MAP 2302 Sample Exam 1, Fall 2017

1. Classify each as an ordinary differential equation (ODE) or a partial differential equation (PDE), give the order, and indicate the independent and dependent variables. If the equation is an ordinary differential equation, indicate whether the equation is linear or nonlinear.

   (a) $y'' + xy' + \sin(x)y = \cos(x)$.

   (b) $\frac{\partial^2 u}{\partial x^2} - c\frac{\partial^2 u}{\partial t^2} = 0$. (The wave equation.)

   (c) $\frac{dp}{dt} = 5p(2 - p)$.

2. In each case determine if the given function ϕ is a solution to the given ODE.

   (a) $\phi(x) = x^2 - x^{-1}$ and the ODE $y^2\frac{d^2y}{dx^2} = 2y$.

   (b) $\phi(x) = e^x$ and the ODE $y'' + 2y' - 4y = 0$.

3. In each determine if the equation $G(x, y) = 0$ determines a solution to the given ODE.

   (a) $G(x, y) = e^{xy}y + x - y + 1$ and the ODE $\frac{dy}{dx} = \frac{e^{-xy} - y}{e^{-xy} + x}$.

   (b) $G(x, y) = x^2 + y^2 - 4$ and the ODE $\frac{dy}{dx} = \frac{x}{y}$.

4. Give a sketch of the isoclines and direction field for $y' = x - y$. Give enough detail to give a rough sketch of the solutions with initial conditions $y(0) = 0$ and $y(0) = 1$. Explain carefully why the graphs of these two solutions can not intersect.

5. Apply the method of Euler with $h = .1$ to the initial value problem $y' = x - y$ and $y(0) = 0$ to approximate $y(.2)$.