The Cyclic Behavior of Cosubnormal Operators

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John Conway Day
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- 1998: Feldman: All pure SNOs have cyclic adjoints!

Theorem (Feldman '98) Every Pure SNO has a cyclic adjoint.

Strategy of Proof:

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Strategy of Proof: Show $\exists$ a one-to-one linear map

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A Multiplicity Theory? If S is a pure SNO, does $\exists$ a 1-1 map A s.t.

$$(S, \mathcal{H}) \xrightarrow[1-1]{A} (M_z, \mathcal{H}_1 \underset{pure}{\subseteq} L^2(\mu_1))?$$

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If $x \in \mathcal{H}$ and $T \in \mathcal{B}(\mathcal{H})$, then the orbit of x under T is

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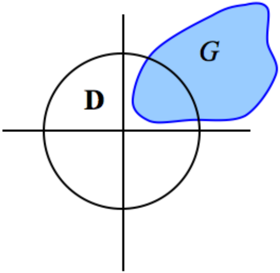
Some Hypercyclic Operators

Theorem (G. Godefrey & J. Shapiro (1991))

If G is a bounded region in $\mathbb{C}$, then M_z^* is hypercyclic on $H^2(G)$ or $L_a^2(G)$ if and only if $G \cap \partial\mathbb{D} \neq \emptyset$.



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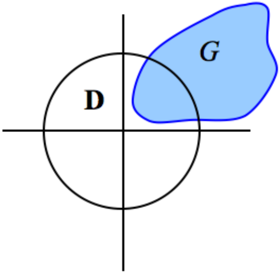


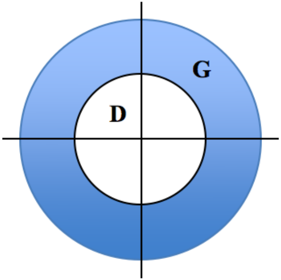
Corollary

If G is any bounded region in $\mathbb{C}$, then M_z^* is supercyclic on $H^2(G)$ or $L_a^2(G)$.

Weakly Hypercyclic Operators

Theorem (K. Chan & R. Sanders (2002))

If $G = \{z \in \mathbb{C} : 1 < |z| < r\}$, then M_z^* on $H^2(G)$ is weakly hypercyclic, but not norm hypercyclic.



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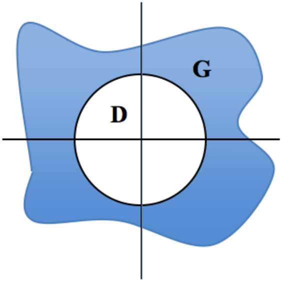

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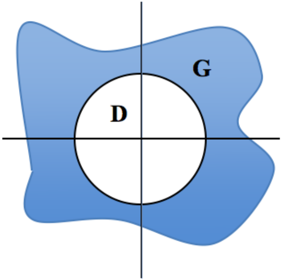
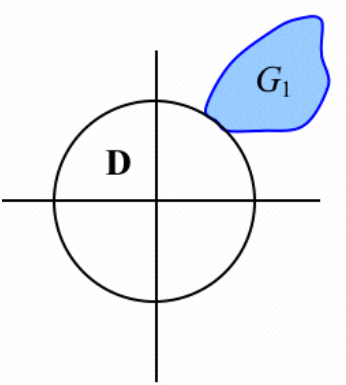


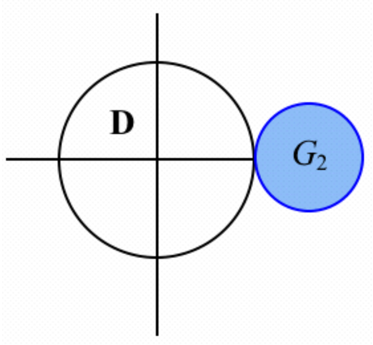
Open Question

For which open sets G is M_z^* weakly hypercyclic on $H^2(G)$?

Weakly Hypercyclic?

What if...





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The Weak Topology

$$X = \text{Banach Space}$$

A basis for the weak topology on X

$$N(x_0, \mathcal{F}, \epsilon) = \{x \in X : |f(x - x_0)| < \epsilon \text{ for all } f \in \mathcal{F}\}$$

where $\mathcal{F} \subseteq X^*$ is a finite set

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A set $E \subseteq X$ is *n-weakly dense* in X if $E \cap N(x_0, \mathcal{F}, \epsilon) \neq \emptyset$

$\forall x_0 \in X, \epsilon > 0,$ and all finite sets $\mathcal{F} \subseteq X^*$ with $|\mathcal{F}| \leq n$

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Definition

1. An operator T is *n*-weakly hypercyclic if $\exists\, x \in \mathcal{H}$ such that $Orb(x, T)$ is n-weakly dense in $\mathcal{H}$.
2. T is *n*-weakly supercyclic if $\exists\, x \in \mathcal{H}$ such that $\mathbb{C} \cdot Orb(x, T)$ is n-weakly dense in $\mathcal{H}$.

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Suppose that $B_1, B_2, \ldots, B_m$ *are each hypercyclic backward weighted shifts and* $1 \leq n \leq m$.

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Backward Unilateral Weighted Shifts

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Corollary (Feldman 2010)

There exist operators that are n-weakly hypercyclic, but not $(n+1)$-weakly hypercyclic, for any $n \geq 1$.

The Matrix Case

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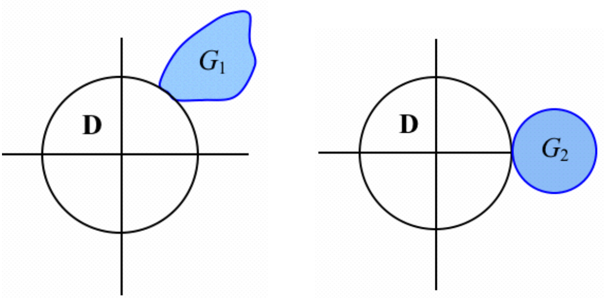
If $\{\pi, \theta_1, \theta_2, \ldots, \theta_n\}$ are linearly independent over $\mathbb{Q}$, then $T = R(\theta_1) \oplus R(\theta_2) \oplus \cdots \oplus R(\theta_n)$ is 2-weakly supercyclic on $\mathbb{R}^{2n}$, where $R(\theta)$ is the 2×2 matrix that rotates by θ.

1-Weakly Hypercyclic?



Open Question

Is M_z^* 1-weakly hypercyclic on $H^2(G)$?

Thanks for your Time!

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Best wishes to John!