1. Suppose \( a \in \mathbb{Z} \). Prove, if \( 5 \mid 2a \), then \( 5 \mid a \).

2. Suppose \( x \in \mathbb{R} \). Prove, if \( x^5 + 7x^3 + 5x \geq x^4 + x^2 + 8 \), then \( x \geq 0 \).

3. Prove \( \sqrt{5} \) is not rational.

4. Prove: If \( k \in \mathbb{Z} \), then \( \{ n \in \mathbb{Z} : n \mid k \} \subset \{ n \in \mathbb{Z} : n \mid k^2 \} \).

Part B. Do one.

For these questions you may, or not, want to use the following Theorem.

**Theorem.** If \( a, b \in \mathbb{N} \), then there exists \( k, \ell \in \mathbb{Z} \) such that \( ak + b\ell = \gcd(a, b) \).

Moreover,

\[
\gcd(a, b) = \min\{ m \in \mathbb{N} : \exists s, t \in \mathbb{Z} \text{ such that } m = as + bt \}.
\]

(a) Suppose \( a, b, c \in \mathbb{Z} \). Show, if \( a \mid bc \) and \( \gcd(a, b) = 1 \), then \( a \mid c \).

(b) Prove, if \( n \in \mathbb{Z} \), then \( \gcd(2n + 1, 4n^2 + 1) = 1 \). (Suggestion: First use \( 4n^2 + 1 = (2n - 1)(2n + 1) \) to show the gcd is either 1 or 2.)