Show all work. Answers given with incomplete reasoning will not receive full credit.

Question 1 (2 points) Calculate the following limit, if it exists. If the limit does not exist, explain why.

\[
\lim_{x \to -1} \frac{x^2 - 6x - 7}{|x + 1|}
\]

The limit DNE as the LHT and RH limits do not agree.

Question 2 (2 points) Sketch a graph of a function:
   a.) \( f(x) \) such that \( \lim_{x \to a} f(x) \) exists but \( f \) is not continuous at \( a \)
   b.) \( g(x) \) such that \( g \) is continuous from the right at \( b \) but not from the left at \( b \)
Question 3 (2 points) Let

\[ f(x) = \begin{cases} 
\frac{x + 3}{|x + 3|} & \text{if } x < -3 \\
-4(x + 4)^2 + 4 & \text{if } x \geq -3 
\end{cases} \]

Find \( \lim_{x \to -3^-} f(x) \) and \( \lim_{x \to -3^+} f(x) \). Is \( f(x) \) continuous at \( x = -3 \)? Explain why or why not.

\[
\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \frac{|x+3|}{x+3} = \lim_{x \to -3^-} -(x+3) = -(-3+3) = 0 \quad \left( \frac{1}{2} \right)
\]

\[
\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} -4(x+4)^2 + 4 = -4(-3+4)^2 + 4 = -4 \cdot 1 + 4 = 0 \quad \left( \frac{1}{2} \right)
\]

\( f(x) \) is continuous at \( x = -3 \) since the above shows that \( \lim_{x \to -3} f(x) = 0 \) AND because \( f(x) \) is defined at \( x = -3 \) and we have

\[
f(-3) = -4(-3+4)^2 + 4 = 0 = \lim_{x \to -3} f(x) \quad (1)
\]