MAT 3503
Functions and Modeling

Terminal Speed Lab
Revised November 14, 2010

Purpose
To estimate the terminal speed of a falling sphere and find the distance the object falls in order to achieve terminal speed.

Required Equipment
- stopwatch
- a long tape measure
- a light weight ball (such as a whiffle or nerf golf ball)

Discussion

The area under a graph of a speed vs. time curve represents the distance traveled by the object. This very powerful idea underlies the mathematics of integral calculus. You will investigate the idea that the area under a graph of speed vs. time can be used to predict a property of the behavior of objects falling under the influence of gravity in the presence of air resistance.

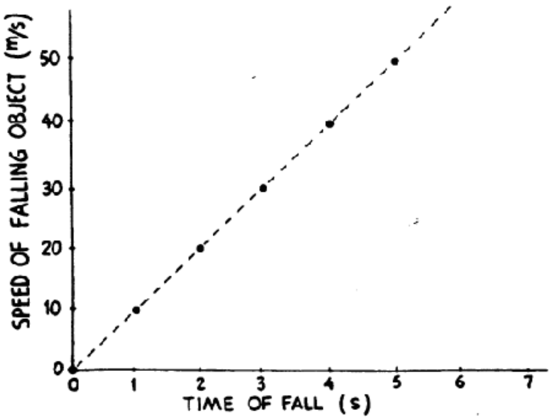
If there is no air friction, a falling whiffle ball or nerf ball falls at constant acceleration g so its change of speed is:

$$V_F - V_o = gt$$

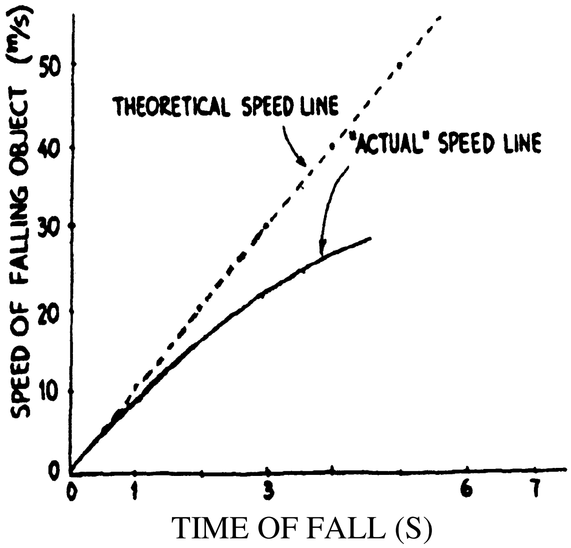
where:

V_F = final speed
V_O = initial speed
g = acceleration due to gravity
t = time of fall

A graph of speed vs. time of fall is shown at the top of the next page, where $V_0 = 0$. The V axis represents the speed V_F of the freely falling object at the end of any time T. The area under the graph of the line is a triangle of base T and height V_F, so the area equals $(1/2)V_F$ T.



If you time a whiffle or nerf ball falling from rest a distance of 43.0 M in Air, the fall takes 3.5 S. This is *longer* than the theoretical time of 2.96 S. Sir friction is *not negligible* for most objects, including whiffle balls. A graph of the actual speed vs. the time of fall looks like the curve in below.



Since air resistance reduces the acceleration of the object to below the theoretical value of 9.8 M/S^2, the falling speed is less than the theoretical speed, the difference is small at first, but grows as air resistance becomes greater and greater with the increasing speed. The graph of actual speed vs. time curve increases more slowly than the theoretical line.

Procedure

Step 1: Dropping Locations
Using the whiffle ball, select five different heights from which to drop the object. Sites should range from 1 meter to 10 meters in height. Clock the ball's time of fall to within 0.1 S or better. Consider various releasing techniques and reaction times associated with the timer you use.

Step 2: Practice
Practice you technique for dropping and timing to produce maximum consistency, any small error in technique will drastically effect your results.

Step 3: Measure Height and Falling Times
Measure the height of each location and determine the falling times for your object at each site. Repeat three times per site and find the average time of fall for each location.

Step 4: Calculate Theoretical Falling Time
Using your measured value for the height, calculate the THEORETICAL TIME of fall for your object at each location. Remember, this is the time it would take the object to reach the ground if there were not air resistance:

$$d = v_0 t + \frac{1}{2} g t^2 \quad where \quad v_0 = 0 \therefore \; d = \frac{1}{2} g t^2$$

$$\Rightarrow t = \sqrt{\frac{2d}{g}}$$

Step 5: Calculate Theoretical Final Velocity
Using your calculated THEORETICAL TIME of fall from Step 5, calculate the THEORETICAL FINAL VELOCITY (speed, in our case) for an object falling without air resistance from each location tested.

Step 6: Graphing Theoretical Speed
On graph paper, plot THEORETICAL VELOCITY vs. THEORETICAL TIME. Draw a best-fit line between your data points. (See Useful Tables)

Step 7: Calculating Actual Speed
Using your recorded actual times, calculate the ACTUAL FINAL VELOCITY of your object for each height:

$$d = \left(\frac{v_0 + v_F}{2} \right) t \;\; where \;\; v_0 = 0 \;\therefore\; d = \left(\frac{v_F}{2} \right) t$$

$$\Rightarrow v_F = \frac{2d}{t}$$

Step 8: Graphing Actual Speed
<u>Using the same graph from Step 7</u>, plot actual final speed vs. time. Starting from the origin, sketch your approximation for the actual speed vs. time curve. Your sketch should begin to level off after a certain amount of time. The limit this curve approaches is known as the terminal speed.

Step 9: Finding an Approximate Limit Using Your Graphing Calculator
Using a graphing calculator, we will attempt to find an approximate value for the limit or the terminal speed. Use your graphing calculator to find a *best-fit regression quadratic equation* for your actual data. (Keep in mind that the actual curve in not really quadratic. However, this method allows you to find a point where the derivative is equal to zero, which will be a good approximation for the time at which your object reached terminal velocity.

Step 10: Approximating Terminal Speed.

Find the time at which the slope is equal to zero. This represents the maximum value for your curve. (This is an approximation for the time at which your object reached terminal velocity. To find the point at which the slope is equal to zero, take the derivative of your best-fit regression equation generated from your calculator in Step 9. Set the derivative equal to zero and solve for time. Once you have found this time you can substitute this value into your original equation to find the terminal speed of your object.

Step 11: Distance to Terminal Velocity

By integrating your best-fit equation from the time zero through time to maximum from step 10, you can determine the fall distance at which your object will reach terminal speed. The distance to terminal speed is the area under the curve from time zero through time to maximum on your graph.

Analysis

1. What can you say about objects whose speed vs. time curves are close to the theoretical speed vs. time line?
2. What does the area under your speed vs. time graph represent?
3. If you dropped a large leaf from the Empire State Building, what would its speed vs. time graph look like? How might it differ from that of a baseball?
4. The terminal speed of a falling object is the speed at which it stops accelerating. How could you tell whether an object had reached its terminal speed by glancing at an actual speed vs. time graph?
5. Search references to find the actual equation that models how a small sphere falls in a medium and state this equation with a brief description. Also, note how terminal speed fits into the equation.

Useful Tables

Theoretical Time	Theoretical Velocity
$t = \sqrt{\frac{2d}{g}}$	$v_F = gt$
0	0

Actual Time	Actual Velocity
t	$v_F = \frac{2d}{t}$
0	0