace Transforn $\mathcal{L}$ [**7.2 Def. 1**]
- (b) Existence of $\mathcal{L}$ for piecewise continuous functions of exponential order [**7.2 Def. 2, Def. 3, Thm. 2**]
- (c) Properties of $\mathcal{L}$ (Linearity [**7.2 Thm. 1**], transforms of derivatives [**7.3 Thm 4, Thm. 5**])
- (d) The inverse transform $\mathcal{L}^{-1}$ [**7.4 Def. 4**]
- (e) Inverse transforms of rational functions (partial fractions) [**7.4**]
- (f) Solving ODEs with $\mathcal{L}$ [**7.5**]
  - Paradigm: transform, manipulate, inverse transform
  - Soltuions of constant coefficient equations [**7.5 Exs. 1-3**] and some variable coefficient equations [**7.5 Ex. 4**]
- (g) Transforms of periodic and discontinuous functions (and solutions of IVPs involving them) [**7.6**]
- (h) Convolutions [**7.7 Def. 9, Thm. 10**] and the convoloution method for finding $\mathcal{L}$ [**7.7 Thm. 11**]
- (i) The transfer and impulse response functions and solutions of IVPs [**7.7 p.401, Thm. 12**], **(7.7) #[23-28]**
- (j) The Dirac Delta function δ [**7.8**]
  - Properties (not a function [**7.8 pp.405-408**] , how to integrate with it [**7.8 Def. 10**])
  - Transform of δ [**7.8 p.406**], solutions of IVPs involving δ [**7.8 Ex. 1**]

(5) Chapter 8: Power Series Solutions
- (a) Bootstrapping Method to compute Taylor Polynomials for Solutions [**8.1**]
- (b) General Power Series Knowledge (Radius/Interval of Convergence, Convergence Tests) [**8.2**]
- (c) Power Series Solutions to Linear ODEs [**8.3,8.4**]
  - (i) Ordinary and Singular Points
  - (ii) Radius of Convergence
  - (iii) Recurrence Relation vs Closed Form