Write final answers on this sheet. Turn in all relevant work on separate sheets. Full work is required for full credit.

(1) [15] Use the method of undetermined coefficients to find the form of a particular solution $y_p$ to the following ODEs. Do not solve the equation or solve for the coefficients!

(a) [5] $y'' - 2y' + 5y = e^{-t}$

(b) [5] $y'' - 2y' + 5y = te^t \sin(2t)$

(c) [5] $y'' - 2y' + 5y = e^{-t} + te^t \sin(2t)$

(2) [10] Is it possible for $y_1 = e^t$ and $y_2 = t + 1$ to both be solutions to $y'' + p(t)y' + q(t)y = g(t)$ on $(-\infty, \infty)$ if $p, q, g$ are all continuous on $(-\infty, \infty)$? **Justify your answer.** *(Hint: Examine how the two functions intersect.)*

(3) [20] Find the general solution to the ODE

$$y'' - 4y' + 4y = te^t.$$ 

(4) [25] Given that $t^2e^t$ and $(t^2 + 1)e^t$ are solutions to

$$ty'' + (1 - 2t)y' + (t - 1)y = 4te^t, \ t > 0,$$

find the general solution to the ODE on $t > 0$.

(5) [30] Solve the following Cauchy-Euler IVP:

$$t^2y'' - ty' + y = t, \ y(-1) = 1, y'(-1) = 2.$$