Name: Key

Score:

Show all work. Answers given with incomplete reasoning will not receive full credit.

Question 1 (2 points) Find the solution set to the inequality \( \frac{2-x}{3x+2} \geq 2 \)

\[
\frac{2-x}{3x+2} - 2 \geq 0
\]

Critical points: \( x = -\frac{2}{3}, -\frac{2}{7} \)

\[
\frac{2-x}{3x+2} - \frac{2(3x+2)}{3x+2} \geq 0
\]

\[
\frac{2-x-6x-4}{3x+2} \geq 0
\]

\[
\frac{-7x-2}{3x+2} \geq 0
\]

So, \( \left(-\frac{2}{3}, -\frac{2}{7}\right] \)

Question 2 (2 points) Solve the equation \( 2x = 1 - \sqrt{2-x} \) for \( x \)

\[
\sqrt{2-x} = 1-2x
\]

\[
(\sqrt{2-x})^2 = (1-2x)^2
\]

\[
x = 4x^2 - 4x + 1
\]

\[
4x^2 - 3x - 1 = 0
\]

\[
(4x+1)(x-1) = 0
\]

So, \( x = -\frac{1}{4}, 1 \)

Check \( x = -\frac{1}{4} \):

\[
2 \cdot \left(-\frac{1}{4}\right) = 1 - \sqrt{2 - \left(-\frac{1}{4}\right)}
\]

\[
-\frac{1}{2} = 1 - \sqrt{\frac{9}{4}}
\]

\[
-\frac{1}{2} = 1 - \frac{3}{2} = -\frac{1}{2}
\]

Check \( x = 1 \):

\[
2 \cdot 1 = 1 - \sqrt{2-1}
\]

\[
1 = 1 - \sqrt{1} = 0
\]

So, \( x = -\frac{1}{4} \) is the only solution.
Question 3 (2 points) If \( f(x) = \frac{2x}{x+1} \), find and simplify the difference quotient \( \frac{f(x+h) - f(x)}{h} \)

\[
\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)}{(x+h)+1} - \frac{2x}{x+1}
\]

\[
= \frac{2x + 2h}{x+h+1} - \frac{2x}{x+1}
\]

\[
= \frac{(2x+2h)(x+1) - 2x(x+h+1)}{(x+1)(x+h+1)}
\]

\[
= \frac{2x^2 + 2x + 2hx + 2h - 2x^2 - 2xh - 2x}{(x+1)(x+h+1)}
\]

\[
= \frac{2h}{(x+1)(x+h+1)}
\]

Thus, \( \frac{f(x+h) - f(x)}{h} = \frac{2h}{(x+1)(x+h+1)} \)