MAA4211 Sample Exam I, Fall 2012

(1) (i) Given $f : X \to Y$ and $B \subset Y$, define $f^{-1}(B)$, the *inverse image* of B under f;

(ii) Define the *least upper bound* of a subset S of $\mathbb{R}$;

(iii) Define *open set* U in a metric space X;

(iv) For sets A and B, define A *is equivalent to* B.

(2) (a) Give an example, if possible, of a subset S of $\mathbb{R}$ which is bounded above but has no least upper bound;

(b) Give an example, if possible, of a subset S of $\mathbb{R}$ which has a least upper bound α, but $\alpha \notin S$;

(c) Give an example, if possible, of a function $f : X \to Y$ and subsets $A, B \subset X$ such that $f(A \cap B) \neq f(A) \cap f(B)$;

(d) Give an example, if possible, of an onto mapping $f : \mathbb{N} \to P(\mathbb{N})$.

(3) Do one of the following.

(i) Prove carefully, if $A \subset B$ are nonempty bounded subsets of $\mathbb{R}$, then $\sup(A) \leq \sup(B)$;

(ii) Prove carefully that a (open) neighborhood $N_\delta(x)$ in a metric space X is an open set.