MAP 2302, Exam III, Spring 2015

Name:_____

Student signature:_____

Write final answers on this sheet. Turn in all relevant work on separate sheets. Full work is required for full credit.

(1) Suppose that f, g are functions defined on $(-\infty, \infty)$. What is the convolution $f * g$? What if f, g are only defined on $[0, \infty)$?

(2) If f is piecewise continuous and $|f(t)| \leq 10e^{5t}$, for what values of s is $F(s)$ guaranteed to exist?

(3) Find the Laplace transform of the following functions

(a) $f(t) = t\sin(t)$

(b) $f(t) = t^2$ if $0 < t < 2$ and f is periodic of period 2.

(c) $f(t) = \begin{cases} e^t & \text{if } 0 \leq t < 5 \\ t & \text{if } t > 5 \end{cases}$

(4) Use the Laplace transform to solve the following IVP

$$y'' - 2y' + y = 6t - 2;\ y(-1) = 3, y'(-1) = 7$$

(5) Use the Laplace transform to solve the following IVP

$$ty'' - ty' + y = 2;\ y(0) = 2, y'(0) = -1$$

(6) Use the Laplace transform to solve the following symbolic IVP

$$y'' + 5y' + 6y = e^{-t}\delta(t-2);\ y(0) = 2, y'(0) = -5$$