Derivational points of Banach bimodules

Z. Patrick Pan Saginaw Valley State University

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Introduction

Derivable maps

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- Clearly, a derivation is derivable at every $c \in \mathcal{A}$.
- Question: If δ is derivable at a fixed $c \in A$, what can one say about δ ? (Sub-question: Why should one care?)

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- helps better understand the structure of the underlying algebra.
- provides simpler mechanisms for construction of these maps.

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- For any m ∈ A', let L_m : A → M be L_m(a) = ma, ∀ a ∈ A then L_m is derivable at c = 0.

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Various results in the literature

Under certain conditions on δ , A, and M, if δ is derivable at 0 then there exists a derivation d from A to M such that $\delta = d + L_m$, where $m = \delta(1) \in A'$.

Image: A = A

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Theorem 1 (Alaminos, Bresar, Extremera, and Villena, 2007)

Let δ be a bounded linear map from a unital C^* -algebra \mathcal{A} to a unital Banach \mathcal{A} -bimodule \mathcal{M} . If δ is derivable at 0 then there exists a derivation d from \mathcal{A} to \mathcal{M} such that $\delta = d + L_m$, where $m = \delta(1) \in \mathcal{A}'$.



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Theorem 2 (J. Li and Z. Pan, 2010)

Let δ be a bounded linear map from a *CSL*-algebra \mathcal{A} in a von Neumann algebra on a Hilbert space H. If δ is derivable at 0 then there exists a derivation d from \mathcal{A} to $\mathcal{B}(H)$ such that $\delta = d + L_m$, where $m = \delta(1) \in \mathcal{A}'$.

An embedding

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- Define $\phi : \mathcal{A} \mapsto \mathcal{B}$ by $\phi(a) = \begin{bmatrix} a & \delta(a) \\ 0 & a \end{bmatrix}, \forall a \in \mathcal{A}.$

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• δ is a derivation iff ϕ is a homomorphism.

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Derivable at c = 0 and zero-product preserving maps

Definition:

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, defined by $\phi(a) = \begin{bmatrix} a & \delta(a) \\ 0 & a \end{bmatrix}$.

• δ is derivable at c = 0 iff ϕ is zero-product preserving.

Various results in the literature

Under certain conditions, if ϕ is a zero-product preserving map from a unital algebra \mathcal{A} to an algebra \mathcal{B} then there exists a homomorphism Φ from \mathcal{A} to \mathcal{B} such that

 $\phi(a) = \phi(1)\Phi(a) \ \forall \ a \in \mathcal{A}.$

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- If δ is a Jordan derivation and \mathcal{M} is a unital \mathcal{A} -bimodule then $\delta(1) = 0$.

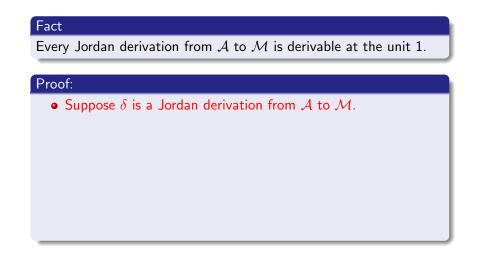
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- If δ is a Jordan derivation then $\delta(bab) = \delta(b)ab + b\delta(a)b + ba\delta(b)$, for all $a, b \in A$.
- There are many papers in the literature studying when Jordan derivations are derivations (dating back to 1957, I.N. Herstein and I. Kaplansky)

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- Suppose δ is a Jordan derivation from \mathcal{A} to \mathcal{M} .
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• It follows $b(\delta(a)b + a\delta(b)) = 0$.

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Theorem 3 (J. Li and Z. Pan, 2011)

Suppose $\forall \{x : \forall x \otimes f^* \in A\} = X$ and δ is a linear map from A to B(X). If δ is derivable at $C \in A$ and ran(C) is dense in X then δ is a derivation.

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Corollary:

If $\forall \{x : \forall x \otimes f^* \in A\} = X$ then every Jordan derivation from A to B(X) is a derivation.

Algebras that satisfy $\lor \{x : \forall x \otimes f^* \in \mathcal{A}\} = X$ include

• Standard algebras

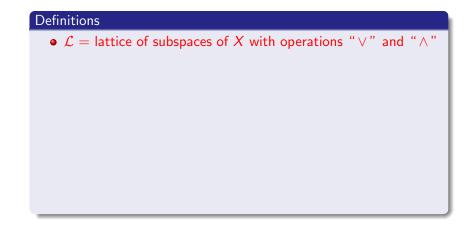


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$$L_- = \lor \{ M \in \mathcal{L} : L \not\subseteq M \}.$$

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• Define $\mathcal{J}(\mathcal{L}) = \{L \in \mathcal{L} : L \neq 0 \text{ and } L_{-} \neq X\}$, i.e.

 $\mathcal{J}(\mathcal{L})$ is the set of L with non-trivial predecessors.

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Lattices that satisfy $\forall \{E : E \in \mathcal{J}(\mathcal{L})\} = X$ include

completely distributive subspace lattice *L*, in particular a nest.
 (*L* is completely distributive iff *E* = ∧{*L*_− : *L* ∈ *L*, *L* ⊈ *E*}.)

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- **3** \mathcal{L} that satisfies $X_{-} \neq X$.
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- **2** \mathcal{J} -subspace lattices.
- 3 \mathcal{L} that satisfies $X_{-} \neq X_{-}$
- \mathcal{L} that satisfies $\wedge \{L_{-} : L \in \mathcal{J}(\mathcal{L})\} = (0).$
- **(a)** \mathcal{L} that satisfies $(0)_+ \neq (0)$.

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Theorem 3 says:

If $\forall \{x : \forall x \otimes f^* \in A\} = X$ then every $C \in A$ such that ran(C) is dense in X is a derivational point of L(A, B(X)).

Characterization of derivable maps on nest algebras

Theorem 4:

If A is a nest algebra on a Hilbert space H then every $0 \neq C \in A$ is a derivational point of L(A, B(H)).



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If A is a nest algebra on a Hilbert space H then every $0 \neq C \in A$ is a derivational point of L(A, B(H)).

Corollary:

Suppose δ is a linear map from a nest algebra \mathcal{A} on a Hilbert space H to B(H) and δ is derivable at $C \in \mathcal{A}$.

(i) If C = 0 then there exists a derivation d from A to B(H) and a scalar λ such that $\delta = d + \lambda I$.

(ii) If $C \neq 0$ then δ must be a derivation.

Thank you for your attention!

