Show all work. Answers given with incomplete reasoning will not receive full credit.

Question 1 (2 points) State Rolle's Theorem. Then, show that the function

\[ f(x) = x^2 + 2x - 3 \]

on the interval \([-3, 1]\) satisfies the hypotheses of Rolle's Theorem. Finally, find a number \(c\) in \([-3, 1]\) that satisfies the conclusion of Rolle's Theorem.

**Statement:** If \( f(x) \) is continuous on \([a, b]\), differentiable on \((a, b)\), and \( f(a) = f(b) \), then there is some \( c \in (a, b) \) with \( f'(c) = 0 \).

Here, \( f(x) \) is a polynomial, \( f(x) \) is continuous on \([-3, 1]\), differentiable on \((-3, 1)\), and \( f(-3) = 0 = f(1) \).

Differentiating \( f(x) = x^2 + 2x - 3 \), we get \( f'(x) = 2x + 2 = 0 \), then \( x = -1 \), which is in \((-3, 1)\).

Question 2 (2 points) For the function \( f(x) = e^{-x^2} \), determine the intervals on which \( f(x) \) is increasing or decreasing as well as the \( x \)-coordinates of any local maximums or local minimums.

\[ f'(x) = e^{-x^2} \cdot (-2x) = -2x \cdot e^{-x^2} = 0 \]

\( \Rightarrow \) \( x = 0 \)

Thus, \( f \) is increasing on \((-\infty, 0)\) and decreasing on \((0, \infty)\). Also, \( x = 0 \) is a local max.
Question 3 (2 points) Find \( \lim_{x \to 0^+} (4x + 1)^{\cot x} \)

Be sure to carefully justify each of your steps.

Let \( L = \lim_{x \to 0^+} (4x + 1)^{\cot x} \). Then we have

\[
\ln(L) = \lim_{x \to 0^+} \ln \left( (4x + 1)^{\cot x} \right)
\]

\[
= \lim_{x \to 0^+} \cot x \ln(4x + 1)
\]

\[
= \lim_{x \to 0^+} \frac{\ln(4x + 1)}{\tan x} = \frac{0}{0} \text{ use L'Hopital's Rule}
\]

\[
\lim_{x \to 0^+} \frac{\ln(4x + 1)}{\tan x} = \lim_{x \to 0^+} \frac{4}{4x + 1} = \frac{4}{1} = 4
\]

Thus, \( \ln(L) = 4 \), so \( e^{\ln(L)} = e^4 \)

\[
\Rightarrow L = e^4
\]

15