MAP 2302, Exam IV, Spring 2015

Name:____________________

Student signature:____________________

Turn in all relevant work with final answers circled on separate sheets. Full work is required for full credit.

(1) [25] Solve the following symbolic IVP

$$y'' + 2y' + 2y = \delta(t - 2\pi)\ ; y(\pi) = 2, y'(\pi) = 1$$

(2) [25] Answer the following questions about the ODE

$$y' + 4xy = 0$$

- [15] Find a recurrence relation that determines the coefficients a_n of a power series for the solution y.
- [5] What is the radius of convergence for that power series?
- [5] Use the recurrence relation to find an explicit formula (closed form) for a_n.

(3) [25] Find a recurrence relation for the coefficients of a power series for a general solution to

$$(x^2 + 1)y'' + y = 0$$

centered around $x = 1$. Use the recurrence relation ([15]) to find the first four nonzero terms ([5]) of the series. What is the minimum radius of convergence ([5]) of the series?

(4) [25] Find the first four nonzero terms of a power series solution centered at $x = 0$ to the IVP

$$y'' - \sin(x)y = \cos(x)\ ; y(0) = 1, y'(0) = 1.$$

Recall that

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}, \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

(5) **[+5]** Use the root test to show that the power series $\Sigma_{n=1}^{\infty} n a_n x^{n-1}$ has the same radius of convergence as the power series for $\Sigma_{n=0}^{\infty} a_n x^n$. In other words, the series for y' has the same radius of convergence as the series for y.

(6) **[+5]** Give an example of a function f for which the nth derivative $f^{(n)}$ is continuous for all n but f is not analytic at 0. For full credit, provide justification.

$f(t)$	$\mathcal{L}\{f\}(s)$
1	$\frac{1}{s}$
$t^n, n = 0, 1, 2, \ldots$	$\frac{n!}{s^{n+1}}$
$\sin(bt)$	$\frac{b}{s^2 + b^2}$
$\cos(bt)$	$\frac{s}{s^2 + b^2}$
$u(t-a)f(t-a)$	$e^{-as}\mathcal{L}\{f(t)\}(s)$
$\delta(t-a)$	e^{-as}
$e^{at}f(t)$	$\mathcal{L}\{f(t)\}(s-a)$
$f'(t)$	$s\mathcal{L}\{f(t)\}(s) - f(0)$
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}\mathcal{L}\{f(t)\}(s)$
$f(t)$ (period T)	$\frac{\mathcal{L}\{f_T(t)\}}{1 - e^{-Ts}}$
$(g * h)(t)$	$\mathcal{L}\{g(t)\}(s)\mathcal{L}\{h(t)\}(s)$