Show all work. Answers given with incomplete reasoning will not receive full credit.

Question 1 (2 points) Calculate the following limit, if it exists. If the limit does not exist, explain why.

\[
\lim_{x \to 4} \frac{x^2 - 3x - 4}{|x - 4|} = \lim_{x \to 4^-} \frac{(x-4)(x+1)}{-(x-4)} = \lim_{x \to 4^-} -(x+1) = -5
\]

The limit DNE so the LH and RH limits do not agree.

Question 2 (2 points) Sketch a graph of a function:

a.) \( f(x) \) such that \( \lim_{x \to a} f(x) \) exists but \( \lim_{x \to a} f(a) \) does not exist

b.) \( g(x) \) such that \( g \) is continuous from the right at \( b \) but \( g \) is not continuous at \( b \)
Question 3 (2 points) Let

\[ f(x) = \begin{cases} 
-(x - 2)^2 + 8 & \text{if } x < 2 \\
x^3 & \text{if } x \geq 2 
\end{cases} \]

Find \( \lim_{x \to 2^-} f(x) \) and \( \lim_{x \to 2^+} f(x) \). Is \( f(x) \) continuous at \( x = 2 \)? Explain why or why not.

\[
\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} -(x-2)^2 + 8 = -(2-2)^2 + 8 = 8 \quad \text{(5)}
\]

\[
\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^3 = (2)^3 = 8 \quad \text{(5)}
\]

\( f(x) \) IS continuous at \( x = 2 \) since the above shows that \( \lim_{x \to 2^-} f(x) = 8 \) AND because \( f \) is defined at \( x = 2 \) and we have \( f(2) = 2^3 = 8 = \lim_{x \to 2} f(x) \).