Technical Appendix to Accompany

"On the Performance of Endogenous Access Pricing"

by

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The key equations from Fjell et al. (2013) are:

$$\pi_0(q_0, q_1, ..., q_N, w, F) \; = \; P(Q)\, q_0 \, + \, w \sum_{i=1}^{N} q_i \, - F \, . \tag{1}$$

$$\pi_i(q_0, q_1, ..., q_N, w) \; = \; [\, P(Q) - w \,]\, q_i \quad \text{for } \; i \in \{1, ..., N\} \; . \tag{2}$$

$$\frac{\partial \pi_0}{\partial q_0} \; = \; 0 \quad \Leftrightarrow \quad P\,(Q) + q_0\, P'\,(Q) - \frac{F}{Q} + \frac{q_0\, F}{Q^2} \; = \; 0\, . \tag{3}$$

$$\frac{\partial \pi_i}{\partial q_i} \; = \; P\,(Q) + q_i\, P'\,(Q) - \frac{F}{Q} + \frac{q_i\, F}{Q^2} \; = \; 0 \quad \text{for } \; i \; = \; 1, ..., N\, . \tag{4}$$

$$\widetilde{q}_0^{\,*} \; = \; \widetilde{q}_i^{\,*} \; = \; \frac{\widetilde{Q}^{\,*}}{N+1} \quad \text{for } \; i \; = \; 1, ..., N\, . \tag{5}$$

$$\widetilde{w} \; \sum_{i=1}^{N} \widetilde{q}_i - F \;\; = \;\; \frac{F}{\widetilde{Q}} \left[\widetilde{Q} - \widetilde{q}_0 \right] - F \;\; = \;\; - \left[\frac{F}{\widetilde{Q}} \right] \widetilde{q}_0 \;\; = \;\; - \; \widetilde{w}\; \widetilde{q}_0\, . \tag{6}$$

$$\widehat{w} \; \sum_{i=1}^{N} \widehat{q}_i^{\,*} - F \;\; = \;\; \frac{F}{Q^e} \left[\, Q^e - \; \widehat{q}_0^{\,*} \,\right] - F \;\; = \;\; - \left[\frac{\widehat{q}_0^{\,*}}{Q^e} \right] F\, . \tag{7}$$

__**Proof of Proposition 1**__.

From (1) and (5), the VIP's equilibrium profit under ENAP is:

$$\pi_0^* \; = \; q_0^*\, P\,(Q^*) + \frac{F}{Q^*} \; \sum_{i=1}^{N} q_i^* - F \; = \; \left[\frac{Q^*}{N+1} \right] P\,(Q^*) + \frac{F}{Q^*} \left[\frac{N\, Q^*}{N+1} \right] - F$$

$$= \; \left[\frac{Q^*}{N+1} \right] P\,(Q^*) + \frac{N\, F}{N+1} - F \; = \; \frac{1}{N+1} \left[\, Q^*\, P\,(Q^*) - F \,\right] . \tag{8}$$

From (3) and (5), equilibrium industry output under ENAP is given by:

$$P\,(Q^*) + \left[\frac{Q^*}{N+1} \right] P'\,(Q^*) - \frac{F}{Q^*} + \frac{F \left[\frac{Q^*}{N+1} \right]}{(Q^*)^2} \; = \; 0$$

$$\Rightarrow \quad P(Q^*) + \left[\frac{Q^*}{N+1}\right] P'(Q^*) - \left[\frac{N}{N+1}\right] \frac{F}{Q^*} \; = \; 0 \tag{9}$$

$$\Rightarrow \quad Q^* [N+1] \, P(Q^*) + (Q^*)^2 \, P'(Q^*) - N\,F \; = \; 0 \tag{10}$$

$$\Rightarrow \quad Q^* \, P(Q^*) \; = \; \frac{N\,F - (Q^*)^2 \, P'(Q^*)}{N+1} \, . \tag{11}$$

(8) and (11) provide:

$$\pi_0^* \; = \; \frac{1}{N+1} \left[\frac{N\,F - (Q^*)^2 \; P'(Q^*)}{N+1} - F \right] \; = \; - \, \frac{1}{[N+1]^2} \left[(Q^*)^2 \, P'(Q^*) + F \right]$$

$$\Rightarrow \quad \frac{d\pi_0^*}{dF} \; = \; - \, \frac{1}{[N+1]^2} \left\{ \left[(Q^*)^2 \; P''(Q^*) + 2\,Q^* P'(Q^*) \right] \frac{\partial Q^*}{\partial F} + 1 \right\} . \tag{12}$$

(12) implies that if $P''(Q^*) \le 0$ and $\frac{\partial Q^*}{\partial F} \le 0$, then $\frac{d\pi_0^*}{dF} < 0$, and so the VIP will set $F = \underline{F}$ under ENAP. To determine when $\frac{\partial Q^*}{\partial F} \le 0$, let:

$$\begin{aligned}
h(Q^*) \; &\equiv \; Q^* [N+1] \, P(Q^*) + (Q^*)^2 \; P'(Q^*) \\
\Rightarrow \; h'(Q^*) \; &= \; [N+1] \, P(Q^*) + Q^* [N+1] \, P'(Q^*) + (Q^*)^2 \; P''(Q^*) + 2\,Q^* \, P'(Q^*) \\
&= \; [N+1] \, P(Q^*) + Q^* [N+3] \, P'(Q^*) + (Q^*)^2 \; P''(Q^*) \\
\Rightarrow \; h''(Q^*) \; &= \; [N+1] \, P'(Q^*) + [N+3] \, P'(Q^*) + Q^* [N+3] \, P''(Q^*) \\
&\qquad + \; (Q^*)^2 \; P'''(Q^*) + 2\,Q^* \, P''(Q^*) \\
&= \; [2\,N+4] \, P'(Q^*) + Q^* [N+5] \, P''(Q^*) + (Q^*)^2 \; P'''(Q^*) \, .
\end{aligned} \tag{13}$$

(13) implies that $h''(\cdot) < 0$, and so $h(\cdot)$ is a concave function of Q^*, under the conditions specified in the proposition. From (10), Q^* is determined by $h(Q^*) = N\,F$, and so (10) will have at least one real root when F is sufficiently small. Furthermore, when (10) has two real roots, the larger root of (10) decreases as F increases, and so $\frac{\partial Q^*}{\partial F} < 0$, when $h(\cdot)$ is a concave function of Q^*.

It remains to verify that the larger root of (10) is the relevant root in cases where (10) has two roots. To do so, let Q_1^* and Q_2^* denote two distinct roots of (10), with $Q_1^* < Q_2^*$. We will show that $\left.\frac{\partial^2 \pi_0}{\partial (q_0)^2}\right|_{Q_1^*} > 0$, and so the smaller root does not correspond to a profit-maximizing level of output for the VIP.

From (9):

$$g(Q^*) \; \equiv \; g_1(Q^*) - g_2(Q^*) \; = \; 0 \, , \tag{14}$$

where:

$$g_1(Q^*) \; = \; P(Q^*) + \left[\frac{Q^*}{N+1}\right] P'(Q^*) \quad \text{and} \quad g_2(Q^*) \; = \; \left[\frac{N}{N+1}\right] \frac{F}{Q^*} \, . \tag{15}$$

Observe that:

$$g_2'(Q^*) \;=\; -\left[\frac{N}{N+1}\right]\frac{F}{(Q^*)^2} \;<\; 0 \quad\Rightarrow\quad g_2''(Q^*) \;=\; \left[\frac{N}{N+1}\right]\frac{2\,F}{(Q^*)^3} \;>\; 0\,. \qquad (16)$$

Therefore, $g_2(Q^*)$ is a decreasing, convex function of Q^*.

Also observe that:

$$g_1'(Q^*) \;=\; \left[1+\frac{1}{N+1}\right]P'(Q^*) + \left[\frac{Q^*}{N+1}\right]P''(Q^*) \;<\; 0 \qquad (17)$$

$$\Rightarrow\; g_1''(Q^*) \;=\; \left[1+\frac{1}{N+1}\right]P''(Q^*) + \left[\frac{1}{N+1}\right]P''(Q^*) + \left[\frac{Q^*}{N+1}\right]P'''(Q^*) \;\leq\; 0\,.$$

Therefore, $g_1(Q^*)$ is a decreasing, concave function of Q^* under the maintained conditions, and so, from (14), $g(Q^*)$ is a concave function of Q^*.

We now establish that $g'(Q_1^*) > 0$. To do so, consider the interval $[Q_1^*,\; Q_1^*+\epsilon\,]$, where $\epsilon > 0$ is arbitrarily small. (14) implies that $g(Q_1^*) = 0$. Furthermore, $g(Q^*) > 0$ for all $Q^* \in (Q_1^*,\; Q_1^*+\epsilon)$ since $g(Q^*)$ is a concave function of Q^*. Therefore, $g'(Q_1^*) > 0$.

From (1) and (5):

$$\frac{\partial \pi_0}{\partial q_0} \;=\; P(Q^*) + q_0\,P'(Q^*) - \frac{F}{(Q^*)^2}\sum_{i=1}^{N} q_i^*$$

$$\Rightarrow\; \frac{\partial^2 \pi_0}{\partial (q_0)^2} \;=\; 2\,P'(Q^*) + q_0\,P''(Q^*) + \frac{2\,F}{(Q^*)^3}\sum_{i=1}^{N} q_i^*$$

$$\Rightarrow\; \left.\frac{\partial^2 \pi_0}{\partial (q_0)^2}\right|_{q_0^* = q_i^* = \frac{Q_1^*}{N+1}} \;=\; 2\,P'(Q_1^*) + \left[\frac{Q_1^*}{N+1}\right]P''(Q_1^*) + \frac{2\,F}{(Q_1^*)^3}\left[\frac{N\,Q_1^*}{N+1}\right]$$

$$=\; 2\,P'(Q_1^*) + \left[\frac{Q_1^*}{N+1}\right]P''(Q_1^*) + \frac{2\,F}{(Q_1^*)^2}\left[\frac{N}{N+1}\right]. \qquad (18)$$

From (14), (16), and (17):

$$g'(Q_1^*) \;=\; \left[1+\frac{1}{N+1}\right]P'(Q_1^*) + \left[\frac{Q_1^*}{N+1}\right]P''(Q_1^*) + \left[\frac{N}{N+1}\right]\frac{F}{(Q_1^*)^2}$$

$$=\; \left[\frac{N+2}{N+1}\right]P'(Q_1^*) + \left[\frac{Q_1^*}{N+1}\right]P''(Q_1^*) + \left[\frac{N}{N+1}\right]\frac{F}{(Q_1^*)^2}$$

$$=\; \frac{N}{N+1}\left[P'(Q_1^*) + \frac{F}{(Q_1^*)^2}\right] + \left[\frac{2}{N+1}\right]P'(Q_1^*) + \left[\frac{Q_1^*}{N+1}\right]P''(Q_1^*)\,. \qquad (19)$$

Since $g'(Q_1^*) > 0$, (19) implies:

$$P'\left(Q_1^*\right) + \frac{F}{\left(Q_1^*\right)^2} \; > \; 0\,. \tag{20}$$

From (18):

$$\left.\frac{\partial^2 \pi_0}{\partial\left(q_0\right)^2}\right|_{q_0^* = q_i^* = \frac{Q_1^*}{N+1}} \; = \; \frac{2\,N}{N+1}\left[P'\left(Q_1^*\right) + \frac{F}{\left(Q_1^*\right)^2}\right] + \left[\frac{2}{N+1}\right] P'\left(Q_1^*\right) + \left[\frac{Q_1^*}{N+1}\right] P''\left(Q_1^*\right). \tag{21}$$

(19) and (21) provide:

$$\left.\frac{\partial^2 \pi_0}{\partial\left(q_0\right)^2}\right|_{q_0^* = q_i^* = \frac{Q_1^*}{N+1}} \; = \; \frac{N}{N+1}\left[P'\left(Q_1^*\right) + \frac{F}{\left(Q_1^*\right)^2}\right] + g'\left(Q_1^*\right)\,. \tag{22}$$

(20) and (22) imply that $\left.\frac{\partial^2 \pi_0}{\partial(q_0)^2}\right|_{q_0^* = q_i^* = \frac{Q_1^*}{N+1}} \; > \; 0$, since $g'\left(Q_1^*\right) > \; 0$. ■

The following Lemmas are instrumental in the proof of Proposition 2.

Lemma 1. *Suppose Assumption 1 holds. Then given access price $\widehat{w}$, the equilibrium output of the VIP under EXAP is $\widehat{q}_0^* \; = \; \frac{a + \widehat{w}N}{b\,[N+2]}$. The equilibrium output of each of the N rivals under EXAP is $\widehat{q}_i^* \; = \; \frac{a - 2\,\widehat{w}}{b\,[N+2]}$ for $i \; = \; 1, ..., N$.*

Proof. Differentiating (1) and (2) provides:

$$\frac{\partial \pi_0}{\partial q_0} \; = \; a - 2\,b\,q_0 - b\sum_{j=1}^{N} q_j \quad \text{and} \quad \frac{\partial \pi_i}{\partial q_i} \; = \; a - b\,q_i - b\,q_0 - b\sum_{j=1}^{N} q_j - w\,. \tag{23}$$

In equilibrium, $\frac{\partial \pi_0}{\partial q_0} \; = \; \frac{\partial \pi_i}{\partial q_i} = 0$. Therefore, from (23):

$$a - 2\,b\,q_0 \; = \; b\sum_{j=1}^{N} q_j \; = \; a - b\,q_i - b\,q_0 - w$$

$$\Leftrightarrow \quad b\,q_i \; = \; b\,q_0 - w \quad \Rightarrow \quad b\sum_{i=1}^{N} q_i \; = \; N\,b\,q_0 - w\,N\,. \tag{24}$$

Since $\frac{\partial \pi_0}{\partial q_0} = 0$ in equilibrium, (23) and (24) provide:

$$a - 2\,b\,q_0 - N\,b\,q_0 + w\,N \; = \; 0 \quad \Rightarrow \quad \widehat{q}_0^* \; = \; \frac{a + w\,N}{b\,[N+2]}\,. \tag{25}$$

(24) and (25) provide:

$$b\,N\,\widehat{q}_i^* \;=\; N\,b\left[\frac{a+w\,N}{b\,(N+2)}\right] - w\,N \;=\; \frac{a\,N + w\,N^2 - w\,N\,[N+2]}{N+2}$$

$$= \frac{a\,N - 2\,w\,N}{N+2} \quad\Rightarrow\quad \widehat{q}_i^* \;=\; \frac{a-2\,w}{b\,[N+2]}\,. \quad \blacksquare \tag{26}$$

Lemma 2. *Suppose Assumption 1 holds. Then when the VIP's fixed cost is F, the access price that will be set under EXAP is $\widehat{w}(F) = \frac{1}{2\,N}\left[a\,(N+1) - \sqrt{\widehat{G}(F)}\right]$ where $\widehat{G}(F) \equiv a^2\,[N+1]^2 - 4\,b\,F\,N\,[N+2]$.*

<u>Proof</u>. (25) and (26) imply:

$$\widehat{Q}^* \;=\; q_0^* + \sum_{i=1}^{N} \widehat{q}_i^* \;=\; \frac{a+w\,N}{b\,[N+2]} + \frac{N\,[a-2\,w]}{b\,[N+2]} \;=\; \frac{a\,[N+1] - w\,N}{b\,[N+2]}\,. \tag{27}$$

Therefore, when $Q^e = \widehat{Q}^*$:

$$w \;=\; \frac{F}{\widehat{Q}^*} \;=\; \frac{b\,F\,[N+2]}{a\,[N+1] - w\,N} \quad\Rightarrow\quad N\,w^2 - a\,[N+1]\,w + F\,[N+2]\,b \;=\; 0$$

$$\Rightarrow\quad \widehat{w}(F) \;=\; \frac{a\,[N+1] - \sqrt{a^2\,[N+1]^2 - 4\,b\,F\,N\,[N+2]}}{2\,N}. \tag{28}$$

The smallest root here reflects the fact that the lower access price generates larger industry output and welfare. A real solution to (28) exists because:

$$a^2\,[N+1]^2 - 4\,N\,F\,[N+2]\,b \;\geq\; 0 \quad\Leftrightarrow\quad F \;\leq\; \frac{a^2\,[N+1]^2}{4\,b\,N\,[N+2]}\,. \tag{29}$$

Observe that $\frac{[a(N+1)]^2}{4\,b\,N[N+2]} > \frac{a^2}{4\,b}$, since $[N+1]^2 > N\,[N+2]$. $\blacksquare$

Lemma 3. *Suppose Assumption 1 holds. Then for a given fixed cost, F, the VIP's equilibrium profit under EXAP is:*

$$\widehat{\pi}_0^*(F) \;=\; \frac{1}{4\,b\,N^2\,[N+2]^2}\left\{2\,a\,N\,[N+4]\sqrt{\widehat{G}(F)} + 4\,b\,F\,N^2\,[N+4]\,[N+2] - 2\,a^2 N\left[N^2+3\,N+4\right]\right\} - F\,.$$

<u>Proof</u>. For expositional ease, we suppress the dependence of $\widehat{w}(\cdot)$ and $\widehat{G}(\cdot)$ on F in the ensuing analysis. From (1), (25), (26), and (27):

$$
\begin{aligned}
\widehat{\pi}_0^* &= \widehat{q}_0^* \left[a - b\,\widehat{Q}^*\right] + \widehat{w}\, \sum_{i=1}^{N} \widehat{q}_i^* - F \\
&= \frac{a + \widehat{w}\,N}{[N+2]\,b} \left[\frac{a + \widehat{w}\,N}{N+2}\right] + \widehat{w} \left[\frac{N\,(a - 2\,\widehat{w})}{b\,(N+2)}\right] - F \;=\; \frac{H}{b\,[N+2]^2} - F \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
\text{where}\quad H &= [a + \widehat{w}\,N]^2 + [N+2]\;\widehat{w}\,N\,[a - 2\,\widehat{w}] \\
&= a^2 + N^2\,\widehat{w}^2 + 2\,a\,N\,\widehat{w} + a\,N^2\,\widehat{w} + 2\,a\,\widehat{w}\,N - 2\,N^2\;\widehat{w}^2 - 4\,\widehat{w}^2\,N \\
&= a^2 + a\,N\,\widehat{w}\,[N+4] - \;\widehat{w}^2\,N\,[N+4]
\end{aligned}
$$

$$
\begin{aligned}
&= a^2 + a\,N\,[4+N] \left[\frac{a\,(N+1) - \sqrt{\widehat{G}}}{2\,N}\right] - N\,[N+4] \left[\frac{a^2\,[N+1]^2 + \widehat{G} - 2\,a\,[N+1]\,\sqrt{\widehat{G}}}{4\,N^2}\right] \\
&= \frac{1}{4\,N^2}\,\{\,4\,N^2\,a^2 + 2\,a\,N^2\,[N+4]\left[a\,(N+1) - \sqrt{\widehat{G}}\,\right] \\
&\qquad\qquad - \;N\,[N+4]\left[a^2\,(N+1)^2 + \widehat{G} - 2\,a\,(N+1)\,\sqrt{\widehat{G}}\,\right]\} \\
&= \frac{1}{4\,N^2}\,\{\,4\,N^2\,a^2 + 2\,a^2\,N^2\,[N+4]\,[N+1] - \left[2\,a\,N^2\,(N+4)\,\sqrt{\widehat{G}}\,\right] \\
&\qquad\qquad - \;a^2\,N\,[N+4]\,[N+1]^2 + 2\,a\,N\,[N+4]\,[N+1]\,\sqrt{\widehat{G}} \\
&\qquad\qquad - \;N\,[N+4]\left[a^2\,(N+1)^2 - 4\,b\,N\,F\,(N+2)\right]\} \\
&= \frac{1}{4\,N^2}\,\{\,4\,N^2 a^2 + 2\,a^2 N^2\,[N+4]\,[N+1] - 2\,a^2 N\,[N+4]\,[N+1]^2 \\
&\qquad\qquad - \;2\,a\,N^2\,[N+4]\,\sqrt{\widehat{G}} + 2\,a\,N\,[N+4]\,[N+1]\,\sqrt{\widehat{G}} + 4\,b\,F\,N^2\,[N+4]\,[N+2]\,\} \\
&= \frac{1}{4\,N^2}\,\{-2\,a^2 N\left[N^2 + 3N + 4\right] + \;2\,a\,N\,[4+N]\,\sqrt{\widehat{G}} + 4\,b\,F\,N^2\,[N+4]\,[N+2]\,\}. \qquad (31)
\end{aligned}
$$

(30) and (31) provide the expression for $\widehat{\pi}_0^*(F)$ specified in the lemma. ■

Proof of Proposition 2.

Differentiating $\widehat{\pi}_0^*(F)$ provides:

$$
\begin{aligned}
\widehat{\pi}_0^{*\prime}(F) &= \frac{1}{4\,b\,N^2\,[N+2]^2}\left[a\,N\,[4+N]\,\frac{\widehat{G}'(F)}{\sqrt{\widehat{G}}} + 4\,b\,N^2\,(N+4)\,(N+2)\right] - 1 \\
&= \left[\frac{1}{4\,b\,N^2\,[N+2]^2}\right]\left[-\,\frac{4\,a\,N^2\,[N+4]\,[N+2]\,b}{\sqrt{\widehat{G}}} + 4\,b\,N^2\,[N+4]\,[N+2]\right] - 1 \\
&= \frac{4\,N^2\,[N+4]\,[N+2]\,b}{4\,b\,N^2\,[N+2]^2}\left[-\,\frac{a}{\sqrt{\widehat{G}}}+1\right] - 1 \;=\; \frac{N+4}{N+2}\left[-\,\frac{a}{\sqrt{\widehat{G}}}+1\right] - 1\,. \qquad (32)
\end{aligned}
$$

(32) implies:

$$
\begin{aligned}
\widehat{\pi}_0^{*\prime}(F) \gtreqless 0 &\Leftrightarrow \left[\frac{N+4}{N+2}\right]\left[-\,\frac{a}{\sqrt{\widehat{G}}}+1\right] \gtreqless 1 \\
&\Leftrightarrow -\,\frac{a}{\sqrt{\widehat{G}}}+1 \gtreqless \frac{N+2}{N+4} \;\Leftrightarrow\; -\,\frac{a}{\sqrt{\widehat{G}}} \gtreqless \frac{N+2}{N+4}-1 \;\Leftrightarrow\; -\,\frac{a}{\sqrt{\widehat{G}}} \gtreqless \frac{-2}{N+4} \\
&\Leftrightarrow \frac{a}{\sqrt{\widehat{G}}} \lesseqgtr \frac{2}{N+4} \;\Leftrightarrow\; \frac{\sqrt{\widehat{G}}}{a} \gtreqless \frac{N+4}{2} \;\Leftrightarrow\; \sqrt{\widehat{G}} \gtreqless \frac{[N+4]\,a}{2} \\
&\Leftrightarrow \widehat{G} \gtreqless \frac{[N+4]^2\,a^2}{4} \;\Leftrightarrow\; [\,a\,(N+1)\,]^2 - 4\,N\,F\,[N+2\,]\,b \gtreqless \frac{[N+4]^2\,a^2}{4} \\
&\Leftrightarrow a^2\left[(N+1)^2 - \frac{(N+4)^2}{4}\right] \gtreqless 4\,b\,N\,F\,[N+2\,] \\
&\Leftrightarrow a^2\left[4\,(N+1)^2 - (N+4)^2\right] \gtreqless 16\,b\,N\,F\,[N+2\,] \\
&\Leftrightarrow a^2\,[\,3\,(N+2)\,(N-2)] \gtreqless 16\,b\,N\,F\,[N+2\,] \;\Leftrightarrow\; F \lesseqgtr \frac{3\,a^2\,[N-2\,]}{16\,b\,N}\,. \qquad (33)
\end{aligned}
$$

(33) implies that $\frac{\partial \pi_0^*}{\partial F} < 0$ (and so $\widehat{F}^* = \underline{F}$) if $N \leq 2$. In contrast, if $N \geq 3$, then $\widehat{F}^* = \min\left\{\max\left(\underline{F}\,,\,\frac{3\,a^2[N-2]}{16\,b\,N}\right)\,,\,\overline{F}\right\}$. Consequently, $\widehat{F}^* > \underline{F}$ if $\underline{F} < \frac{3\,a^2[N-2]}{16\,b\,N}$. This will be the case if $\underline{F} < \frac{a^2}{16\,b}$, since $z(N) \equiv \frac{N-2}{N}$ is an increasing function of N with $z(3) = \frac{1}{3}$. ■

Proof of Proposition 3.

The incumbent's profit in the setting with variable access costs is:

$$\pi_0 \;=\; q_0 \, P\,(Q) + w \sum_{i=1}^{n} q_i - F \; - c(F)\, Q \, . \tag{34}$$

Differentiating (34) provides:

$$\frac{\partial \pi_0}{\partial F} \;=\; \frac{\partial}{\partial F} \left\{ q_0 \, P\,(Q) + w \sum_{i=1}^{n} q_i - F \right\} - \frac{\partial}{\partial F} \{ \, c(F)\, Q \, \} \tag{35}$$

where:

$$\frac{\partial}{\partial F} \{ \, c(F)\, Q \, \} \;=\; Q \left[\frac{\partial c(F)}{\partial F} \right] + c(F) \left[\frac{\partial Q}{\partial F} \right] \;=\; -\, Q \; r'(F) + c(F) \left[\frac{\partial Q}{\partial F} \right] . \tag{36}$$

From Proposition 3 in Fjell et al. (2010), the equilibrium value of Q is the same under EXAP and ENAP for a given F. Therefore, it must be the case that both Q and the rate at which Q varies with F are the same at each F under EXAP and ENAP. Consequently, (36) implies that for any given F, $\frac{\partial}{\partial F} \{ \, c(F)\, Q \, \}$ is the same under exogenous access pricing and endogenous access pricing.

Under the conditions specified in Proposition 3, $\frac{\partial}{\partial F} \{ q_0 \, P(Q) + w \sum_{i=1}^{n} q_i - F \}$ is strictly positive under EXAP for $F \in [\, 0, \, \widehat{F}^* \,)$ (where $\widehat{F}^* > 0$) and strictly negative under ENAP for all $F \geq 0$. Therefore, (35) implies that for each F, $\frac{\partial \pi_0}{\partial F}$ is larger under EXAP than under ENAP, and so the VIP will implement a larger level of F under EXAP than under ENAP in the setting with variable access costs. ■

Reference

Fjell, K., Pal, D., and Sappington, D. (2013). On the Performance of Endogenous Access Pricing. *Journal of Regulatory Economics*, (forthcoming).