SEAM 38, 2021

Abstracts

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The range of the Berezin transform and Brown-Halmos type theorems on the upper half-plane

Farouq N. Alshormani
King Saud University

In the framework of the complex upper half-plane, we investigate the range of the Berezin transform and aim to establish a Brown-Halmos type theorem for Toeplitz operators with harmonic symbols on the Bergman space. This famous theorem consists in characterizing when the product of two Toeplitz operators with bounded harmonic symbols on the Bergman space is again a Toeplitz operator whose symbol is the product of their symbols. The original Hardy space Brown-Halmos theorem is due to A. Brown and P.R. Halmos [5]. A more involved case is related to the Bergman space situation, and has been considered by P. Ahern and Ž. Čučković [2]. P. Ahern [1] studied the range of the Berezin transform, which extends the Bergman space Brown-Halmos theorem as a wonderful consequence. N.V. Rao [11] presented an alternative method based on distribution theory and Luecking's [10] characterization of finite rank Toeplitz operators, see also [3, 12]. Many interesting results in this direction have been obtained in [4, 6, 7, 8, 9], and the references therein. In this talk, we make use of the Berezin transform techniques, Luecking's characterization of finite rank Toeplitz operators, to establish the analogue of these results in the framework of the upper half-plane, and we further generalize some of them to a finite sum of products of two Toeplitz operators.

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Matricial Archimedean Order Unit Spaces and Quantum Correlations

Roy M. Araiza

Purdue University

During this talk I will introduce the notion of a k-AOU space, which we may think of as a matricial Archimedean order unit space. I will then describe the relationship between the category of k-AOU spaces and k-positive maps, and the category of operator systems and completely positive maps. Our goal will be to see that quantum correlations are completely determined by actions of states on generators, of a particular class of k-AOU spaces. Along with previous work, this yields a reformulation of Tsirelson's Conjecture.

An optimal approximation problem for noncommutative polynomials

Palak Arora

University of Florida

Motivated by recent work on optimal approximation by polynomials in the unit disk, we consider the following noncommutative approximation problem: for a polynomial f in d noncommuting arguments, find an nc polynomial p_n, of degree at most n, to minimize

$$c_n := \|p_n f - 1\|^2.$$

(Here the norm is the ℓ^2 norm on coefficients.) We show that $c_n \to 0$ if and only if f is nonsingular in a certain nc domain (the *row ball*). As an application we give a new, elementary, proof of a theorem of Jury, Martin, and Shamovich on cyclic vectors for the d-shift. (This is joint work with M. Augat, M. Jury, and M. Sargent.)

A survey of optimal polynomial approximants

Catherine A. Bénéteau

University of South Florida

In the last few years, the notion of optimal polynomial approximant has appeared in the mathematics literature in connection with Hilbert spaces of analytic functions of one or more variables. In the 70s, researchers in engineering and applied mathematics introduced least squares inverses in the context of digital filters in signal processing, and these turn out to be the same objects in say the Hardy space of the disk H^2. Recently, several young mathematicians have been working on extending these objects to other settings. In this talk, I will give an overview of known results, focusing in particular on zeros and convergence, and will discuss some open problems.

Noncommutative rationals, their skew fields, and Jacobian matrices

Meric L. Augat

Washington University in St. Louis

Contemporary advances in Free Analysis have yielded remarkable analogs of classical theorems in analysis, algebra and geometry. In particular, the Free Inverse Function Theorem has a stronger conclusion than its classical counterpart while the Free Jacobian Conjecture is true and its proof is quite tractable.

Recently, there has been success in tying algebraic results about noncommutative (nc) polynomials to evaluation conditions on the polynomials. A natural byproduct is to ask whether an evaluation condition exists for nc rational functions as well. Specifically, given a nc rational mapping (a g-tuple of nc rationals) in g freely noncommuting variables, we ask whether there exists an injectivity condition that guarantees the mapping has a nc rational inverse.

In this talk we generalize a Jacobian conjecture for endomorphisms of the free algebra to endomorphisms of the free skew field. Motivated by the positive results in the algebraic setting, we ask whether evaluations on matrices can detect when a given nc rational is contained in the skew field generated by a tuple of nc rationals.

On the boundedness of oscillating singular integrals

Duvan Cardona

Ghent University

It was proved by Fefferman [1] and Fefferman and Stein [2], the weak (1,1) boundedness of oscillating singular integrals on Rn, and the boundedness from the Hardy space H1 into L1, respectively. The aim of this talk is to discuss the recent extension of these results in the Euclidean setting and on Lie groups of polynomial growth. In view of the solution of the Rockland conjecture by Helffer and Nourrigat [3], and of the Hormander theorem of sums of squares [4], our criteria are presented in terms of the analysis of sub-Laplacians and of Rockland operators. Joint work with Michael Ruzhansky.

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Zeros of optimal polynomial approximants in ℓ^p_A

Ray Cheng

Old Dominion University

The study of inner functions and cyclic vectors in ℓ^p_A spaces requires a better understanding of the roots of optimal polynomial approximants (OPAs).

The main result is that a point of the complex plane is the root of an OPA for some element of ℓ^p_A if and only if it lies outside a closed disk (centered at the origin) of a certain radius, which depends on the value of p. It was known that when $p = 2$, a nontrivial OPA has no roots in the unit disk. However, this radius is less than 1 when $p \neq 2$, $1 < p < \infty$.

The solution method is somewhat unusual. We will conduct a sequence of reductions to the problem of locating the roots of an OPA: to the linear case, to a simpler equivalent extremal problem, and finally to a real-variables problem. This reduction enables us to bring in the Lagrange multiplier method, and finally an associated dynamical system.

In collaboration with Bill Ross and Daniel Seco.

Optimal Polynomial Approximates in L^p

Ray Centner

University of South Florida

Over the past several years, optimal polynomial approximants (OPAs) have been studied in many different function spaces. In these settings, numerous papers have been devoted to studying the properties of their zeros. In this talk, I will introduce the notion of optimal polynomial approximant in the space L^p, $1 \leq p \leq \infty$. In the first half of the talk, I will focus the discussion on the location of their zeros. As an example, I will show that if $1 < p < \infty$, $f \in H^p$, and $f(0) \neq 0$, then there exists a disk, centered at the origin, in which all the associated OPAs are zero-free. In the second half of the talk, I will shed light on an orthogonality condition in L^p. This allows one to study the zeros of OPAs through the lens of the Hilbert space L^2. To inspire further research in the general theory, I will pose some open questions throughout the discussions.

Free group factor problem and Popa's MV-Property

Sayan Das
University of California Riverside

One of the most important outstanding problems in von Neumann algebras asks if the group von Neumann algebra of the free group on two generators, denoted by $L(F_2)$, is isomorphic to the group von Neumann algebra of the free group on infinitely many generators, denoted by $L(F_\infty)$. Recently, S. Popa established a roadmap for showing the nonisomorphism of $L(F_2)$ and $L(F_\infty)$. The first step of the proposed roadmap is to establish the so-called mean value property (abbreviated MV-property) for $L(F_2)$.

In this talk I shall describe the proof of the result that $L(F_2)$ has the MV-property, thereby establishing the first step of Popa's roadmap. This talk is based on a recent joint work with Prof. Jesse Peterson.

Hermitian projections on some Operator spaces

Priyadarshi Dey
Virginia Tech

We say that a projection P is a Hermitian projection if e^{itP} is a surjective isometry for every $t \in \mathbb{R}$. One of the main problems is to give an explicit description of the Hermitian projections on different Banach spaces. It has been well studied for many Banach Spaces and for algebras as well. In this talk, forms of such projections will be mentioned for important Banach spaces. Also, in specific, I will talk about the form of such projection on the space $B(\mathcal{H}, \mathcal{K})$ for distinct Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$. This is a joint work with Fernanda Botelho & Dijana Ilišević.

Toeplitz operators on the Bergman spaces of quotient domains

Gargi Ghosh
Indian Institute of Science

Let G be a finite pseudoreflection group and $\Omega \subseteq \mathbb{C}^d$ be a G-space. We establish identities using representation theory of the group G which connect Toeplitz operators on the Bergman spaces of Ω/G and Ω. Consequently, we study the algebraic properties of Toeplitz operators (such as the generalized zero-product problem, characterization of commuting Toeplitz operators) on the Bergman space of Ω/G. In the proceedings, we state an analytic version of the Chevalley-Shephard-Todd theorem (aCST). Apart from being interesting in its own right, aCST theorem plays an important role in our discussion.

This is a joint work with E.K. Narayanan.

On Popa's Factorial Commutant Embedding Problem

Isaac Goldbring
University of California Riverside

The famous Connes Embedding Problem (CEP) asks whether or not every II_1 factor (a particular kind of von Neumann algebra) admits an embedding into an ultrapower of a very particular II_1 factor, namely the hyperfinite II_1 factor R. (Incidentally, a negative resolution to the CEP was claimed in early 2020.) In an attempt to approach the CEP, Sorin Popa asked whether every II_1 factor that satisfied the conclusion of the CEP must admit such an embedding which is in a sense "ergodic" in that the relative commutant of the embedding is itself a factor; we call the statement that this always holds Popa's Factorial Commutant Embedding Problem (FCEP). Moreover, Popa believed that this should at least hold for all II_1 factors satisfying a certain rigidity property known as property (T). We show how a "Poor Man's" version of this latter problem holds in that there is some II_1 factor M for which all II_1 factors with property (T) admit an embedding into an ultra power of M with factorial commutant. Moreover, the proof shows that Popa's original version of the FCEP has a positive solution for property (T) factors provided two natural conjectures have positive solutions. A novel aspect of our work is that essential use is made of techniques from model theory, a branch of mathematical logic. We will define all model-theoretic prerequisites and most von Neumann algebraic notions needed.

Wavelet Representation of Smooth Calderon-Zygmund Forms

Walton Green
Washington University in St. Louis

We represent smooth CZ forms in terms of so-called wavelet forms and smooth paraproducts forms which readily imply sharp weighted Sobolev space bounds through sparse domination. These methods can also be used to analyze truncated CZOs on Lipschitz domains. One application is to the regularity theory of quasiconformal maps.

Spaces of Continuous and Measurable Functions Invariant Under a Group Action

Samuel A. Hokamp
Sterling College

In this talk we characterize spaces of continuous and L^p-functions on a compact Hausdorff space that are invariant under a transitive and continuous group action. This work generalizes Nagel and Rudin's 1976 results concerning unitarily and Möbius invariant spaces of continuous and measurable functions defined on the unit sphere in $\mathbb{C}^n$.

Polynomially convex sets whose union is not polynomially convex

Alexander J. Izzo
Bowling Green State University

Polynomial convexity, a fundamental notion in the study of Banach algebras and polynomial approximation, will be defined. Simple examples show that the union of two disjoint polynomially convex sets need not be polynomially convex. But what if topological restrictions are placed on the sets? For instance, is the union of two disjoint polynomially convex arcs polynomially convex? Is the union of two disjoint Cantor sets polynomially convex? These questions will be answered. Applications to polynomial convexity of limits of curves, the speaker's original motivation for considering polynomial convexity of disjoint unions, will also be given.

Feature Cross-Correlation

Austin Jacobs
University of Florida

Cross-correlation is a common integral operator used in "block matching" and other pattern recognition and location methods. Here we will give a brief analysis of using the derivative as a pre-processing operator, which is roughly equivalent to doing block matching on the boundary of an object instead of on the object. We will also be showing basic criteria for when an operator on a function space can be used before applying cross-correlation without altering the result.

Local and multilinear restriction theorems for non-commutative Fourier multipliers

Amudhan Krishnaswamy-Usha
TU Delft

For a function m on the real line, it's Fourier multiplier T_m is the operator which acts on a function f by first multiplying the Fourier transform of f by m, and then taking the inverse Fourier transform of the product. These are well-studied objects in classical harmonic analysis. Of particular interest is when the Fourier multiplier defines a bounded operator on L_p. Fourier multipliers can be generalized to arbitrary locally compact groups. If the group is non-abelian, the L_p spaces involved are now the non-commutative L_p spaces associated with the group von Neumann algebra. Fourier multipliers also have a natural extension to the multilinear setting. However their behavior can differ markedly from the linear case, and there is much that is unknown even about multilinear Fourier multipliers on the reals.

One question of interest is this : If m is a function on a group G which defines a bounded L_p multiplier, is the restriction of m to a subgroup H also the symbol of a bounded L_p multiplier on H? De Leeuw proved that the answer is yes, when

G is $\mathbb{R}^n$. This was extended to the commutative case by Saeki and to the non-commutative case (provided the group G is sufficiently nice) by Caspers, Parcet, Perrin and Ricard. In this talk, I will present a local version of this theorem in the linear case and a global version in the multilinear case. This is part of joint work with Martijn Caspers, Bas Janssens and Lukas Miaskiwskyi.

Similarity of Cowen-Douglas Operator Tuples

Hyun-Kyoung Kwon

University of Albany

We present some recent results on the similarity of Cowen-Douglas operator tuples. As in the single operator case, the curvature of the eigenvector bundle associated with a Cowen-Douglas operator tuple comes into play in the similarity characterization. This talk is based on joint work with Kui Ji and his graduate students Shanshan Ji and Jing Xu.

Interaction Between Occupation Kernels and Multiplication operators and resulting Transforms

John Kyei

University of South Florida

The Occupation kernel was introcduced in a joint work by Joel A. Rosenfeld, Rushikesh Kamalapurkar, and Benjamin Russo. Coupled with densely defined Liouville operators, the occupation kernel was used for system Identification and it has been demonstrated through their work to provide a more efficient way for computing dynamic modes of a dynamical system. In this talk, we discuss an analogous notion on the interaction of the multiplication operator with an occupation kernel.

We will investigate properties of the Hilbert Space obtained through the interaction between the operator and the occupation kernel. Introducing a bilinear map on the symbols of densely defined multiplication operators in terms of an inner product, a kernelized Fourier Transform will be realized. We will discuss Properties of the transform born out of the action of the adjoint multiplication operator on the kernel functions.

Caffarelli-Kohn-Nirenberg inequalities for curl-free vector

Nguyen Lam

Memorial University of Newfoundland-Grenfell Campus

In this talk, we discuss the uncertainty principles and the Caffarelli-Kohn-Nirenberg inequalities for vector fields. We will mainly focus on the sharp constants and optimizers of the Caffarelli-Kohn-Nirenberg inequalities for curl-free vector fields. This is joint work with Cristian Cazacu and Joshua Flynn.

The classification problem for arclength null quadrature domains

Erik Lundberg

Florida Atlantic University

A planar domain Ω is referred to as an arclength null quadrature domain if the integral along the boundary of any analytic function in the Smirnov space $E^1(\Omega)$ vanishes. We use classical results of Havinson-Tumarkin and Denjoy-Carleman-Ahlfors in order to prove the existence of a roof function (a positive harmonic function whose gradient coincides with the inward pointing normal along $\partial\Omega$) for arclength null quadrature domains having finitely many boundary components. This bridges a gap toward classification of arclength null quadrature domains by removing an a priori assumption from previous classification results. This result also strengthens an existing connection to free boundary problems for Laplace?s equation and the hollow vortex problem in fluid dynamics. We conclude by discussing the current status of the classification problem for arclength null quadrature domains. This is joint work with Dmitry Khavinson.

Invariant subspaces of the direct sum of forward and backward shifts on vector-valued Hardy spaces

Shuaibing Luo

Hunan University

Let S_E be the shift operator on vector-valued Hardy space H^2_E. We study the invariant subspaces of $S_E \oplus S_F^*$. We establish a one-to-one correspondence between the invariant subspaces of $S_E \oplus S_F^*$ and a class of invariant subspaces of bilateral shift $B_E \oplus B_F$ which were described by Helson and Lowdenslager. As applications, we express invariant subspaces of $S_E \oplus S_F^*$ as kernels or ranges of mixed Toeplitz operators and Hankel operators with partial isometry-valued symbols. Our approach extends and gives different proofs of the results of Câmara and Ross, and Timotin where the case with one dimensional E and F was considered.

Harmonic mappings with fixing its analytic part as starlike

M Manivannan
Kalasalingam Academy of Research and Education

In this paper, we study a family of sense-preserving harmonic mappings whose analytic part is starlike. First we prove that the *sense-preserving* harmonic mappings $f = h + \overline{g}$ are injective and close-to-convex in $\mathbb{D}$ under certain conditions. Next we obtain radius of injectivity for non *sense-preserving* harmonic mappings $f = h + \overline{g}$ such that h is starlike in $\mathbb{D}$ and $g(z) = \varphi(z)h(z)$, where $|\varphi(z)| \leq 1$. Finally we obtain radius of fully starlike and fully convex of order β under the hypothesis $g(z) = \varphi(z)h(z)$ such that $|\varphi(z)| \leq 1$ and h is starlike in $\mathbb{D}$.

A Constructive Definition of the Fourier Transform over a Separable Banach Space

Timothy Myers
Howard University

Gill and Myers proved that every separable Banach space, denoted $\mathcal{B}$, has an isomorphic, isometric embedding in $\mathbb{R}^\infty = \mathbb{R} \times \mathbb{R} \times \cdots$. They used this result and a method due to Yamasaki to construct a sigma-finite Lebesgue measure $\lambda_\mathcal{B}$ for $\mathcal{B}$ and defined the associated integral $\int_\mathcal{B} \cdot \, d\lambda_\mathcal{B}$ in a way that equals a limit of finite-dimensional Lebesgue integrals.

The objective of this talk is to apply this theory to developing a constructive definition of the Fourier transform on $L^1[\mathcal{B}]$. Our approach is constructive in the sense that this Fourier transform is defined as an integral on $\mathcal{B}$, which, by the aforementioned definition, equals a limit of Lebesgue integrals on Euclidean space as the dimension $n \to \infty$. Thus with this theory we may evaluate infinite-dimensional quantities, such as the Fourier transform on $\mathcal{B}$, by means of finite-dimensional approximation. As an application, we will apply the familiar properties of the transform to solving the heat equation on $\mathcal{B}$.

The uncertainty principle in finite dimensions

Shahaf Nitzan
Georgia Tech

I will give a survey of some results related to the talks title, and discuss a couple of new observations in the area. The talk is based on joint work with Jan-Fredrik Olsen and Michael Northington.

Multiplane lensing ensembles with an abundance of images

Sean Perry
Florida Atlantic University

The multiplane lensing system is a model of the phenomenon of gravitational lensing. Masses situated between a light source and an observer may cause the appearance of multiple images of that source. The locations of these apparent images may be calculated via a system of rational equations of a complex variable and its conjugate. In the case of a single lensing plane a quadratic (in the number of point masses) upper bound based on algebraic geometric methods was later improved to a linear one by the use of complex dynamics. This upper bound, $5(g-1)$ where g is the number of lensing point masses, turned out to be the maximal number as it is attained by certain examples constructed by the astronomer S. Rhie. On the other hand, the problem of determining the maximal number of lensed images in the general multiplane case remains open. Though an upper bound for generic multiplane ensembles has been established using algebraic methods, it seems likely that this upper bound has room for improvement. We present multiplane examples that produce $\prod 5(g_i - 1)$ images where g_i is the number of point masses in the ith plane. The method for constructing these examples makes use of Rhie's single-plane examples to construct a non-physical example having the desired number of zeros, and this example is then perturbed to a physically meaningful system while preserving the number of zeros (lensed images). This is joint work with Charles R. Keeton and Erik Lundberg.

The Incredible Occupation Kernel

Joel A. Rosenfeld
University of South Florida

This talk will give a survey of several recent results in operator theoretic methods for learning dynamical systems which involves the usage of what is called the occupation kernel corresponding to a trajectory in a reproducing kernel Hilbert space. We will go over the definition of the occupation kernel and its variants, and we will demonstrate several nice properties and interactions with a variety of operators. Occupation kernels have been used in Dynamic Mode Decompositions, Regression Problems via the Representer Theorem, and can be leveraged to give a norm on symbols for densely defined operators. Bring some popcorn, and learn about a nice collection of basis functions.

The square root of the square of the shift

William T. Ross
University of Richmond

In this joint work with J. Mashreghi and M. Ptak, I present a description of the bounded operators on the Hardy space whose square is the square of the unilateral shift.

Weighted Composition Operator over the Mittag-Leffler Spaces of Entire functions

Himanshu Singh
University of South Florida

Hai and Rosenfeld studied the weighted composition operator over the Mittag-Leffler space of entire functions $ML2(C, q)$ in 2021. Certain operator theoretic characterizations were made for the $0 < q < 2$. In this announcement, we made an attempt to study the same and further the study for positive q. The important reference during the study is Weighted Composition Operators on the Mittag-Leffler Spaces of Entire Functions by Hai and Rosenfeld.

A distance formula for tuples of operators

Sushil Singla
Shiv Nadar University

For a tuple of operators $\boldsymbol{A} = (A_1, \ldots, A_d)$, $\operatorname{dist}(\boldsymbol{A}, \mathbb{C}^d \boldsymbol{I})$ is defined as $\min\limits_{\boldsymbol{z} \in \mathbb{C}^d} \|\boldsymbol{A} - \boldsymbol{z}\boldsymbol{I}\|$ and $\operatorname*{var}\limits_{x}(\boldsymbol{A})$ as $\|\boldsymbol{A}x\|^2 - \sum\limits_{j=1}^{d} \left|\langle x|A_j x\rangle\right|^2$. F. Ming showed that if $\boldsymbol{A}$ is a tuple of commuting normal operators on a Hilbert space $\mathcal{H}$, then

$$\sup_{\|x\|=1} \operatorname*{var}_{x}(\boldsymbol{A}) = R_{\boldsymbol{A}}^2,$$

where $R_{\boldsymbol{A}}$ is the radius of the smallest disc containing the Taylor spectrum of $\boldsymbol{A}$. We have $R_{\boldsymbol{A}} = \operatorname{dist}(\boldsymbol{A}, \mathbb{C}^d \boldsymbol{I})$.

For tuples of doubly commuting matrices and tuples of Toeplitz operators, we will prove the following.

$$\operatorname{dist}(\boldsymbol{A}, \mathbb{C}^d \boldsymbol{I})^2 = \sup_{\|x\|=1} \operatorname*{var}_{x}(\boldsymbol{A}). \tag{1}$$

We also give some equivalent conditions for any tuple of operators on any Hilbert space to satisfy (1). Let $\boldsymbol{z^0} = (z_1^0, \ldots, z_d^0) \in \mathbb{C}^d$ be the unique element for which $\operatorname{dist}(\boldsymbol{A}, \mathbb{C}^d \boldsymbol{I}) = \|\boldsymbol{A} - \boldsymbol{z^0}\boldsymbol{I}\|$. The convexity of the joint numerical range of $\boldsymbol{A} - \boldsymbol{z^0}\boldsymbol{I}$ is sufficient for $\boldsymbol{A}$ to satisfy (1). We will also give an example to show that the convexity of the joint numerical range of $\boldsymbol{A} - \boldsymbol{z^0}\boldsymbol{I}$ is not necessary for $\boldsymbol{A}$ to satisfy (1).

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Application of Fixed Point Iterations in generation of Fractals

Anita Tomar

Sri Dev Suman Uttarakhand University, Pt. L. M. S. Campus

In this talk , the novel escape criteria for a function of the type $\sin(z^n) - az + c$, $a, c \in \mathbb{C}$, and $n \geq 2$ (z is a complex variable), will be discussed utilizing fixed point iterations to explore some new variants of the Mandelbrot sets. Our concern is to utilize the lesser number of iterations that are necessary to attain the fixed point of the underlying complex-valued sine function. Also, we discuss the variation in images and examine the impact of parameters on the deviation of dynamics, color, and appearance of fractals. It is fascinating to notice that some obtained fractals represent the Swastika (a symbol of spirituality and divinity in Indian religions), Shivling (an abstract representation of the Hindu God Shiva), and some are similar to beautiful objects found in our surroundings like flowers, spiders, butterflies, and so on. Some of these fractals represent the stunning art on glass and Rangoli (made in different parts of India, especially during the festive season) which are useful in interior decoration.

The annulus as a K-spectral set

Georgios Tsikalas
Washington University in St. Louis

Suppose X is a compact subset of the complex plane and T a Hilbert space operator with spectrum $\sigma(T) \subset X$. X is said to be a K-spectral set for T, for some constant $K \geq 1$, if

$$||f(T)|| \leq K \max_{x \in X} ||f(x)||,$$

for any rational function f with poles off X.
A fundamental problem in operator theory is to find simple necessary and sufficient conditions on T for X to be a 1-spectral (or simply "spectral") set for T. In the case when X is the closed unit disk $\overline{\mathbb{D}}$, a complete answer is available: a famous inequality due to von Neumann tells us that an operator T is a contraction if and only if $\overline{\mathbb{D}}$ is a spectral set for T. Unfortunately, the situation becomes considerably more complicated in more general domains.

In this talk, we work in the annulus $A_R = \{1/R \leq |z| \leq R\}$. A class of operators that occurs naturally in the study of the annulus as a K-spectral set is the class $\mathcal{C}_R = \{T \in \mathcal{B}(H) : (1/R^2) \leq T^*T \leq R^2\}$. We show that the smallest constant $K(R)$ such that A_R is a $K(R)$-spectral set for T whenever $T \in \mathcal{C}_R$ is at least 2, improving previous known estimates. We also investigate a smaller class $\mathcal{C}_\alpha \subsetneq \mathcal{C}_R$, defined using a simple positivity condition, and show that A_R is a $\sqrt{2}$-spectral set for every $T \in \mathcal{C}_\alpha$, the constant $\sqrt{2}$ being optimal. Our results about $\mathcal{C}_\alpha$ have also been obtained independently by Bello and Yakubovich.

On Asymptotic Moments of Patterned Random Matrices

Tapesh Yadav
University of Florida

For a sufficiently nice 2 dimensional shape, we define its approximating matrix (or patterned matrix) as a random matrix with iid entries arranged according to the given pattern. For large approximating matrices, we observe that the eigenvalues roughly follow an underlying distribution. This phenomenon is similar to the classical observation on Wigner matrices. We prove that the moments of such matrices converge asymptotically as the size increases and equals to the integral of a combinatorially-defined function which counts certain paths on a finite grid.

On the convergence of frame series

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Let H be a separable infinite-dimensional Hilbert space equipped with inner product $\langle\ ,\ \rangle$, $\{x_n\}$ be a frame for H. It is well-known that there exists at least one sequence $\{y_n\}$ in H such that the reconstruction formula $x = \sum \langle x, y_n\rangle\, x_n$ holds for all $x \in H$, where the infinite series converges in the norm of H. An interesting question regarding the reconstruction formula is whether it converges unconditionally for every x and $\{y_n\}$. In this talk, we give a counterexample by constructing a frame for which the reconstruction formula converges conditionally for some x and $\{y_n\}$. Moreover, we found that the unconditional convergence of reconstruction formula is tightly connected to the amount of redundant elements contained in $\{x_n\}$.

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