(1) The direction field for the ODE \( \frac{dy}{dx} = y^2 - x^2 - 2x \) is shown below.

Answer the following questions about the ODE and its direction field.

(a) [8] What are all of the linear solutions \( y = ax + b \) for some \( a, b \) of the ODE?

(b) [6] Sketch the solutions to the IVPs with initial conditions \( y(-4) = 2 \) and \( y(-2) = -2 \).

(c) [6] Is there a solution to the ODE which satisfies both \( y(-3) = 0 \) and \( y(-2) = -2 \)? Justify your answer.

(2) [15] Use separation of variables to solve the IVP

\[
\frac{dy}{dx} = \frac{\sin(x)}{\cos(y)}, \quad y(0) = \pi.
\]
(3) [15] Find the general solution to
\[
\frac{dy}{dx} + \frac{3}{x}y + 2 = 3x.
\]

(4) [20] Show that the equation \( \frac{dy}{dx} = \frac{x^3 + xy^2 - x}{y} \) is not homogeneous.
Solve the ODE using the substitution \( v = x^2 + y^2 \).

(5) [30] Show that there is no integrating factor which depends only on \( x \) for the ODE (10)
\[
(3y + 2xy^2) \, dx + (x + 2x^2y) \, dy = 0
\]
Then use the integrating factor \( \mu(x, y) = \frac{1}{xy} \) to solve the ODE (20).