Show all work. Answers given with incomplete reasoning will not receive full credit.

Question 1 (2 points) State Rolle's Theorem. Then, show that the function

\[ f(x) = x^2 - 3x \]

on the interval \([0, 3]\) satisfies the hypotheses of Rolle's Theorem. Finally, find a number \(c\) in \([0, 3]\) that satisfies the conclusion of Rolle's Theorem.

**Statement:** If \(f(x)\) is continuous on \([a, b]\), differentiable on \((a, b)\), and \(f(a) = f(b)\), then there is some \(c\) in \((a, b)\) with \(f'(c) = 0\).

Here, \(f(x)\) is a polynomial, so \(f(x)\) is continuous on \([0, 3]\), differentiable on \((0, 3)\), and \(f(0) = 0 = f(3)\). Thus, \(f'(x) = 2x - 3 = 0\), then \(x = \frac{3}{2}\), which is in \((0, 3)\).

Question 2 (2 points) For the function

\[ f(x) = x^2e^x \]

determine the intervals on which \(f(x)\) is increasing or decreasing as well as the \(x\)-coordinates of any local maximums or local minimums.

\[ f'(x) = x^2e^x + 2xe^x = xe^x(x + 2) = 0 \Rightarrow x = 0, -2 \]

Thus, \(f(x)\) is increasing on \((-\infty, -2)\) and \((0, \infty)\) and decreasing on \((-2, 0)\). Also, \(x = -2\) is a local max and \(x = 0\) is a local min.
Question 3 (2 points) Find \( \lim_{x \to \infty} \frac{\ln^2 x}{1 + \ln x} \).

Be sure to carefully justify each of your steps.

Let \( L = \lim_{x \to \infty} \frac{\ln^2 x}{1 + \ln x} \). Then we have

\[
\ln(L) = \lim_{x \to \infty} \ln \left( x \frac{\frac{\ln^2 x}{1 + \ln x}}{x} \right)
\]

\[
= \lim_{x \to \infty} \frac{\ln^2 x}{1 + \ln x}
\]

\[
= \frac{\infty}{\infty}, \quad \text{so use L'Hôpital's Rule}
\]

\[
\lim_{x \to \infty} \frac{\ln^2 x}{1 + \ln x} = \lim_{x \to \infty} \frac{\frac{2 \ln x}{1 + \ln x} \times x}{1}
\]

\[
= \lim_{x \to \infty} \frac{2 \ln x}{1 + \ln x}
\]

\[
= \ln^2 2 = \ln 2
\]

Thus, \( \ln(L) = \ln(2) \), so \( e^{\ln(L)} = e^{\ln(2)} \)

\[
= e^L = 2
\]