MAA 4212 Exam 1, Section 16G0, Fall 2014

1. (40 points) Define the following terms with precision and in a form suitable for using in a proof.

   (i) The *greatest lower bound* of a subset $S \subset \mathbb{R}$.

   (ii) The *closure* of a subset S of a metric space X.

   (iii) The *inverse image,* $f^{-1}(E)$, of a subset E of a set Y under a mapping $f : X \to Y$.

   (iv) A *metric* d on a set X.

2. (30 points) Do three of four. In each case, give an example, if possible.

   (a) A bounded subset S of $\mathbb{R}$ which has no least upper bound.

   (b) Nonempty sets $A, B \subset \mathbb{R}$ such that for each $a \in A$ there is a $b \in B$ such that $a < b$, but for which A is not bounded above and B is not bounded below.

   (c) Sets X, Y and $C \subset X$ and a function $f : X \to Y$ such that $f^{-1}(f(C)) \neq C$.

   (d) A metric space X a point $p \in X$ and $r > 0$ such that the closure of $N_r(p)$ is not $B_r(p)$.

3. (30 points) Do one of two.

   (a) Let (X, d) be a metric space and suppose $p \in X$ and $r \geq 0$. Prove that the set

   $$B_r(p) = \{x \in X : d(x, p) \leq r\}$$

   is a closed set.

   (b) Suppose $S \subset \mathbb{R}$ is nonempty, bounded below and that $t \in \mathbb{R}$. Show that

   $$T = S + t = \{x \in \mathbb{R} : \text{ there exists } s \in S \text{ such that } x = s + t\}.$$

   has a greatest lower bound and moreover $\inf(T) = \inf(S) + t$.