Name: Key

Score:

Show all work. Answers given with incomplete reasoning will not receive full credit.

Question 1 (2 points) Evaluate the integral

$$\int_{-2}^{0} (x + \sqrt{4 - x^2}) \, dx$$

by interpreting it in terms of areas.

$$= \int_{-2}^{0} x \, dx + \int_{-2}^{0} \sqrt{4 - x^2} \, dx = (-2) + (\pi) = \pi - 2 \quad (5)$$

Question 2 (2 points) If

$$h(x) = \int_{1}^{3x+2} \frac{1}{1+t^2} \, dt$$

find $h'(x)$.

By FTC,

$$h'(x) = \frac{d}{dx} \left[ 3x+2 \right] \left( \frac{1}{1+(3x+2)^2} \right)$$

$$= (3) \left( \frac{1}{9x^2+12x+5} \right)$$

$$= \frac{3}{9x^2+12x+5} \quad (5)$$
Question 3 (2 points) Evaluate the limit by first recognizing the sum as a Riemann sum for a function defined on [0, 1]

\[
\lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{1}{n} \left( \sqrt{\frac{i}{n}} + 1 \right) \right)
\]

\[
= \lim_{n \to \infty} \frac{1}{n} \left( \sum_{i=1}^{n} \sqrt{\frac{i}{n}} \cdot \frac{1}{n} + \sum_{i=1}^{n} \frac{1}{n} \right)
\]

\[
\Delta x = \frac{1 - 0}{n} = \frac{1}{n}
\]

\[
x^* = 0 + i \Delta x = \frac{i}{n}
\]

\[
f(x^*) = \sqrt{\frac{i}{n}} \to \infty
\]

\[
f(x) = 5x
\]

\[
= \int_{0}^{1} 5x \, dx + \int_{0}^{1} 1 \, dx
\]

\[
= \frac{2}{3} x^{3/2} \bigg|_{0}^{1} + x \bigg|_{0}^{1}
\]

\[
= \left( \frac{2}{3} - 0 \right) + (1 - 0)
\]

\[
= \frac{2}{3} + 1 = \sqrt{\frac{5}{3}}
\]