

MAP 2302, Exam III, Spring 2015

Name: _____

Student signature: _____

Write final answers on this sheet when able. Turn in all relevant work on separate sheets. Full work is required for full credit.

- (1) [15] Find the Laplace transform of the following function:
 $f(t) = 1 - (t - 1)^2$ if $0 < t < 2$ and f is periodic of period 2.
- (2) [20] Find the inverse Laplace transform of the following functions:
- (a) [12] $F(s) = \frac{2s - 3}{s^2 - 4s + 8}$
- (b) [8] $F(s) = e^{-2s} \frac{2s - 3}{s^2 - 4s + 8}$
- (3) [25] Solve the following IVP:
 $y'' + 3y' + 2y = \delta(t - 2); y(1) = 0, y'(1) = 1$
[+5] Is the solution continuous? Is its derivative continuous?
- (4) Answer the following related questions:
- (a) [15] Take the Laplace transform of the following IVP:
 $t^2 y'' + 4ty' + 2y = t + 2; y(0) = 2, y'(0) = 1$
- (b) [10] Find y if $\frac{d^2}{ds^2} Y(s) = \frac{1}{s^4} + \frac{2}{s^3}$
- (c) [+5] Why isn't y from part (b) a solution to the IVP from part (a)?
- (5) [5] Write a formula for computing $\mathcal{L}\{y'''\}$ in terms of $\mathcal{L}\{y\}$.
- (6) [5] Suppose that f is continuous and periodic. For what values of s is $F(s)$ guaranteed to exist? **Justify your answer.**
- (7) [5] Given a function f which is PWC and of exponential order find $(f * \delta)(t)$. **Hint:** Either use the definition of convolution or use \mathcal{L} .
- (8) [+5] Find a continuous function f defined on $[0, \infty)$ which is not of exponential order α for any choice of α but $\mathcal{L}\{f\}(s)$ exists for $s > 0$.