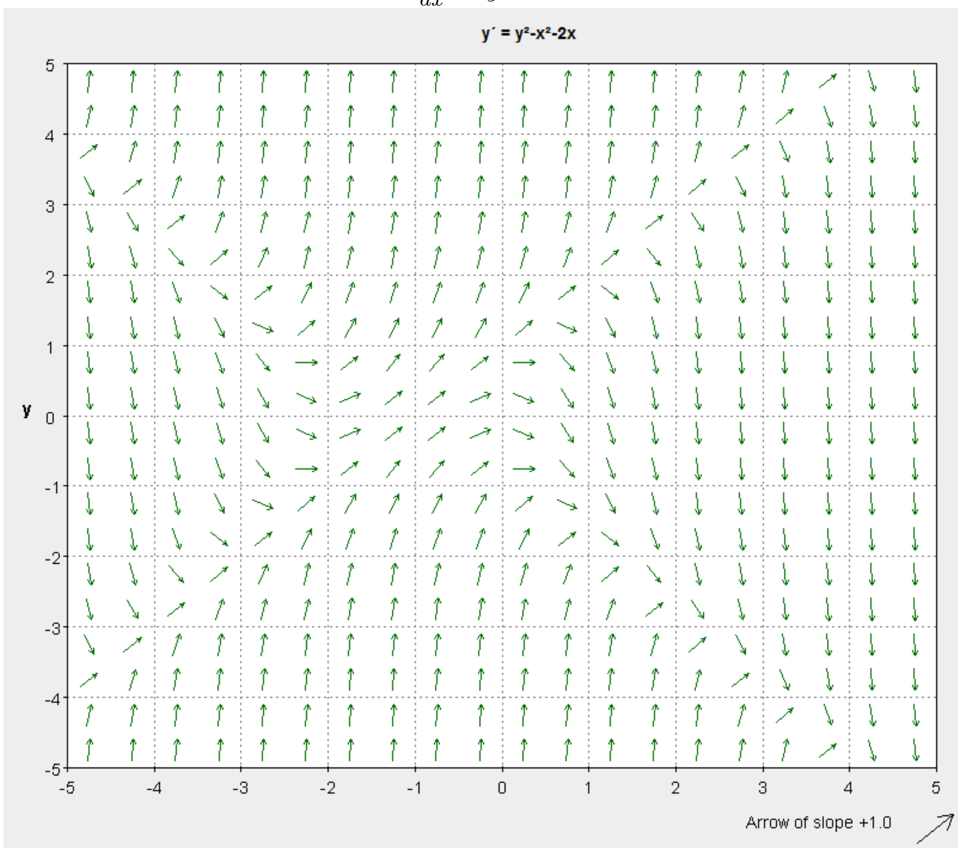
MAP 2302, Exam I, Spring 2015

Name:____________________

Student signature:____________________

Write final answers on this sheet. Turn in all relevant work on separate sheets.

(1) The direction field for the ODE $\frac{dy}{dx} = y^2 - x^2 - 2x$ is shown below.



Answer the following questions about the ODE and its direction field.

(a) [8] What are all of the linear solutions($y = ax + b$ for some a, b) of the ODE?

(b) [6] Sketch the solutions to the IVPs with initial conditions $y(-4) = 2$ and $y(-2) = -2$.

(c) [6] Is there a solution to the ODE which satisfies both $y(-3) = 0$ and $y(-2) = -2$? Justify your answer.

(2) [15] Use separation of variables to solve the IVP

$$\frac{dy}{dx} = \frac{\sin(x)}{\cos(y)} \quad , y(0) = \pi.$$

(3) [15] Find the general solution to

$$\frac{dy}{dx} + \frac{3}{x}y + 2 = 3x.$$

(4) [20] Show that the equation $\frac{dy}{dx} = \frac{x^3+xy^2-x}{y}$ is **not** homogeneous. Solve the ODE using the substitution $v = x^2 + y^2$.

(5) [30] Show that there is no integrating factor which depends only on x for the ODE (10)

$$(3y + 2xy^2)\,dx + (x + 2x^2y)\,dy = 0$$

Then use the integrating factor $\mu(x, y) = \frac{1}{xy}$ to solve the ODE (20).