1. Write the MATLAB function $B = \text{badgauss}(A)$ and test this on $A = \text{rand}(5)$.

2. Write the MATLAB function $x = \text{backsub}(A, b)$ and test this on $A = \text{triu}(\text{rand}(5))$, $b = \text{rand}(5, 1)$. Check against MATLAB's $A \backslash b$.

3. Write a MATLAB function $x = \text{mysolve}(A, b)$ which finds the solution to $Ax = b$ using $\text{badgauss}$, and following with $\text{backsub}$. Test your $\text{mysolve}$ on $A = \text{magic}(7)$, $b = A \ast \text{ones}(7, 1)$. Note: You know the true solution to the system $Ax = b$. What is it? Compare your $\text{mysolve}$ answer with the outcome of MATLAB's $A \backslash b$.

4. Write the MATLAB function $B = \text{grmsch}(A)$ and test this on $A = \text{rand}(6, 4)$. Check that $B^T B = I_n$ Which is a better way to check this: $B^T B = \text{eye}(4)$ or $B^T B - \text{eye}(4)$? Why?

We will be using two important matrices. The Givens rotation, $G$, which is defined by five parameters $n, i, j, c, s$, where $n$ is the size, $c$ and $s$ are the cosine and sine of some angle, and $i < j$ specify four locations, $G(i, i), G(i, j), G(j, i), \text{and } G(j, j)$.

$$G = \text{eye}(n); \quad G(i, i) = c; \quad G(j, j) = s; \quad G(i, j) = s; \quad G(j, i) = -s;$$

5. Write the MATLAB function $G = \text{givrot}(n, i, j, c, s)$ which makes the Givens matrix defined above. Check your $\text{givrot}$ using $n = 3$, $i = 1$, $j = 3$, $c = \sin(pi/6)$ and $s = \cos(pi/6)$. Show that $G$ is an orthogonal matrix. Now using $\text{givrot}$ form all possible $3 \times 3$ Givens rotations with $c = \sin(pi/6)$ and $s = \cos(pi/6)$.

The other matrix is the Householder reflection. This matrix depends on an $n \times 1$ column vector $w$ as a parameter and is defined by

$$H_w = I_n - 2/(w^T w)(w \ast w^T)$$

6. Write the MATLAB function $H = \text{house}(w)$ which will make the Householder matrix defined above. Let $w = \text{rand}(5, 1)$

$$H = \text{house}(w)$$

Show the following properties of Householder matrices for this $H$.

1. $H = H^T$
2. $H^2 = I_n$
3. $H$ is orthogonal

7. Write a MATLAB function $p = \text{proj}(A, b)$ which computes the projection of $b$ into Col($A$). You can assume that rank($A$) = $n$. Let $A = \text{rand}(5, 2)$ and $b = \text{ones}(5, 1)$.

1. Is $b \in \text{Col}(A)$? Note: this is equivalent to the question, is $Ax = b$ consistent?
2. Compute $p = \text{proj}(A, b)$. Make sure rank($A$) = $n$.
3. Check that $p$ is the projection of $b$ into $\text{Col}(A)$. This entails showing $p \in \text{Col}(A)$ and $b - p \in \text{Col}(A)^\perp$.

8. Write a MATLAB function $x = \text{lstsq}(A, b)$ which computes the least squares solution by solving the normal equations $A^T A x = A^T b$. You may assume rank($A$) = $n$. Let $A = \text{rand}(5, 2)$ and $b = \text{ones}(5, 1)$ and compute $x = \text{lstsq}(A, b)$. You can compute the residual with norm($A x - b$). Check that $x$ is the least squares solution by showing $A \ast x - b \in \text{Col}(A)^\perp$.

9. MATLAB's solver $A \backslash b$ computes the least squares solution. Compare your $\text{lstsq}$ with $A \backslash b$ on these systems. Which least squares solver gives the better residual $\|Ax - b\|$ in each of these?

1. $A = \text{rand}(6, 4); \quad b = \text{ones}(6, 1)$;
(2) \( A = \text{hilb}(6); A(:,1:4); b = \text{ones}(6,1) \);  
(3) \( A = \text{vander}(1:8); A(:,1:5); b = \text{ones}(8,1) \);

10. Write MATLAB functions
   (1) \( E = \text{ele1}(n, i, j) \) which switches the \( i^{th} \) and \( j^{th} \) rows of an identity matrix \( I_n \)
   (2) \( E = \text{ele2}(n, r, i) \) which multiplies the \( i^{th} \) row of an identity matrix \( I_n \) by \( r \).
   (3) \( E = \text{ele3}(n, r, i, j) \) which multiplies the \( i^{th} \) row of an identity matrix \( I_n \) by \( r \) and adds it to the \( j^{th} \) row.

Let \( A = \text{magic}(5) \). Describe the outcome of each of the following multiplications:
   \( \text{ele1}(5, 2, 4)*A \)
   \( \text{ele2}(5, \pi, 3)*A \)
   \( \text{ele2}(5, -1, 3, 5)*A \)