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Rudyak, Yuli B. (D-HDBG)
★**On Thom spectra, orientability, and cobordism.**
With a foreword by Haynes Miller.
Springer Monographs in Mathematics.
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This book provides an excellent and thorough treatment of various topics related to cobordism. It should become an indispensable tool for advanced graduate students and workers in algebraic topology.

The prerequisites are about three semesters of algebraic topology, including ordinary homology and cohomology and a bit of homotopy theory, including some familiarity with spectral sequences and the Steenrod algebra. The book is not self-contained. For example, it quotes the Bott periodicity theorem and other results about homotopy groups of classifying spaces, providing references to the literature for the proofs.

The three books with which it might most closely be compared are R. M. Switzer's 1975 *Algebraic topology—homotopy and homology* [Springer, New York, 1975; MR0385836 (52 #6695)], R. E. Stong's 1968 *Notes on cobordism theory* [Princeton Univ. Press, Princeton, N.J., 1968; MR0248858 (40 #2108)], and W. S. Wilson's 1982 *Brown-Peterson homology: an introduction and sampler* [Conf. Board Math. Sci., Washington, D.C., 1982; MR0655040 (83j:55005)]. Rudyak's book is more sophisticated and less elementary than Switzer's. They overlap considerably regarding spectra and generalized (co)homology and some aspects of cobordism, but Rudyak goes deeply into issues of phantom maps, orientability theorems (especially K- and KO-orientability), and cobordism with singularities, three of his research interests. These topics are given little, if any, mention in Switzer.

The approach to cobordism in Stong's book is much more computational than Rudyak's. Whereas a major thrust of Stong was the computation of various bordism rings, Rudyak's book offers a bare minimum of computation. For example, Rudyak refers to Stong for an account of the determination of $\pi_* M\mathrm{U}$. Topics such as determination of the spin cobordism ring, which was thoroughly treated in Stong, are not even mentioned by Rudyak.

A number of topics related to cobordism have been developed since the publication of Stong's book and are dealt with thoroughly by Rudyak. These include the relationship with formal groups, cohomology operations in cobordism, Brown-Peterson (BP) theory, Landweber's filtration theorems involving invariant prime ideals, and spectra such as $P(n)$, BP$\langle n\rangle$, and Morava K-theory defined from manifolds with singularities. These topics are all included in Wilson's book, but Rudyak's book contains much more regarding the algebraic topology of manifolds than does Wilson's. Rudyak's book is four or five times as large as Wilson's; it contains much more basic material about spectra, and is much more thorough in all regards.

Chapter 1 gives preliminaries, while the 100-page Chapter 2 deals with spectra and (co)homology theories. The approach to spectra is similar to that of Switzer. The smash product of spectra is not defined (referring to Switzer or Adams), but the properties are stated. The important recent approach to spectra of Elmendorf, Kriz, Mandell, and May is barely mentioned.

Chapter 3 deals with phantom maps. It includes a proof of several versions of the Brown representability theorem, which fits here because of the importance of inverse limits in its proof. Other results proved here are Anderson's condition for the absence of phantom maps and Wilkerson's classification of spaces of the same n-type for all n.

Thom spectra are the topic of the 110-page Chapter 4. This includes construction of classifying spaces, proofs that certain universal Thom spectra are graded Eilenberg-Mac Lane spectra, a detailed discussion of the Pontryagin-Thom theorem relating cobordism and homotopy groups of Thom spectra, and theorems about realizing homology classes by various types of manifolds. Great care is taken to explain the concept of structures on a bundle.

Next come two chapters on orientability, especially K- and KO-orientability. One includes basic results such as the Thom isomorphism for E-oriented bundles, Poincaré duality for E-oriented manifolds, and classifying spaces for E-oriented bundles. An emphasis here is on the question of which Postnikov invariants of a spectrum E can appear as the obstruction to orientability for some bundle (vector, PL, topological, or spherical). This work appeared originally in several of Rudyak's papers.

A chapter on complex cobordism presents mostly basic material, such as appears in Wilson's book. There is a nice section on Steenrod-tom Dieck operations. The expressed purpose of this chapter is to present facts to be used in the final two chapters, which deal with cobordism with singularities.

One of these chapters concentrates on the geometric theory of manifolds with singularities, while the second concentrates on the various spectra, such as $P(n)$ and BP$\langle n\rangle$, that can be obtained as cobordism of complex manifolds with certain types of singularities. Results of Johnson and Wilson relating homological dimension of $\mathrm{BP}_*(X)$ to the structure of $\mathrm{BP}\langle n\rangle_*(X)$ are proved. A tie-in with earlier sections consist of results about which Postnikov invariants of Morava K-theory can appear as obstructions to orientabilty for bundles of various types.

The last chapter includes exactness theorems, due to Landweber and the author, characterizing the BP_*/I_m-modules M for which $P(m)_*(-)\otimes_{\mathrm{BP}_*/I_m} M$ is a homology theory. There is also a nice characterization of Morava K-theories, showing them independent of the precise singularities used to define them, and showing how they are completely characterized by their formal group.

In addition to his research interests, the author also puts his personality into the book, offering his opinions about some unsolved problems. He discusses the historical development of various topics, and is careful in his attributions of theorems and ideas.

Reviewed by *Donald M. Davis*