Show all work. Answers given with incomplete reasoning will not receive full credit.

Question 1 (2 points) Find the equation of the tangent line to the curve \[ f(x) = \frac{1}{x^2 + 1} \]

at the point \((1, \frac{1}{2})\).

\[
f'(x) = \frac{(x^2 + 1)' \cdot 0 - 1 \cdot (2x)}{(x^2 + 1)^2} = \frac{-2x}{(x^2 + 1)^2} \tag{1}
\]

\[
f'(1) = \frac{-2 \cdot 1}{(1^2 + 1)^2} = \frac{-2}{4} = \frac{-1}{2} \tag{5}
\]

Use point-slope form with \(m = \frac{-1}{2}, \quad P = (1, \frac{1}{2})\):

\[y - \frac{1}{2} = \frac{-1}{2} (x - 1) = \frac{-1}{2} x + \frac{1}{2} \]

\[\therefore \quad y = \frac{-1}{2} x + 1 \tag{5}\]

Question 2 (2 points) Evaluate the limit

\[
\lim_{x \to 0} \frac{\sin(x^2)}{x}
\]

\[= \lim_{x \to 0} \frac{0 \cdot \sin(x^2)}{x} \tag{5}\]

\[= \lim_{x \to 0} \frac{x \cdot \sin(x^2)}{x^2} = \lim_{x \to 0} x \cdot \lim_{x \to 0} \frac{\sin(x^2)}{x^2} \tag{5}\]

\[= 0 \cdot 1 = 0 \tag{1}\]
Question 3 (2 points) Find the derivative of the function

\[ f(x) = (e^x + x^3)^5 \]

Let \( u(x) = e^x + x^3 \). Then,

\[ f(x) = u^5 \]

\[ u(x) = e^x + x^3 \]

\[ u'(x) = e^x + 3x^2 \]

\[ f'(x) = 5u^4 \cdot u'(x) \]

Therefore,

\[ f'(x) = 5(e^x + x^3)^4 \cdot (e^x + 3x^2) \]

\[ = (5e^x + 15x^2)(e^x + x^3)^4 \]