On the Merits of Antitrust Liability in Regulated Industries

by

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Abstract

We examine the merits of subjecting an incumbent supplier of regulated services to antitrust review. We show that antitrust review can harm consumers even when the review entails no direct costs of implementation. The consumer harm arises in part because imperfect antitrust review can “crowd out” more effective regulatory oversight. More generally, antitrust review can usefully complement regulatory oversight, but affects the nature of the optimal regulatory policy.

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1 Introduction

In its *Trinko* Decision, the U. S. Supreme Court identifies little role for antitrust review in regulated industries where regulators are well equipped to design and enforce industry policy. The Court (2004, § IV) observes that in settings where “a regulatory structure designed to deter and remedy anticompetitive harm ... exists, the additional benefit to competition provided by antitrust enforcement will tend to be small.” More generally, though, antitrust review might usefully complement (or perhaps even substitute for) regulatory oversight in settings with emerging industry competition and limited regulatory resources (Levy and Spiller, 1996). A central purpose of this research is to assess the merits of subjecting an incumbent regulated supplier to antitrust review in settings where regulatory and court oversight are both imperfect. We also examine how the presence of court oversight affects the optimal design of regulatory policy.

To explore these issues formally, we analyze a model that admits varying degrees of imperfect oversight by a regulator and an antitrust court. Both entities have limited ability to assess whether a vertically-integrated incumbent supplier has undertaken an anticompetitive action designed to raise the costs of a retail competitor. The regulator also can set the price the incumbent supplier charges its rival for access to its network.

The regulator chooses her policy instruments optimally in light of industry characteristics and the nature of the prevailing antitrust review. The regulator’s choice of an access price is complicated by the fact that the price has several effects. The price affects the incumbent supplier’s upstream revenue, and thus its willingness to operate in the industry. The access price also affects the entrant’s production cost, and thus the extent to which the retail competitor can impose meaningful discipline on the incumbent supplier. By influencing the incumbent’s profit margin on sales of access to the entrant, the access price also affects the incumbent’s incentive to raise its rival’s cost (and thereby reduce the entrant’s demand for network access).

We find that the regulator optimally increases the accuracy of her industry oversight and
reduces the access price as the incumbent supplier’s ability to raise its rival’s cost increases. The more accurate oversight, which helps to deter the incumbent from undertaking the anticompetitive action, reduces the regulatory penalty the incumbent expects to incur in equilibrium. The reduced penalty enables the regulator to deliver the requisite upstream profit to the incumbent supplier even as she reduces the access price, thereby securing a lower retail price for consumers.

We also find that regulatory oversight and antitrust review are substitutes in the sense that the regulator optimally reduces the resources she devotes to improving the accuracy of her industry oversight as antitrust review becomes better able to impose effective discipline on the incumbent supplier. Furthermore, the regulator increases the price the incumbent charges for access to its network as the incumbent’s potential antitrust liability increases and as antitrust review becomes better able to detect the incumbent’s anticompetitive action. In the face of increased antitrust liability, a higher access charge helps to offset the increased antitrust penalty the incumbent anticipates in equilibrium (even when it refrains from anticompetitive activity).\footnote{The Supreme Court (2004, § IV) notes that “[m]istaken inferences and the resulting false condemnations ‘are especially costly, because they chill the very conduct the antitrust laws are designed to protect’.”} A higher access charge also helps to counteract the increased regulatory penalty the supplier anticipates in equilibrium when the regulator conserves resources by reducing the accuracy of her industry oversight in response to a more accurate antitrust review.

We consider settings where, as is common in practice, the penalties the regulator can impose on the incumbent supplier are limited. Consequently, it is not surprising that antitrust review often can usefully complement regulatory oversight in our model. However, even when antitrust review entails no direct costs, it can reduce consumer welfare. It will do so, for example, when the antitrust review is not particularly adept at inferring the intent and impact of the incumbent supplier’s actions, when the regulator has substantial ability to penalize the incumbent supplier for anticompetitive behavior, when a large fraction of regulatory penalties accrue to consumers, and when the regulator faces relatively low costs
of implementing accurate industry oversight.

Our derivation and discussion of these findings proceeds as follows. Section 2 describes the key elements of the simple model we analyze. Section 3 reviews how the optimal regulatory policy responds to selected changes in industry conditions (e.g., the impact of the incumbent’s anticompetitive action on its rival’s cost), taking the prevailing antitrust review as given. Section 4 examines how the optimal regulatory policy changes in response to changes in the prevailing antitrust review and examines when antitrust review serves as a useful complement to regulatory oversight. Section 5 concludes and suggests directions for further research. The Appendix provides the proofs of all formal conclusions.²

Before proceeding, we note that the literature provides many useful policy discussions of the benefits and costs of antitrust regulation in the presence of regulatory oversight (e.g., Bourreau and Dogan, 2001; Cave, 2004; Katz, 2004; Rey, 2004; Gérardin and Sidak, 2005; Kahn, 2006; Ginsburg, 2009; Weiser, 2009). However, the literature offers few rigorous, detailed formal models of the relevant trade-offs. Formal models of related issues focus on different considerations. For instance, Garoupa and Gomez-Pomar (2004) demonstrate that criminal sanctions can usefully accompany regulatory fines when regulatory sanctions are insufficient to deter harmful behavior and when the firm can bribe the regulator not to impose the sanctions. Schwartzstein and Shleifer (2013) demonstrate how regulatory oversight can reduce (imperfect) litigation that deters valuable industry activity. Ottaviani and Wickelgren (2010, 2011) analyze the optimal timing of regulatory reviews (of mergers, for example) when better information is available at later reviews. Innes (2004) examines the optimal interplay between ex ante and ex post regulations that seek to limit the harm from accidents,³ focusing on the role that ex ante investigation can play in reducing enforcement costs.

Tirole (2004) and Weiser (2005), among others, observe that the best policy regarding

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²Bose et al. (2015) provide more detailed proofs.

³Encinosa and Sappington (1995) also analyze the interaction between ex ante and ex post regulatory investigations in a setting where no antitrust review is present.
antitrust review in regulated industries typically entails case-specific economic analysis rather than broad, uniform, rigid rules. Our analysis supports this conclusion, given our finding that a complete assessment of the merits of antitrust review in the presence of regulatory oversight entails many subtleties even in the simplest of economic models. Our analysis also provides a first step in demonstrating formally how some of the key considerations in any case-specific economic analysis are likely to affect the merits of antitrust liability.

2 Elements of the Model

We consider a setting in which a vertically-integrated incumbent supplier (V) produces a critical input (e.g., “access” to its network infrastructure) and supplies a retail product. A rival supplier (E for “entrant”) also produces the homogeneous retail product. E must secure access from V in order to produce the retail product. One unit of access is required to produce each unit of the retail product.

E’s cost of producing \( x_e \) units of output is \( F_e + [w + c_e] x_e \), where \( F_e \) is a fixed cost of production, \( w \) is the unit price of access, and \( c_e \) is an additional (downstream) marginal production cost. V’s cost of supplying \( x_e \) units of access to E and supplying \( x_v \) units of the retail product is \( F_u + F_d + u x_e + [u + c_v] x_v \). \( F_u \) is an (upstream) fixed cost of supplying access. \( F_d \) is a (downstream) fixed cost of producing the retail product. \( u \) is a marginal cost of supplying access and \( c_v \) is a marginal cost of supplying the retail product.

We consider a setting where, as is presently the case in many telecommunications sectors, the regulator sets the price of access (\( w \)) but does not directly regulate the price of the retail product. The retail price is determined by Cournot competition between V and E which

\footnote{Tardi\textsuperscript{f} and Taylor (2003, pp. 348-9) note that as early as 2003, “the services to which any form of price regulation is applied [in the U.S. telecommunications industry] have been narrowed primarily to access services for smaller customers [and] the remaining services have been removed from price regulation.” Tardi\textsuperscript{f} (2007, Table 3, p. 121) summarizes the widespread deregulation of retail telecommunications services that U.S. states had undertaken by 2007. Lichtenberg (2012, p. 4) reviews the additional deregulation that was implemented between 2010 and 2012, noting that “[b]y the end of April 2012, more than one third of the nation (21 states) had deregulated its incumbent wireline carriers, and all had adopted language ensuring that broadband transport and VoIP services would remain outside commission jurisdiction.” The prices of most retail telecommunications services were deregulated in the UK in 2006, when Ofcom (2006, p. 1) announced its “landmark decision” to “allow Retail price controls (RPC) to lapse on their expiry on 31 July 2006.”}
takes place after \( w \) and production costs are determined. For analytic simplicity, we assume
the market demand for the retail product is linear, so \( P(X) = a - bX \), where \( a > 0 \) and
\( b > 0 \) are constants, \( X = x_v + x_e \) is the total output of the retail product, and \( P(\cdot) \) denotes
the corresponding price of the retail product.\(^5\)

In addition to setting the price of access \( (w) \), the regulator engages in ongoing oversight
of \( V \)’s behavior toward \( E \). \( V \) can undertake either a competitive action \( (\alpha) \) or an anti-
competitive action \( (\overline{\alpha}) \). When \( V \) undertakes action \( \alpha \in \{ \alpha, \overline{\alpha} \} \), \( E \)’s expected downstream
marginal cost of production is \( q(\alpha) c_H + [1 - q(\alpha)] c_L \), where \( 0 < c_L < c_H \) and \( q(\alpha) \in (0, 1) \).
\( V \)’s anticompetitive action raises \( E \)’s expected cost by increasing the likelihood that \( E \) has the higher of the two possible downstream marginal costs \( (c_H) \), i.e., \( q(\overline{\alpha}) > q(\alpha) \).\(^6\)

\( r \in [\frac{1}{2}, 1] \) denotes the probability that the regulator assesses accurately \( V \)’s action toward
its retail rival. The regulator’s cost of ensuring “accuracy” \( r \) is \( K(r) = k \left[ r - \frac{1}{2} \right]^2 \), where
\( k > 0 \) is a constant.\(^7\) The regulator imposes penalty \( D_R \) on \( V \) if her ongoing industry
oversight leads her to conclude that \( V \) has undertaken the anticompetitive behavior.\(^8\) \( D_R \)
cannot exceed \( D_R \) due to legislated limits on regulatory fines, for example. The fraction
\( f_R \in [0, 1] \) of the imposed penalty is awarded to \( E \). The remaining fraction, \( 1 - f_R \), is awarded
to consumers. We will consider the arguably realistic setting where it is prohibitively costly
for the regulator to implement perfect oversight, so \( r \in (\frac{1}{2}, 1) \) in equilibrium.\(^9\)

To illustrate regulatory penalties that have been imposed in practice, consider the state
performance remedy plans that were imposed on the regional Bell operating companies
(RBOCs) in the early 2000’s as a condition for providing long distance (interLATA) telephone

\(^5\)Recall from Kreps and Scheinkman (1983) that Cournot competition can be viewed as reflecting outcomes
that arise in settings where suppliers first choose production capacities and then engage in price competition.

\(^6\)“Downstream” costs include any costs associated with employing access to produce the retail product. \( V \)
can increase \( E \)’s downstream costs by, for example, requiring \( E \) to incur expenses that are truly unnecessary,
but are alleged to be essential to ensure reliable, secure access to \( V \)’s network.

\(^7\)\( K(r) \) can be viewed as a fixed cost of establishing and implementing the capabilities, policies, and protocols
required to provide an ongoing assessment of \( V \)’s actions that is accurate with probability \( r \).

\(^8\)The regulator imposes no penalty on \( V \) if her industry oversight leads her to conclude that \( V \) has not
undertaken the anticompetitive action.

\(^9\)Conclusion 2 in the Appendix provides a sufficient condition for \( r \in (\frac{1}{2}, 1) \) in equilibrium.
service in the state. The plans specified performance standards that the relevant RBOC was expected to achieve in providing wholesale services to competitive local exchange carriers (CLECs). The plans typically required the RBOCs to make a specified per-incident payment (often on the order of $75) to a CLEC for sub-standard performance that directly hindered the CLEC’s ability to serve its retail customers (e.g., a relatively lengthy delay in restoring lost service to the CLEC). Some plans also required the RBOC to make payments to the State Treasury if the RBOC was found to have delivered sub-standard performance on activities that affected industry competition more broadly (e.g., a malfunction of the RBOC’s operational support service) or if the quality of service the RBOC delivered to all CLECs on average was judged to be sub-standard.\(^\text{10}\)

\(V\) may also face antitrust sanctions for allegedly having engaged in anticompetitive behavior. \(D_C\) will denote the financial penalty \(V\) must pay if the antitrust court concludes that \(V\) has undertaken the anticompetitive action. We capture antitrust review most simply by letting \(d(\alpha) \in (0, 1)\) denote the exogenous probability that \(V\) incurs court penalty \(D_C\) when it undertakes action \(\alpha\). We assume \(d(\overline{\alpha}) > d(\underline{\alpha})\), so the court review is more likely to conclude that \(V\) has undertaken the anticompetitive action when, in fact, it has done so. \(f_C \in [0, 1]\) is the fraction of any court penalty imposed on \(V\) that is awarded to \(E\). The residual fraction, \(1 - f_C\), is awarded to consumers.\(^\text{11,12}\)

\(x_v(w, c_i)\) and \(x_e(w, c_i)\) will denote the equilibrium retail outputs of \(V\) and \(E\), respectively, when the access price is \(w\) and \(E\)’s downstream marginal cost is \(c_i \in \{c_L, c_H\}\). \(Y(r, D_R; \alpha)\) will denote the regulatory penalty that \(V\) expects to incur when it undertakes


\(^{11}\text{\(f_C\) will be 1 if all court activity reflects lawsuits filed by \(E\). \(f_C\) can be less than 1 if the court activity reflects actions by, say, the Department of Justice, the Federal Trade Commission, or a group explicitly representing industry consumers. For simplicity, we abstract from litigation costs, so \(V\) always faces a court review. Alternative formulations are discussed in the concluding section.}\)

\(^{12}\text{This formulation avoids the need to model formally the decision of \(E\), a government agency, or other interested party to initiate antitrust action against \(V\). The formulation thereby avoids the need to specify precisely the objectives, costs, and information that underlie decisions to pursue antitrust actions in practice. These data can be viewed as influencing the exogenous \(d(\underline{\alpha})\) and \(d(\overline{\alpha})\) probabilities in our model.}\)
action $\alpha \in \{ \alpha, \overline{\alpha} \}$, the regulatory oversight accuracy is $r$, and the regulatory penalty for anticompetitive behavior is $D_R$. This expected penalty is larger when $V$ undertakes the anticompetitive action than when it undertakes the competitive action. Formally:

$$Y(r, D_R; \overline{\alpha}) = r D_R \geq [1 - r] D_R = Y(r, D_R; \alpha). \quad (1)$$

$V$’s expected upstream profit when it undertakes action $\alpha$, given $w$, $r$, and $D_R$, is:

$$\pi_u(w, r, D_R; \alpha) = [w - u] [q(\alpha) x_e(w, c_H) + (1 - q(\alpha)) x_e(w, c_L)] - F_u - Y(r, D_R; \alpha) - d(\alpha) D_C. \quad (2)$$

$V$’s corresponding expected total profit is:

$$\pi_v(w, r, D_R; \alpha) = \pi_u(w, r, D_R; \alpha) + q(\alpha) [P(x_v(w, c_H) + x_e(w, c_H)) - u - c_v] x_v(w, c_H) + [1 - q(\alpha)] [P(x_v(w, c_L) + x_e(w, c_L)) - u - c_v] x_v(w, c_L) - F_d. \quad (3)$$

The regulator seeks to maximize the difference between expected consumer welfare and regulatory monitoring costs while ensuring that $V$ anticipates a normal profit from its upstream operations.\(^{13}\) This profit is normalized to 0. Expected consumer welfare is the sum of expected consumer surplus and the expected revenue from penalties imposed on $V$ that is awarded to consumers. Let $S(w, c_e)$ denote equilibrium consumer surplus when the access price is $w$ and $E$’s downstream unit production cost is $c_e$. Then when $V$ undertakes the competitive action $\overline{\alpha}$, expected consumer welfare, given $w$, $r$, and $D_R$, is:

$$S(w, r, D_R; \overline{\alpha}) \equiv q(\overline{\alpha}) S(w, c_H) + [1 - q(\overline{\alpha})] S(w, c_L) + [1 - r] [1 - f_R] D_R + d(\overline{\alpha}) [1 - f_C] D_C. \quad (4)$$

Observe from expression (4) that $V$ may be penalized for allegedly having engaged in anticompetitive behavior even when it has not done so because regulatory oversight and antitrust review are imperfect (i.e., because $r < 1$ and $d(\overline{\alpha}) > 0$).

We will focus on the setting of primary interest where the regulator induces $V$ to un-

\(^{13}\)As Carlton (2007) observes, “The Department of Justice and the Federal Trade Commission, the two federal antitrust agencies [in the United States], often state that their focus is on consumers, which seems to imply a focus on consumer surplus.”
dertake the competitive action rather than the anticompetitive action.\textsuperscript{14} In this setting, the regulator’s problem, [RP], is:

\[
\begin{align*}
\text{Maximize} & \quad W \equiv S(w, r, D_R; \alpha) - K(r) \\
\text{subject to:} & \quad \pi^u_v(w, r, D_R; \alpha) \geq 0 \quad \text{and} \quad \pi_v(w, r, D_R; \alpha) \geq \pi_v(w, r, D_R; \bar{\alpha}). \quad (5)
\end{align*}
\]

The first constraint in expression (5) – the participation constraint – ensures that \( V \) anticipates a normal profit from its upstream operations when it undertakes the competitive action. The second constraint in expression (5) – the incentive compatibility constraint – ensures that \( V \) will undertake the competitive action rather than the anticompetitive action.

The ensuing analysis will focus on the setting of primary interest where both of the constraints in expression (5) bind at the solution to [RP]. The participation constraint will bind when \( V \)’s upstream fixed cost of production \( (F_u) \) is sufficiently large. The incentive compatibility constraint will bind when: (i) \( F_e \) and \( c_H \) are sufficiently small, so \( E \) will operate in equilibrium; (ii) \( a \) and \( [q(\bar{\pi}) - q(\alpha)] [c_H - c_L] \) are relatively large, so the impact of the anticompetitive action is pronounced; and (iii) \( [d(\bar{\alpha}) - d(\alpha)] D_C \) is not too large, so court oversight alone is insufficient to deter \( V \) from undertaking the anticompetitive action.\textsuperscript{15}

The timing in the model is as follows. First, the regulator chooses the accuracy \( (r) \) of her oversight and sets the access price \( (w) \) and the regulatory penalty \( (D_R) \) to maximize net consumer welfare, \( W \). Second, \( V \) determines whether to undertake the competitive or the anticompetitive action. Third, \( E \)’s downstream unit cost of production \( (c_e \in \{c_L, c_H\}) \) is determined and the two suppliers choose their retail outputs simultaneously and non-

\textsuperscript{14} The regulator will optimally induce \( V \) to undertake action \( \alpha \) when \( q(\bar{\pi}) - q(\alpha) \), \( c_H - c_L \), and \( a \) are sufficiently large (so the anticompetitive action reduces expected surplus substantially) and when \( F_e \), \( c_L - c_e \), and \( K(1) \) are sufficiently small (so \( E \) is a relatively efficient producer and regulatory monitoring costs are not excessive). In a richer model that admitted a broad range of behaviors in which \( V \) might engage, the regulator would choose the optimal extent to which \( V \)’s anticompetitive behavior should be limited.

\textsuperscript{15} Conclusion 2 in the Appendix specifies the conditions under which both constraints in expression (5) bind at the solution to [RP]. The conditions include the requirement that \( a \) (the intercept of the industry inverse demand curve) is sufficiently large. See Assumption 1 in the Appendix, which is maintained throughout the ensuing analysis.
cooperatively. Fourth, the market-clearing price is determined and consumer demand for
the retail product is fulfilled. Fifth, the regulator and the court assess V’s behavior, and any
resulting penalties are assessed.

3 Regulatory Policy Design

Before assessing the incremental value of antitrust review and the impact of antitrust re-
view on regulatory policy, we briefly examine selected determinants of the optimal regulatory
policy, taking the nature of the prevailing antitrust review as given.

First consider the penalty \( D_R \) the regulator imposes on V when her ongoing industry
oversight leads her to conclude that V has undertaken the anticompetitive action. It is
conceivable that the regulator might set \( D_R \) below its maximum feasible level, \( \overline{D}_R \). This is
the case because, due to the regulator’s imperfect industry oversight \( (r < 1) \), V may incur
a regulatory penalty even when it refrains from the anticompetitive action. To ensure V’s
participation, the regulator must compensate V for this equilibrium expected penalty. To
limit this (costly) compensation, the regulator might conceivably choose to impose less than
the maximum feasible penalty on V (i.e., to set \( D_R \) below \( \overline{D}_R \) at the solution to \([RP]\)).

In practice, though, the penalties that regulators are authorized to implement can be quite
limited. To reflect these common institutional constraints, we focus on settings where the
regulator imposes the maximum feasible penalty on V whenever her oversight indicates that
V has undertaken the anticompetitive action. Observation 1 provides a sufficient condition
for this policy to be optimal.

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\[\text{16} \text{ We assume } V \text{ and } E \text{ both serve retail customers in equilibrium. Sufficient conditions are provided in}
\text{Lemma 2 in the Appendix.}\]

\[\text{17} \text{ Critics routinely charged that the penalties specified in the state performance remedy plans that were}
\text{imposed on RBOCs in the early 2000s were insufficient to deter anticompetitive behavior. See, for example,}
\text{Southwestern Bell Telephone Company (2000, p. 84).}\]

\[\text{18} \text{ This assumption facilitates a solution to } [RP] \text{ by effectively reducing the number of instruments the regu-}
\text{lator controls from three to two.}\]
Observation 1. \( D_R = \overline{D}_R \) at the solution to \([RP]\) if:

\[
[a - u + c_v - 2\zeta]^2 > \left[ \frac{2a - 2u - \zeta - c_v}{3(1 - f_R)} \right]^2 + 24b \left[ \frac{\overline{D}_R}{2} + d(\alpha) D_C + F_u \right]. \tag{6}
\]

Observation 1 identifies four elements of the environment – in addition to a relatively small magnitude of \( \overline{D}_R \) – that lead the regulator to systematically impose the maximum feasible penalty on \( V \). First, \( f_R \) is close to 0, so most of the regulatory penalty is awarded to consumers, rather than to \( E \). Second, \( d(\alpha) D_C \) is small, so imperfections in the antitrust review process do not impose large equilibrium expected penalties on \( V \). Third, \( F_u \) is small, so the regulator can set \( w \) close to \( u \) without reducing \( V \)'s expected upstream profit below 0. The resulting relatively low upstream profit margin \( (w - u) \) increases \( V \)'s incentive to raise \( E \)'s costs because the associated reduction in \( E \)'s demand for the input does not reduce \( V \)'s upstream profit substantially. In this setting, imposing the maximum feasible regulatory penalty on \( V \) can be valuable in precluding \( V \) from undertaking the anticompetitive action. Fourth, \( c_v \) is large, so \( V \) is relatively inclined to raise \( E \)'s cost to help offset \( V \)'s limited cost advantage (or its cost disadvantage).

We now consider selected determinants of the regulator’s other policy instruments – the access price \( (w) \) and the accuracy of industry oversight \( (r) \). Observation 2 reports how the regulator adjusts these instruments as \( V \)'s ability to raise its rival’s cost increases, i.e., as \( \overline{q} \) increases or \( \underline{q} \) decreases, so \( V \)'s pursuit of the anticompetitive action increases \( E \)'s expected unit cost of production more substantially.

**Observation 2.** As \( V \)'s ability to raise its rival’s cost increases, the regulator increases the accuracy of her oversight and reduces the access price \( (i.e., \frac{\partial w}{\partial \overline{q}} > 0, \frac{\partial r}{\partial \underline{q}} < 0, \frac{\partial w}{\partial \overline{q}} < 0, \text{ and } \frac{\partial w}{\partial \underline{q}} > 0 \) at the solution to \([RP]\)).

As \( V \)'s anticompetitive action becomes more effective at raising \( E \)'s cost, the action becomes more profitable for \( V \) to pursue, *ceteris paribus*. To induce \( V \) to refrain from this otherwise relatively profitable action, the regulator increases the accuracy of her oversight.
The increased accuracy reduces the expected regulatory penalty that \( V \) anticipates in equilibrium (when it undertakes the competitive action). Consequently, the regulator is able to reduce \( w \) while still ensuring nonnegative upstream profit for \( V \).

It can be shown that the regulator will also increase \( r \) and reduce \( w \) as consumer demand for the retail product \( (a) \) increases or as \( V \)'s upstream production cost \( (u \text{ or } F_u) \) declines. When \( a \) increases, \( E \) produces more output, and so purchases more of the input from \( V \). Consequently, the regulator can reduce \( w \) without reducing \( V \)'s expected upstream profit below 0. The reduction in \( w \) and the increase in the scale of the retail market increases \( V \)'s incentive to increase \( E \)'s unit cost of production. To ensure that \( V \) refrains from the anticompetitive action, the regulator increases the accuracy of her oversight.

When \( u \) or \( F_u \) declines, the regulator can reduce \( w \) without reducing \( V \)'s upstream profit below 0. Furthermore, because a reduction in \( w \) increases \( E \)'s equilibrium output, the regulator can reduce \( w \) by more than \( u \) declines while still ensuring nonnegative upstream profit for \( V \). The reduction in \( w - u \) makes sales of the input to \( E \) less profitable for \( V \), which reduces \( V \)'s opportunity cost of reducing \( E \)'s retail output. The regulator increases the accuracy of her industry oversight to counteract \( V \)'s increased incentive to raise its rival’s cost.

One might suspect that the regulator would increase the accuracy of her industry oversight as the accuracy become less costly to improve. However, as Observation 3 reports, this is not the case in the present setting.

**Observation 3.** *Changes in the cost of improving the accuracy of regulatory oversight do not affect either the optimal access charge or the optimal accuracy of the industry oversight (i.e., \( \frac{\partial r}{\partial k} = \frac{\partial w}{\partial k} = 0 \) at the solution to [RP]).*

Observation 3 reflects the fact that \( k \) does not directly affect \( V \)'s expected profit, regardless of the action \( V \) undertakes. Consequently, changes in \( k \) do not affect the trade-offs the regulator faces as she chooses \( w \) and \( r \) to induce \( V \) to undertake the competitive action while ensuring zero upstream profit for \( V \). In particular, there is a single \((w, r)\) pair that contin-
ues to solve both the participation constraint and the incentive compatibility constraint (in expression (5)) as \( k \) changes. The maximum level of net consumer welfare \( (W) \) the regulator can achieve declines as \( k \) increases, but the best way to achieve the (now lower) \( W \) does not change.\(^{19}\)

Having examined how the regulator adjusts her policy instruments in response to selected changes in the environment in which she operates, we now consider the interaction between the optimal regulatory policy and the prevailing antitrust policy, and assess the merits of subjecting \( V \) to both antitrust review and regulatory oversight.

## 4 Regulatory and Antitrust Policy Interactions

We begin by considering how the optimal regulatory policy changes as key elements of the antitrust review change. Proposition 1 characterizes the relevant changes in the accuracy of the regulator’s industry oversight. The proposition refers to \( d = d(\alpha) \) and \( \overline{d} = d(\pi) \).

**Proposition 1.** Antitrust review and regulatory oversight are substitutes in the sense that the regulator reduces the accuracy of her industry oversight as either the court penalty increases or the court’s ability to detect \( V \)’s anticompetitive action accurately increases. In contrast, the regulator increases the accuracy of her industry oversight as the court becomes less able to verify \( V \)’s competitive action accurately (i.e., \( \frac{\partial r}{\partial D_C} < 0 \), \( \frac{\partial r}{\partial d} < 0 \), and \( \frac{\partial r}{\partial d} > 0 \) at the solution to \([RP]\)).

Proposition 1 reflects the following considerations. When the court penalty \( (D_C) \) or the court’s ability to detect \( V \)’s anticompetitive behavior accurately \( (\overline{d}) \) increases, \( V \)’s incentive to undertake the anticompetitive action declines, *ceteris paribus*. Consequently, the regulator can reduce the costly resources she devotes to improving the accuracy of her industry oversight \( (r) \) without inducing \( V \) to undertake the anticompetitive action.

When \( \overline{d} \) increases, antitrust review becomes less reliable in the sense that \( V \) becomes more

\(^{19}\)Observation 3 reflects the binary nature of \( V \)’s action. More generally, a regulator might well increase \( r \) as \( k \) declines in settings where \( V \)’s equilibrium action can change as industry conditions change.
likely to incur the court penalty even though it has refrained from the anticompetitive action. The less reliable antitrust review reduces $V$’s incentive to undertake the competitive action. To enhance this incentive (and thereby ensure that $V$ refrains from the anticompetitive action), the regulator increases the accuracy of her oversight.

Changes in the key elements of the antitrust review also affect the optimal access charge, as Proposition 2 reports.

**Proposition 2.** The regulator increases the access charge when the court penalty increases, the court’s ability to detect $V$’s anticompetitive action accurately increases, or the court’s ability to verify $V$’s competitive action accurately declines (i.e., $\frac{\partial w}{\partial D_c} > 0$, $\frac{\partial w}{\partial d} > 0$, and $\frac{\partial w}{\partial d} > 0$ at the solution to $[RP]$).

When the court penalty increases, $V$ faces a higher expected court penalty even when it undertakes the competitive action. The associated reduction in $r$ also increases the (regulatory) penalty that $V$ anticipates in equilibrium. (Recall Proposition 1.) The regulator increases $w$ to ensure that $V$ continues to earn a normal upstream profit in equilibrium.

Recall from Proposition 1 that the regulator reduces the accuracy of her industry oversight as the court’s ability to detect $V$’s anticompetitive action accurately increases. The reduced regulatory accuracy increases the equilibrium regulatory penalty that $V$ anticipates. The regulator increases $w$ to ensure $V$ continues to earn a normal profit from its upstream operations in equilibrium. The regulator also increases the access charge ($w$) to help compensate $V$ for the increased court penalty it anticipates in equilibrium as the court’s ability to verify $V$’s competitive action accurately decreases.

Having determined how the optimal regulatory policy changes as the prevailing antitrust review changes, it remains to assess when antitrust review is a useful complement to regulatory oversight. To do so, we consider how changes in the magnitude of the court penalty ($D_c$) affect net consumer welfare. An increase in $D_c$ can be viewed as a source of increased antitrust discipline. Propositions 3 and 4 examine how this increased discipline affects net
consumer welfare.

Proposition 3. Net consumer welfare increases as the court penalty increases (i.e., \( \frac{dW}{dD_C} > 0 \) at the solution to \([RP]\)) if \( D_R \) is sufficiently small or if \( f_R \geq f_C \).

Proposition 3 reveals that increased antitrust discipline in the form of a higher court penalty increases net consumer welfare either when the maximum feasible regulatory penalty (\( D_R \)) is sufficiently small or when a relatively large fraction of the court penalty is awarded to consumers (i.e., when \( 1 - f_C \) exceeds \( 1 - f_R \)). When \( D_R \) is small, regulatory oversight is unable to impose substantial discipline on \( V \). Consequently, increased antitrust discipline is useful in deterring \( V \) from undertaking the anticompetitive action.

When consumers receive a relatively large fraction of the court penalty, they benefit directly from increased expected penalty revenue as \( D_C \) increases. Consumers also benefit as increases in \( D_C \) induce the regulator to reduce the accuracy of her industry oversight (\( r \)). (Recall Proposition 1.) Consumers gain from both the reduced regulatory oversight costs and the associated increase in equilibrium regulatory penalty revenue.

To illustrate the gains that consumers can experience as \( D_C \) increases, consider:

Example 1. \( a = 10, b = 1, c_v = 4, c_L = 1, c_H = 3, u = 0, k = 1, F_u = 2, \)
\[
q = 0.2, \text{ and } \overline{q} = 0.8.
\]

Suppose \( D_C \) increases from 0 (in which case there is effectively no antitrust discipline) to 2 in the setting of Example 1. Net consumer welfare (\( W \)) increases by 3.7\% (from 11.626 to 12.059) when \( d = 0.4, \overline{d} = 0.6, D_R = 2, \) and \( f_C = f_R = 0.1 \). The corresponding increase in \( W \) is 3.8\% (from 11.626 to 12.062) when \( d = 0.35, \overline{d} = 0.65, D_R = 2, \) and \( f_C = f_R = 0.1 \), so the antitrust review discerns \( V \)'s action with greater accuracy. The corresponding increase in \( W \) is 5.5\% (from 11.237 to 11.850) when \( d = 0.4, \overline{d} = 0.6, D_R = 1, \) and \( f_C = f_R = 0.1 \), so the maximum potential regulatory discipline is more limited. The corresponding increase in \( W \) is 4.4\% (from 11.219 to 11.714) when \( d = 0.4, \overline{d} = 0.6, D_R = 1, f_C = 0.1, \) and \( f_R = 0.5 \), so a relatively large fraction of the court
penalty accrues to consumers.

The fraction of the court penalty that accrues to consumers \((1 - f_C)\) may be relatively small when antitrust review primarily results from private lawsuits filed by industry competitors. In such settings, consumers can be harmed from increased antitrust discipline in the form of a larger court penalty.

**Proposition 4.** Net consumer welfare declines as the court penalty increases (i.e., \(\frac{dW}{dD_C} < 0\) at the solution to [RP]) if \(f_C > f_R\), and

\[
\left[ \frac{k}{D_R} + 1 - f_R \right] \left[ \frac{d - d}{d} \right] < 2 \left[ f_C - \frac{2}{3} \right].
\]  

Proposition 4 confirms that increased antitrust discipline in the form of a higher court penalty can reduce net consumer welfare when a relatively small fraction of the court penalty accrues to consumers (so \(f_C\) is sufficiently large). In this event, consumers receive little of the additional court penalty revenue and are harmed by the higher retail price that results from the increased access charge the regulator establishes to compensate \(V\) for the increase in its equilibrium expected court penalty. (Recall Proposition 2.)

Proposition 4 also indicates that, *ceteris paribus*, a higher court penalty is more likely to reduce net consumer welfare when: (i) the antitrust review is inaccurate in the sense that \(\frac{d - d}{d}\) is small; (ii) the cost of increasing the accuracy of regulatory oversight \((k)\) is small; and (iii) potential regulatory discipline \((D_R)\) is pronounced. When antitrust review is inaccurate, a higher court penalty provides little incremental deterrence because \(V\) may well incur the large penalty even when it undertakes the competitive action. Furthermore, the increase in \(D_C\) increases the court penalty that \(V\) expects to incur in equilibrium and thereby increases the extent to which the regulator must increase \(w\) in order to ensure nonnegative upstream profit for \(V\).

When \(k\) is small and \(D_R\) is large, it is not very costly for the regulator, acting alone, to create strong incentives for \(V\) to undertake the competitive action. When the cost of
establishing accurate industry oversight is low, the regulator will do so. Consequently, $V$ recognizes that it is likely to be penalized severely if (and only if) if it undertakes the anticompetitive action. Consequently, $V$ will refrain from this action even in the absence of antitrust discipline.

Proposition 4 holds for any value of $D_C$. Consequently, net consumer welfare declines under the identified conditions identified in the proposition whenever non-trivial antitrust review is added to regulatory oversight. This conclusion is stated formally, for emphasis.

**Corollary.** Suppose $D_C > 0$ and the conditions identified in Proposition 4 hold. Then net consumer welfare is higher in the absence of antitrust review than in its presence.

To illustrate the magnitude of the losses that antitrust review can introduce, consider:

**Example 2.** $a = 10$, $b = 1$, $c_v = 4$, $c_L = 1$, $c_H = 3$, $u = 0$, $k = 1$, $F_u = 2$, $\bar{D}_R = 2$, $q = 0.2$, $\bar{q} = 0.8$, $d = 0.5$, $\bar{d} = 0.6$, $f_R = 0.1$, and $f_C = 0.9$.

Example 2 has two distinguishing features. First, little of the court penalty accrues to consumers whereas most of the regulatory penalty accrues to consumers. Second, the antitrust review entails considerable inaccuracy, as the likelihood that the court concludes $V$ has undertaken the anticompetitive action does not vary substantially with $V$’s actual behavior.

Expected net consumer welfare ($W$) declines by 4.4% (from 11.139 to 10.645) in the setting of Example 2 when court review is introduced and the court penalty ($D_C$) is 2.0. The reduction in $W$ arises when the increased court penalty induces the regulator to reduce the accuracy of her industry oversight (from $r = 0.65$ to $r = 0.53$) and increase the access price (from $w = 0.85$ to $w = 1.41$).\(^{20}\) It bears emphasis that this reduction in $W$ arises under arguably plausible conditions even though antitrust review entails no direct costs.

Although antitrust review can reduce net consumer welfare, any antitrust review that is implemented generates a higher level of net consumer welfare as its ability to detect $V$’s

\(^{20}\) The increased access price causes the equilibrium retail price to increase from 5.42 to 5.60.
anticompetitive action accurately increases.

**Proposition 5.** *Net consumer welfare increases as antitrust review becomes better able to detect V’s anticompetitive action accurately (i.e., $\frac{dW}{dd} > 0$ at the solution to [RP]).*

Proposition 5 reflects the fact that V’s incentive to undertake the anticompetitive action declines as the court becomes better able to detect V’s anticompetitive action accurately. Consequently, the regulator can conserve on oversight costs by reducing the accuracy of regulatory oversight without inducing V to undertake the anticompetitive action. The reduced oversight costs enhance net consumer welfare.

Proposition 6 reports the related conclusion that net consumer welfare often declines as the ability of the antitrust review to detect V’s competitive action declines.

**Proposition 6.** *Suppose $f_C > \max\{f_R - \frac{k}{DR}, \frac{1}{2} f_R + \frac{1}{6}\}$. Then net consumer welfare declines as the court becomes less able to detect V’s competitive action accurately (i.e., $\frac{dW}{dd} < 0$ at the solution to [RP]).*

As $d$ increases, $V$ becomes more likely to incur the court penalty even when it undertakes the competitive action. Although consumers benefit from the increased court penalty revenue, they receive little of the increased revenue when $f_C$ is sufficiently large, as specified in Proposition 6. Furthermore, consumers are harmed by the higher retail price that results from the higher access charge the regulator sets to offset the increase in V’s expected court penalty. (Recall Proposition 2.)

Perhaps more surprisingly, consumers can benefit from a reduction in the court’s ability to detect V’s competitive action accurately.

**Proposition 7.** *Suppose $D_C > 0$ and $f_C < f_R - \frac{k}{DR}$. Then net consumer welfare increases as the court becomes less able to detect V’s competitive action accurately (i.e., $\frac{dW}{dd} > 0$ at the solution to [RP]).*

As $d$ increases, $V$ becomes more likely to incur the court penalty in equilibrium. Con-
sumers benefit from the increase in equilibrium expected court penalty revenue. Proposition 7 reports that when the fraction of the court penalty that accrues to consumers \((1 - f_C)\) is sufficiently large relative to the fraction of the regulatory penalty that accrues to consumers \((1 - f_R)\), this direct gain for consumers can outweigh the deleterious effects of an increase in \(d\). These deleterious effects include a reduction in the equilibrium regulatory penalty revenue that consumers anticipate as the regulator increases the accuracy of her oversight as \(d\) increases. (Recall Proposition 1).

In essence, Proposition 7 indicates that when consumers are awarded a relatively large fraction of the revenue generated by court penalties, efforts to enhance the accuracy of antitrust oversight can be counterproductive because they can reduce the incidence of court penalties that flow to consumers in equilibrium.\(^{21,22}\)

5 Conclusions

We have analyzed a simple model to begin to assess the merits of antitrust review in regulated industries. We found that antitrust review can enhance consumer welfare in many plausible settings. However, antitrust review can be counterproductive in some settings even when the review entails no direct costs. Antitrust review can reduce consumer welfare, for example, when the review is not particularly adept at distinguishing between competitive and anticompetitive behavior, when the regulator is empowered to impose large financial penalties if she detects anticompetitive behavior, when a relatively large fraction of these penalties accrue to consumers, and when the regulator faces relatively low costs of implementing accurate industry oversight.

Our finding that antitrust review can either enhance or reduce consumer welfare in reg-

\(^{21}\)This conclusion reflects our focus on net consumer welfare. Total expected surplus generally increases as the court’s ability to detect \(V\)’s competitive action accurately increases.

\(^{22}\)Net consumer welfare can increase as \(d\) increases even when \(f_C = f_R\) if the court review is substantially less accurate than the prevailing regulatory oversight. To illustrate, suppose \(\bar{d} = 0.6, D_C = \bar{D}_R = 2\), and \(f_C = f_R = 0.1\) in the setting of Example 1. Then \(W\) increases from 12.027 to 12.087 as \(d\) increases from 0.3 to 0.5. Expected total welfare (the sum of net consumer welfare and industry profit) declines from 24.713 to 24.514.
ulated industries supports the view that the merits of such review are best assessed on a case-by-case basis (e.g., Tirole, 2004; Weiser, 2005). Antitrust review may be valuable in settings when regulators have limited authority to penalize the incumbent supplier for anticompetitive behavior or when limited resources or other institutional constraints make it difficult for regulators to establish accurate industry oversight. In contrast, antitrust review may be counterproductive in rapidly changing industries where only regulators possess the sophisticated expertise required to assess the true intent of the actions of incumbent suppliers.

Our analysis also explored how regulatory policy optimally adapts to changes in the characteristics of prevailing antitrust policy. We showed that regulatory and antitrust oversight are substitutes in the sense that the regulator optimally reduces the resources she devotes to improving the accuracy of her industry oversight as antitrust review becomes better able to assess the true intent of the incumbent supplier’s actions. We also determined how regulated access prices optimally respond to changes in the characteristics of the prevailing antitrust review process.

We found that the interactions between antitrust and regulatory policy can be subtle and varied even in the streamlined model that we examined. Additional subtlety and complexity can emerge more generally. To illustrate, in practice, the accuracy of antitrust review may vary with the accuracy of the established regulatory oversight, in part because the facts presented in court may reflect findings by industry regulators. The regulator’s decision about the optimal accuracy of her industry oversight will become more complex when the accuracy affects both regulatory governance and the outcomes of antitrust reviews.

In practice, of course, both the antitrust review process and the regulatory oversight

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23 This conclusion is also consistent with the U.S. Supreme Court (2004, § IV)’s observation that “Antitrust analysis must always be attuned to the particular structure and circumstances of the industry at issue.”

24 We have not explicitly modeled the sequencing of the regulatory and court monitoring. Spulber and Yoo (2007, p. 1855) observe that “[a]lthough the presence of regulatory schemes may not provide antitrust immunity, it may nonetheless forestall judicial consideration of the merits of claims until after the relevant agency has had the opportunity to address the issues in the first instance.”
process are endogenous. We did not model the former endogeneity explicitly for analytic simplicity. Additional strategic issues can arise when the entrant can exercise explicit control over the antitrust review of the incumbent supplier’s actions. Moderate litigation costs could encourage the entrant to sue the incumbent only when it has undertaken an anticompetitive action, and could thereby enhance the efficacy of antitrust review. However, if the court is prone to substantial Type I error and potential court-awarded damages substantially exceed relevant litigation costs, the entrant may sue the incumbent for anticompetitive behavior even when the entrant knows the incumbent has refrained from such behavior.\textsuperscript{25}

Future research should account for additional actions by the regulator. For example, the regulator might influence the technology employed in the industry.\textsuperscript{26} In settings where antitrust review would harm consumers, the regulator might encourage the incumbent to adopt an inefficient technology that precludes the operation of additional industry competitors.\textsuperscript{27} By doing so, the regulator may be able to enhance consumer welfare by avoiding counterproductive antitrust reviews.

A broader array of actions by the incumbent supplier also merit consideration.\textsuperscript{28} In practice, incumbent suppliers can choose among different types of anticompetitive activities and may have considerable flexibility in determining the extent to which each activity is pursued.\textsuperscript{29} In such settings, the regulator will face the additional task of determining the extent to which various anticompetitive activities should be deterred.

Future research should also account for the more severe information asymmetries that

\textsuperscript{25}McAfee et al. (2008) provide a formal analysis of such considerations. Spulber and Yoo (2007, p. 1863) observe that “[p]ermitting courts to entertain antitrust suits might simply invite disappointed parties who failed to obtain relief from the agency to try to take a second bite of the apple.”

\textsuperscript{26}Alternatively, the regulator might be able to choose the short-term investigative effort she devotes to assessing the merits of specific complaints lodged by potential or actual industry competitors.

\textsuperscript{27}For example, the regulator may direct the incumbent supplier to install a network infrastructure that does not readily admit interconnection with other suppliers. The Supreme Court (2004, §IV) observes that judicial oversight in regulated industries “would seem destined to distort investment.”

\textsuperscript{28}Alternative forms of industry competition (e.g., price competition among suppliers of differentiated products in the presence of entry costs) also merit investigation.

\textsuperscript{29}To illustrate, an incumbent supplier might undertake actions that raise a rival’s fixed cost of production and/or reduce the demand for the rival’s services.
regulators often possess in practice. The regulator’s comprehensive industry knowledge in our model allowed her to predict the incumbent’s action perfectly and to deter the anticompetitive action at minimum social cost. In practice, regulators typically face more extensive uncertainty about the range of actions available to industry suppliers, the effects of these actions, and the precise manner in which industry suppliers will react to the incentives fashioned by regulatory (and antitrust) policy.

The role of antitrust review in counteracting regulatory capture (e.g., Stigler, 1971) also merits formal investigation. We modeled the regulator as a faithful representative of consumer interests. To the extent that regulators are motivated to promote the interests of industry suppliers, antitrust review may provide a useful check on supplier actions that regulators have explicitly or implicitly condoned.30

30 Antitrust review might also provide a useful check on a regulator’s actions when the regulator may be inclined to pursue unsustainable competition or to promote universal service or diversity excessively, for example. See Bourreau and Dogan (2001), Katz (2004), Rey (2004), Moss (2008), and Ginsburg (2009) for related discussions.
Appendix

Define \( \bar{d} \equiv d(\bar{\alpha}) \), \( \bar{\bar{d}} \equiv d(\bar{\alpha}) \), \( \bar{q} \equiv q(\bar{\alpha}) \), \( \bar{\bar{q}} \equiv q(\bar{\alpha}) \), and \( \bar{c} \equiv q c_H + [1 - \bar{q}] c_L \).

Conclusion 1. The regulator’s problem \([RP]\) is the following:

$$\text{Maximize} \quad W$$

subject to:

\[
\begin{align*}
\frac{w}{3b} [a + 3u + c_v - 2c] - \frac{2w^2}{3b} - \phi & \geq 0 \quad \text{and} \\
- \frac{1}{9b} [\bar{q} - \bar{q}] [c_H - c_L] [2a + 2u + c_H + c_L - 4c_v - 4w] \\
& + [2r - 1] D_R + [\bar{d} - \bar{d}] D_C \geq 0, \quad (8)
\end{align*}
\]

where

\[
W \equiv \frac{q}{18b} [2a - w - u - c_H - c_v]^2 + \left[ \frac{1 - \bar{q}}{18b} \right] [2a - w - u - c_L - c_v]^2 \\
- k \left[ r - \frac{1}{2} \right]^2 + [1 - r] D_R [1 - f_R] + [1 - f_C] \bar{d} D_C, \quad \text{and} \quad (10)
\]

\[
\phi \equiv \frac{u}{3b} [a + u + c_v - 2c] + [1 - r] D_R + \bar{d} D_C + F_u > 0. \quad (11)
\]

\textbf{Proof}. Standard techniques reveal that the (interior) profit-maximizing outputs of \( E \) and \( V \) are, respectively:

\[
x_e = \frac{1}{3b} [a + u + c_v - 2w - 2c] \quad \text{and} \quad x_v = \frac{1}{3b} [a + w + c_L - 2u - 2c_v]. \quad (12)
\]

Straightforward calculations then reveal that \( V \)’s expected profit when it undertakes the competitive action and the anticompetitive action are, respectively:

\[
\pi_v = \bar{q} \left\{ \frac{w - u}{3b} [a + u + c_v - 2w - 2c_H] + \frac{1}{9b} [a + w + c_H - 2u - 2c_v]^2 \right\} \\
+ [1 - \bar{q}] \left\{ \frac{w - u}{3b} [a + u + c_v - 2w - 2c_H] + \frac{1}{9b} [a + w + c_L - 2u - 2c_v]^2 \right\} \\
- F_u - F_d - [1 - r] D_R - \bar{d} D_C, \quad \text{and} \quad (13)
\]

\[\text{This Appendix outlines the proofs of the formal conclusions in the text. Bose et al. (2015) provide more detailed proofs.}\]
\[ \bar{\pi}_v = \bar{q} \left\{ \frac{w-u}{3b} \left[ a + u + c_v - 2w - 2c_H \right] + \frac{1}{9b} \left[ a + w + c_H - 2u - 2c_v \right]^2 \right\} \\
+ [1 - \bar{q}] \left\{ \frac{w-u}{3b} \left[ a + u + c_v - 2w - 2c_L \right] + \frac{1}{9b} \left[ a + w + c_L - 2u - 2c_v \right]^2 \right\} \\
- F_u - F_d - r D_R - \underline{d} D_C. \]

(12) implies that consumers’ surplus when \( E \) unit downstream cost is \( c_i \) is:

\[ S(c_i) = \frac{1}{2} X \left[ a - (a - b X) \right] = \frac{b}{2} X^2 = \frac{1}{18b} \left[ 2a - w - u - c_i - c_v \right]^2. \]

(13) implies that \( V \)'s upstream profit when it undertakes the competitive action is:

\[ \pi_v^u = \frac{w}{3b} \left[ a + 3u + c_v - 2c \right] - \frac{2w^2}{3b} - \phi, \]

so \( V \)'s participation constraint (PC) is:

\[ \frac{w}{3b} \left[ a + 3u + c_v - 2c \right] - \frac{2w^2}{3b} - \phi \geq 0. \]

(13) also implies that \( V \)'s expected profit when it undertakes the competitive action is:

\[ \pi_v = \frac{1}{9b} \left\{ w \left[ 5a + 5u - c_v - 4c \right] - 3u \left[ a + u + c_v - 2c \right] - 5w^2 \right\} \\
+ q \left[ a + c_H - 2u - 2c_v \right]^2 + [1 - q] \left[ a + c_L - 2u - 2c_v \right]^2 \\
- F_u - F_d - [1 - r] D_R - \underline{d} D_C. \]

Tedious calculations then reveal:

\[ \pi_v - \pi_v = - \frac{1}{9b} \left[ \bar{q} - \bar{q} \right] \left[ c_H - c_L \right] \left[ 2a + 2u + c_L + c_H - 4c_v - 4w \right] \\
+ \left[ 2r - 1 \right] D_R + \left[ \bar{d} - \bar{d} \right] D_C. \quad \square \]

Let \( a_1, a_2, a_3, \) and \( a_4 \) be defined by the following equations:

\[ \left[ a_1 - u + c_v - 2c \right]^2 = 24b \left[ \frac{D_R}{2} + \underline{d} D_C + F_u \right] + \frac{4}{9} \left[ \bar{q} - \bar{q} \right]^2 \left[ c_H - c_L \right]^2. \quad (15) \]

\[ \left[ \frac{1}{3} (\bar{d} + \underline{d}) - \left( \frac{k}{D_R} + 1 - f_R \right) (\bar{d} - \underline{d}) - 2\underline{d} \left( 1 - f_C \right) \right] \left[ 2a_2 - u - c_v - \underline{c} \right] \\
= \frac{4}{3} \underline{d} \left[ f_C + \frac{k}{D_R} - f_R \right] \left[ \bar{q} - \bar{q} \right] \left[ c_H - c_L \right]. \quad (16) \]
\[ 7a_3 - 7u - 5c_v - 2c_L = 16 \left( \frac{k}{D_R} + f_C - f_R \right) [q - \theta] [c_H - c_L]. \]  
(17)

\[ a_4 = \frac{1}{2f_C - f_R - \frac{1}{3}} \left\{ 3 \left[ f_R - 2f_C + \frac{10}{9} \right] u + \left[ f_R - 2f_C + \frac{4}{3} \right] c_v \right. \]
\[ - 2 \left[ f_R - 2f_C + \frac{5}{6} \right] c - \frac{4}{3} [\theta - q] [c_H - c_L] \left[ f_R - f_C - \frac{k}{D_R} \right] \}. \]  
(18)

Assumption 1 is maintained throughout the analysis.

**Assumption 1.** \( a > \max\{a_1, a_2, a_3, a_4, 7u + 2c_v, u + 2c_L - c_v\} \).

The following lemmas are employed to prove Conclusion 2 (which specifies the focus of the analysis in the text) and the formal conclusions in the text.

**Lemma 1.** If the incentive compatibility constraint binds at the solution to \([RP]\) and either \(D_R > 0\) or \(D_C > 0\), then:
\[ 2a + 2u + c_L + c_H - 4c_v - 4w > 0. \]  
(19)

**Proof.** (9) implies that if \(2a + 2u + c_L + c_H - 4c_v - 4w \leq 0\), then the incentive compatibility constraint cannot bind when \(D_R > 0\) and/or \(D_C > 0\).  

**Lemma 2.** \(E\) will produce strictly positive output in equilibrium if \(c_v > c_H\). \(V\) will produce strictly positive output in equilibrium if \(a > 2\left[ u + c_v - \frac{c_L + c_H}{2} \right] \).

**Proof.** From Lemma 1:
\[ 4w < 2a + 2u + c_L + c_H - 4c_v \Rightarrow 2w < a + u + \frac{c_L + c_H}{2} - 2c_v. \]
Therefore:
\[ a + u + c_v - 2w - 2c_H > a + u + c_v - a - u - \frac{c_L + c_H}{2} + 2c_v - 2c_H \]
\[ = 3c_v - \frac{c_L + c_H}{2} - 2c_H > 3[c_v - c_H] > 0 \text{ if } c_v > c_H. \]
Consequently, (12) implies that \(x_e > 0\) if \(c_v > c_H\).

Since \(w \geq 0\) and \(c_L < c_H\), (12) implies that if \(a > 2\left[ u + c_v - \frac{c_L + c_H}{2} \right]\), then:
\[ x_v = \frac{1}{3b} \left[ a + w + c_i - 2u - 2c_v \right] > \frac{1}{3b} \left[ a + c_L - 2u - 2c_v \right] > 0. \]  

Lemma 3. \( w \leq \frac{1}{4} [a + 3u + c_v - 2c] \equiv \bar{w} \) at a solution to [RP].

Proof. Define: \( g(w) \equiv \frac{w}{3b} [a + 3u + c_v - 2c] - \frac{2w^2}{3b} - \phi. \) \( \quad (20) \)

Observe that \( g(0) = -\phi < 0 \) and
\[ g'(w) = \frac{1}{3b} [a + 3u + c_v - 2c - 4w] \quad \Rightarrow \quad g''(w) = -\frac{4}{3b} < 0. \]

Therefore, \( g(w) \) is a concave function of \( w \). Consequently, since \( g(0) < 0 \), it must be the case that \( g'(0) > 0 \) if a real solution to the equation \( g(w) = 0 \) is to exist.

It is apparent from (20) that \( g(w) \) reaches a maximum at \( \bar{w} \equiv \frac{1}{4} [a + 3u + c_v - 2c] \).

Hence, \( g(w) = 0 \) has a unique solution if:
\[ \frac{\bar{w}}{3b} [a + 3u + c_v - 2c] - \frac{2\bar{w}^2}{3b} = \phi. \] \( \quad (21) \)

When \( \phi > \frac{\bar{w}}{3b} [a + 3u + c_v - 2c] - \frac{2\bar{w}^2}{3b} \), the equation \( g(w) = 0 \) does not have a real solution.

When \( \phi < \frac{\bar{w}}{3b} [a + 3u + c_v - 2c] - \frac{2\bar{w}^2}{3b} \), the equation \( g(w) = 0 \) has two solutions at \( \bar{w}_1 \) and \( \bar{w}_2 \), where \( \bar{w}_1 < \bar{w}_2 \). If the participation constraint (PC) is binding, then the regulator chooses \( w = \bar{w}_1 \) in this case to maximize aggregate welfare. If the PC is not binding, then the regulator chooses \( w \in (\bar{w}_1, \bar{w}) \). Therefore, \( w \leq \bar{w} \) at a solution to [RP]. \( \blacksquare \)

Lemma 4. If the participation constraint binds at the solution to [RP], then:
\[ 4w \leq a + 3u + c_v - 2c \]
\[ -\sqrt{[a + 3u + c_v - 2c]^2 - 24b \left[ \frac{u}{3b} (a + u + c_v - 2c) + \frac{DR}{2} + d D_C + F_u \right]} \] \( \quad (22) \)

Proof. See Bose et al. (2015).

Conclusion 2. If \( D_R \) is specified exogenously, then \( r \in (\frac{1}{2}, 1) \) and the participation and the incentive compatibility constraints both bind at a solution to [RP] if:
\[ \frac{1}{2b} \left[ \frac{\bar{d} - d}{\bar{q} - q} [c_H - c_L] \right] - \left( [a - u + c_v - 2c]^2 - 24b \left[ \frac{DR}{2} + d D_C + F_u \right] \right)^{\frac{1}{2}} \]
\[ < a - u + c_L + c_H - 5c_v + 2c \]
\[
< \frac{D_R + [d - \bar{d}] D_C}{\frac{1}{9b} [\bar{q} - \bar{q}] [c_H - c_L]} - \left( [a - u + c_v - 2 \bar{z}]^2 - 24b [d D_C + F_u] \right)^{\frac{1}{2}}, \text{ and} \tag{23}
\]

\[
[a - u + c_v - 2 \bar{z}]^2 > \left[ \frac{2a - 2u - \bar{z} - c_v}{3(1 - f_R)} \right]^2 + 24b \left[ \frac{D_R}{2} + d D_C + F_u \right]. \tag{24}
\]

**Proof.** See Bose et al. (2015).

The ensuing analysis will consider solutions to [RP] in which \( r \in \left( \frac{1}{2}, 1 \right) \) and the participation and incentive compatibility constraints both bind.

Lemma 5 is employed in the proofs of Observation 1 and Proposition 3.

**Lemma 5.** Suppose \( D_R \) is exogenous and the participation and incentive compatibility constraints both bind. Then \( \frac{\partial r}{\partial D_R} < 0 \) and \( \frac{\partial w}{\partial D_R} > 0 \) at the solution to [RP].

**Proof.** Differentiating (8) and (9) with respect to \( D_R \) provides:

\[
\Lambda \left[ \begin{array}{c}
\frac{\partial r}{\partial D_R} \\
\frac{\partial w}{\partial D_R}
\end{array} \right] = \begin{bmatrix}
1 - r \\
1 - 2r
\end{bmatrix} \text{ where } \Lambda \equiv \begin{bmatrix}
D_R & \frac{a + 3u + c_v - 2\bar{z} - 4w}{3b} \\
2D_R & \frac{4}{9b} [\bar{q} - \bar{q}] [c_H - c_L]
\end{bmatrix}. \tag{25}
\]

It is readily verified that \(| \Lambda | < 0 \) when Assumption 1 holds. Therefore:

\[
\frac{\partial w}{\partial D_R} = \frac{|\Omega_2|}{|\Lambda|} s = - |\Omega_2| \text{ where } \Omega_2 \equiv \begin{bmatrix}
D_R & 1 - r \\
2D_R & 1 - 2r
\end{bmatrix} \tag{26}
\]

\[
\Rightarrow \quad |\Omega_2| = |1 - 2r| D_R - 2|1 - r| D_R = -D_R. \tag{27}
\]

(26) and (27) imply \( \frac{\partial w}{\partial D_R} s = - |\Omega_2| = D_R > 0. \)

(25) implies \( \frac{\partial r}{\partial D_R} < 0 \), since \( \frac{\partial w}{\partial D_R} > 0 \) and \( r > \frac{1}{2}. \) \( \blacksquare \)

**Proof of Observation 1.**

(22) implies:

\[
\begin{align*}
a + 3u + c_v - 2\bar{z} - 4w &\geq \frac{2}{3} \left[ \bar{q} - \bar{q} \right] [c_H - c_L] \\
&\geq a + 3u + c_v - 2\bar{z} - \frac{2}{3} \left[ \bar{q} - \bar{q} \right] [c_H - c_L] - [a + 3u + c_v - 2\bar{z}]
\end{align*}
\]
\[ + \sqrt{[a + 3u + c_v - 2 \zeta]^2 - 24b \left[ \frac{u}{3b}(a + u + c_v - 2 \zeta) + \frac{D_R}{2} + d D_C + F_u \right]} \]

\[ > - \frac{2}{3} \left[ q - \bar{q} \right] [c_h - c_l] + \sqrt{\frac{4}{9} \left[ q - \bar{q} \right]^2 [c_h - c_l]^2} = 0. \quad (28) \]

The last inequality in (28) holds because:

\[ [a + 3u + c_v - 2 \zeta]^2 - 24b \frac{u}{3b} [a + u + c_v - 2 \zeta] \]

\[ = [a + u + c_v - 2 \zeta - 2u]^2 = [a - u + c_v - 2 \zeta]^2. \quad (29) \]

Define \( \alpha_1 \equiv \frac{2}{3} \left[ q - \bar{q} \right] [c_h - c_l] \) and \( \alpha_2 \equiv a + 3 + c_v - 2 \zeta - 4w. \) \quad (30)

(28) implies \( \alpha_2 > \alpha_1. \) Since \( |\Lambda| < 0, \) (26) and (27) imply:

\[ \frac{\partial w}{\partial D_R} = \left| \frac{\Omega_2}{\Lambda} \right| = \frac{-D_R}{\frac{2D_R}{3b} [\alpha_1 - \alpha_2]} = - \frac{3b}{2[\alpha_1 - \alpha_2]} > 0. \quad (31) \]

From (25):

\[ \frac{\partial r}{\partial D_R} = \left| \frac{\Omega_3}{\Lambda} \right| \quad \text{where} \quad \Omega_3 \equiv \begin{bmatrix} 1 - r & \frac{a + 3u + c_v - 2 \zeta - 4w}{3b} \\ 1 - 2r & \frac{4}{9b} \left[ q - \bar{q} \right] [c_h - c_l] \end{bmatrix} \quad (32) \]

\[ \frac{\partial r}{\partial D_R} < 0 \text{ from Lemma 5. Therefore, since } |\Lambda| < 0: \]

\[ \left| \Omega_3 \right| = \left| 1 - r \right| \frac{4}{9b} \left[ q - \bar{q} \right] [c_h - c_l] \]

\[ - \left| 1 - 2r \right| \left[ \frac{a + 3u + c_v - 2 \zeta - 4w}{3b} \right] > 0. \quad (33) \]

(30) and (33) imply:

\[ \left| \Omega_3 \right| = \frac{2 \left( 1 - r \right) \alpha_1 - \left[ 1 - 2r \right] \alpha_2}{3b} > 0 \quad (34) \]

\[ \Rightarrow 2 \left( 1 - r \right) \alpha_1 - \left[ 1 - 2r \right] \alpha_2 = 2 \left[ 1 - r \right] \left[ \alpha_1 - \alpha_2 \right] + \alpha_2 > 0. \quad (35) \]

(31), (32), and (34) provide:

\[ \frac{\partial r}{\partial D_R} = \frac{2 \left( 1 - r \right) \alpha_1 - \left[ 1 - 2r \right] \alpha_2}{\frac{2D_R}{3b} \left[ \alpha_1 - \alpha_2 \right]} = \frac{2 \left[ 1 - r \right] \alpha_1 - \left[ 1 - 2r \right] \alpha_2}{2D_R \left[ \alpha_1 - \alpha_2 \right]}. \quad (36) \]
Let $W^*$ denote the value of $W$ at the solution to [RP]. From (10):

$$
\frac{dW^*}{dD_R} = -\frac{1}{9b} \left[ 2a - w - u - \xi - c_v \right] \frac{\partial w}{\partial D_R} \\
- \left[ k (2r - 1) + D_R (1 - f_R) \right] \frac{\partial r}{\partial D_R} + [1 - r] [1 - f_R]
$$

$$
\Rightarrow \frac{dW^*}{dD_R} \geq 0 \Leftrightarrow \frac{1}{3} \left[ 2a - w - u - \xi - c_v \right] \\
- \frac{k [2r - 1]}{D_R} \left[ 2 (1 - r) (\alpha_1 - \alpha_2) + \alpha_2 \right] [1 - f_R] \alpha_2 \leq 0. \quad (37)
$$

Using results from the proof of Conclusion 2, it is readily verified that the expression in (37) is negative under the specified condition. Therefore, $\frac{dW^*}{dD_R} > 0$, so the regulator will set $D_R = \overline{D}_R$. ■

**Proof of Observation 2.**

Differentiating (8) and (9) with respect to $q$ provides:

$$
[\Lambda] \begin{bmatrix} \frac{\partial r}{\partial q} \\ \frac{\partial w}{\partial q} \end{bmatrix} = \begin{bmatrix} \frac{2}{3b} [w - u] [c_H - c_L] \\ -\frac{1}{9b} [c_H - c_L] [2a + 2u + c_L + c_H - 4c_v - 4w] \end{bmatrix}. \quad (38)
$$

Since $|\Lambda| < 0$, (38) implies:

$$
\frac{\partial r}{\partial q} = \frac{\left| \Upsilon_{r_2} \right|}{|\Lambda|} \triangleq -\left| \Upsilon_{r_2} \right|
$$

where

$$
\Upsilon_{r_2} \equiv \begin{bmatrix} \frac{2}{3b} [w - u] [c_H - c_L] & \frac{a + 3u + c_v - 2\xi - 4w}{3b} \\ -\frac{1}{9b} [c_H - c_L] [2a + 2u + c_L + c_H - 4c_v - 4w] & \frac{4}{9b} [\bar{q} - q] [c_H - c_L] \end{bmatrix}
$$

$$
\Rightarrow \left| \Upsilon_{r_2} \right| = \frac{1}{27b^2} [c_H - c_L] \left[ 8 (\bar{q} - q) (w - u) (c_H - c_L) \\
+ (2a + 2u + c_L + c_H - 4c_v - 4w) (a + 3u + c_v - 2\xi - 4w) \right] > 0. \quad (39)
$$

The inequality in (39), which reflects Lemmas 1 and 3, implies $\frac{\partial r}{\partial q} < 0$. 

28
Similarly, since $|\Lambda| < 0$, (38) implies:

$$\frac{\partial w}{\partial q} = \left| \frac{\Upsilon_{wq}}{|\Lambda|} \right| = -\left| \Upsilon_{wq} \right|$$

where

$$\Upsilon_{wq} \equiv \begin{bmatrix} D_R & \frac{2}{3b} [w - u] [c_H - c_L] \\ 2 D_R & - \frac{1}{9b} [c_H - c_L] [2a + 2u + c_L + c_H - 4c_v - 4w] \end{bmatrix}$$

$$\Rightarrow \left| \Upsilon_{wq} \right| = - \frac{D_R}{9b} [c_H - c_L] [2a + c_L + c_H - 4c_v + 8w - 10u]. \quad (40)$$

From Lemma 1:

$$2a + c_L + c_H - 4c_v + 8w - 10u \geq 4w - 2u + 8w - 10u = 12[w - u] > 0. \quad (41)$$

(40) and (41) imply $|\Upsilon_{wq}| < 0$, so $\frac{\partial w}{\partial q} > 0$.

The proofs that $\frac{\partial r}{\partial q} > 0$ and $\frac{\partial w}{\partial q} < 0$ are analogous. ■

**Proof of Observation 3.**

Differentiating (8) and (9) with respect to $k$ provides:

$$[\Lambda] \begin{bmatrix} \frac{\partial r}{\partial k} \\ \frac{\partial w}{\partial k} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (42)$$

Since $|\Lambda| < 0$, (42) implies:

$$\frac{\partial r}{\partial k} = \left| \frac{\Upsilon_{rk}}{|\Lambda|} \right| = - \left| \Upsilon_{rk} \right| \quad \text{where} \quad \Upsilon_{rk} \equiv \begin{bmatrix} 0 & \frac{a + 3u + c_v - 2c - 4w}{3b} \\ 0 & \frac{4}{9b} [q - q] [c_H - c_L] \end{bmatrix}$$

$$\Rightarrow \left| \Upsilon_{rk} \right| = 0 \Rightarrow \frac{\partial r}{\partial k} = 0.$$

The proof that $\frac{\partial w}{\partial k} = 0$ is analogous. ■

**Proof of Proposition 1.**

Differentiating (8) and (9) with respect to $D_C$ provides:

$$[\Lambda] \begin{bmatrix} \frac{\partial r}{\partial D_C} \\ \frac{\partial w}{\partial D_C} \end{bmatrix} = \begin{bmatrix} \frac{d}{d} \\ - (\frac{a}{d} - \frac{d}{d}) \end{bmatrix}. \quad (43)$$
Since \(|\Lambda| < 0\), (43) implies:

\[
\frac{\partial r}{\partial D_C} = \frac{|\Lambda_1|}{|\Lambda|} \overset{s}{=} -|\Lambda_1| \quad \text{where} \quad \Lambda_1 \equiv \begin{bmatrix} \frac{d}{3b} & \frac{a + 3u + c_v - 2\xi - 4w}{3b} \\ -[d - d] & \frac{4}{9b} [\bar{q} - q] [c_H - c_L] \end{bmatrix}
\]

\[\Rightarrow |\Lambda_1| = \frac{4d}{9b} [\bar{q} - q] [c_H - c_L] + [d - d] \left[ \frac{a + 3u + c_v - 2\xi - 4w}{3b} \right] > 0. \tag{44} \]

The inequality in (44), which follows from Lemma 3, implies \(\frac{\partial r}{\partial D_C} < 0\).

Differentiating (8) and (9) with respect to \(\bar{d}\) provides:

\[
[\Lambda] \begin{bmatrix} \frac{\partial r}{\partial \bar{d}} \\ \frac{\partial w}{\partial \bar{d}} \end{bmatrix} = \begin{bmatrix} 0 \\ -D_C \end{bmatrix}. \tag{45} \]

Since \(|\Lambda| < 0\), (45) implies:

\[
\frac{\partial r}{\partial \bar{d}} = \frac{|\Upsilon_1|}{|\Lambda|} \overset{s}{=} -|\Upsilon_1| \quad \text{where} \quad \Upsilon_1 \equiv \begin{bmatrix} 0 & \frac{a + 3u + c_v - 2\xi - 4w}{3b} \\ -D_C & \frac{4}{9b} [\bar{q} - q] [c_H - c_L] \end{bmatrix}
\]

\[\Rightarrow |\Upsilon_1| = D_C \left[ \frac{a + 3u + c_v - 2\xi - 4w}{3b} \right] > 0 \Rightarrow \frac{\partial r}{\partial \bar{d}} < 0. \tag{46} \]

Differentiating (8) and (9) with respect to \(d\) provides:

\[
[\Lambda] \begin{bmatrix} \frac{\partial r}{\partial d} \\ \frac{\partial w}{\partial d} \end{bmatrix} = \begin{bmatrix} D_C \\ D_C \end{bmatrix}. \tag{47} \]

Since \(|\Lambda| < 0\), (47) implies:

\[
\frac{\partial r}{\partial d} = \frac{|\Sigma_1|}{|\Lambda|} \overset{s}{=} -|\Sigma_1| \quad \text{where} \quad \Sigma_1 \equiv \begin{bmatrix} D_C & \frac{a + 3u + c_v - 2\xi - 4w}{3b} \\ D_C & \frac{4}{9b} [\bar{q} - q] [c_H - c_L] \end{bmatrix}
\]

\[\Rightarrow |\Sigma_1| = D_C \left[ \frac{4}{9b} [\bar{q} - q] (c_H - c_L) - \frac{a + 3u + c_v - 2\xi - 4w}{3b} \right]. \tag{48} \]

From (22):

\[a + 3u + c_v - 2\xi - 4w - \frac{4}{3} [\bar{q} - q] [c_H - c_L]\]

\[30\]
\[
\geq \sqrt{\left[a + 3u + c_v - 2\xi \right]^2 - 24b \left[ \frac{u}{3b} (a + u + c_v - 2\xi) + \frac{D_R}{2} + dD_C + F_u \right]}
\]
\[
- \frac{4}{3} \left[ q - q \right] \left[c_H - c_L \right].
\]

From (29):
\[
\left[a + 3u + c_v - 2\xi \right]^2 - 24b \left[ \frac{u}{3b} (a + u + c_v - 2\xi) + \frac{D_R}{2} + dD_C + F_u \right]
\]
\[
\Rightarrow \left[a + 3u + c_v - 2\xi \right]^2 - 24b \left[ \frac{u}{3b} (a + u + c_v - 2\xi) + \frac{D_R}{2} + dD_C + F_u \right]
\]
\[
= \left[a - u + c_v - 2\xi \right]^2 - 24b \left[ \frac{D_R}{2} + dD_C + F_u \right].
\]

(50) and (51) imply:
\[
a + 3u + c_v - 2\xi - 4w - \frac{4}{3} \left[ q - q \right] \left[c_H - c_L \right]
\]
\[
> \sqrt{\frac{16}{9} \left[ q - q \right]^2 \left[c_H - c_L \right]^2 - \frac{4}{3} \left[ q - q \right] \left[c_H - c_L \right]} = 0.
\]

(49) and (52) imply:
\[
|\Sigma_1| = \frac{D_C}{3b} \left[ \frac{4}{3} \left( q - q \right) \left(c_H - c_L \right) - (a + 3u + c_v - 2\xi - 4w) \right] < 0.
\]

(48) and (53) imply \( \frac{\partial r}{\partial d} > 0. \)

**Proof of Proposition 2.**

Since \(|\Lambda| < 0\), (43) implies:
\[
\frac{\partial w}{\partial D_C} = \frac{|\Lambda_2|}{|\Lambda|} s - |\Lambda_2| \quad \text{where} \quad \Lambda_2 = \begin{bmatrix}
D_R \\ 2D_R \\
\end{bmatrix}
\]
\[
\Rightarrow |\Lambda_2| = -D_R \left[ d - d \right] - 2D_R d = -D_R \left[ d + d \right] < 0.
\]

(54) and (55) imply \( \frac{\partial w}{\partial d} > 0. \)

The proofs that \( \frac{\partial w}{\partial a} > 0 \) and \( \frac{\partial w}{\partial d} > 0 \) are analogous. \(
\)
Proof of Proposition 3.

Let \( \lambda \) and \( \mu \) denote the Lagrange multipliers associated with constraints (8) and (9), respectively. Also let \( \mathcal{L} \) denote the Lagrangean function associated with [RP]. Then at an interior solution to [RP]:

\[
\frac{\partial \mathcal{L}}{\partial r} = -2k \left[ r - \frac{1}{2} \right] - D_R [1 - f_R] + \lambda D_R + 2\mu D_R = 0
\]

\[ \Rightarrow \lambda + 2\mu = \frac{2k}{D_R} \left[ r - \frac{1}{2} \right] + 1 - f_R. \]  

(56)

Furthermore:

\[
dW^* \frac{d}{dC} = \frac{\partial \mathcal{L}}{\partial D_C} = [1 - f_C] d - [\lambda + 2\mu] d + \mu \left[ \bar{d} + d \right].
\]

(57)

(56) and (57) provide:

\[
dW^* \frac{d}{dC} \leq M \left| \frac{2D_R}{3b} \right| [\alpha_2 - \alpha_1], \quad \alpha_3 \equiv 2a - w - u - c - c_v, \quad \text{and}
\]

\[
|M_\mu| = \frac{D_R}{3b} \left\{ \alpha_2 \left[ \frac{2k}{D_R} \left( r - \frac{1}{2} \right) + (1 - f_R) \right] - \frac{1}{3} \alpha_3 \right\}.
\]

(59), (60) imply:

\[
dW^* \frac{d}{dC} = -\frac{4}{3b} \left[ \alpha_2 - \alpha_1 \right] d k \left[ r - \frac{1}{2} \right] + \frac{2k}{3b} \left[ r - \frac{1}{2} \right] \alpha_2 \left[ \bar{d} + d \right]
\]

\[
+ \frac{2}{3b} \left[ \alpha_2 - \alpha_1 \right] d D_R \left[ f_R - f_C \right] + \frac{D_R}{3b} \left[ \alpha_2 (1 - f_R) - \alpha_3 \right] \left[ \bar{d} + d \right].
\]

(61)

Since \( \alpha_2 > \alpha_1 \) and \( \alpha_2 \left[ 1 - f_R \right] - \frac{\alpha_3}{3} > 0 \), (61) implies that \( \frac{dW^*}{dC} > 0 \) if \( f_R \geq f_C \).

\( \alpha_1 \) is independent of \( D_R \) and \( \alpha_2 \) and \( \alpha_3 \) are bounded above. Consequently, \( \left[ \alpha_2 - \alpha_1 \right] D_R \to 0 \) and \( \left[ \alpha_2 (1 - f_R) - \frac{\alpha_3}{3} \right] D_R \to 0 \) as \( D_R \to 0 \). Therefore, as \( D_R \to 0 : \)

\[
\frac{2}{3b} \left[ \alpha_2 - \alpha_1 \right] d D_R \left[ f_R - f_C \right] + \frac{D_R}{3b} \left[ \alpha_2 (1 - f_R) - \frac{\alpha_3}{3} \right] \left[ \bar{d} + d \right] \to 0.
\]

(62)

Recall that \( \frac{\partial w}{\partial D_R} > 0 \) and \( \frac{\partial r}{\partial D_R} < 0 \), from Lemma 5. Therefore, \( \frac{\partial \alpha_1}{\partial D_R} = 0 \) and \( \frac{\partial \alpha_2}{\partial D_R} < 0 \). Consequently, when \( D_R \) is sufficiently close to 0, the first two terms in (61) are strictly positive, and so (61) and (62) imply that \( \frac{dW^*}{dC} > 0 \).
Proof of Proposition 4.

From (10), using (44) and (54):

\[
\frac{dW^*}{dD_C} = \frac{H}{|\Lambda|} \quad \text{where} \quad H \equiv -\frac{1}{9b} \left[ 2a - w - u - c_v \right] |\Lambda_2| \nonumber
\]

\[
- \left\{ 2k \left[ r - \frac{1}{2} \right] + D_R \left[ 1 - f_R \right] \right\} |\Lambda_1| + \left[ 1 - f_C \right] d |\Lambda| . \quad (63)
\]

Since \( |\Lambda_1| > 0 \) from (44), \( H \geq G(w) \), where:

\[
G(w) \equiv \frac{\alpha_3}{9b} D_R \left[ \bar{d} + d \right] - \left[ 2k \left( r - \frac{1}{2} \right) + D_R \left( 1 - f_R \right) \right] \left\{ \frac{4d}{9b} \left[ \bar{q} - q \right] \left[ c_H - c_L \right] + \left[ \bar{d} - d \right] \frac{\alpha_2}{3b} \right\} \nonumber
\]

\[
- \left[ 1 - f_C \right] d \left[ \frac{2D_R}{3b} \right] \left[ \alpha_2 - \frac{2}{3} \left( \bar{q} - q \right) \left( c_H - c_L \right) \right]. \quad (64)
\]

Since \( G(\cdot) \) is linear in \( w \), \( G(w^*) > 0 \) if: (i) \( G(0) > 0 \); (ii) \( G(\bar{w}) > 0 \); and (iii) \( w^* \in [0, \bar{w}] \), where \( \bar{w} = \frac{1}{4} \left[ a + 3u + c_v - 2\xi \right] \).

(22) implies that \( w^* \leq \bar{w} \). To determine when \( G(0) > 0 \), note that because \( r \leq 1 \):

\[
G(w) \geq \frac{\alpha_3}{9b} D_R \left[ \bar{d} + d \right] - \frac{D_R}{3b} \left[ \frac{k}{D_R} + 1 - f_R \right] \left[ 2d \alpha_1 + \left[ \bar{d} - d \right] \alpha_2 \right] \nonumber
\]

\[
- \left[ 1 - f_C \right] d \left[ \frac{2D_R}{3b} \right] \left[ \alpha_2 - \alpha_1 \right] = \frac{D_R}{3b} \tilde{G}(w) , \quad (65)
\]

where \( \tilde{G}(w) \equiv \frac{\alpha_3}{3} \left[ \bar{d} + d \right] - \left[ \frac{k}{D_R} + 1 - f_R \right] \left[ 2d \alpha_1 + \left[ \bar{d} - d \right] \alpha_2 \right] \nonumber
\]

\[
- 2d \left[ 1 - f_C \right] \left[ \alpha_2 - \alpha_1 \right] \quad \text{(66)}
\]

\[
\Rightarrow \quad \tilde{G}(0) = \frac{1}{3} \left[ \bar{d} + d \right] \left[ 2a - u - c_v - \xi \right] - \left[ \frac{k}{D_R} + 1 - f_R \right] \left[ \bar{d} - d \right] \left[ a + 3u + c_v - \xi \right] \nonumber
\]

\[
- 2d \left[ 1 - f_C \right] \left[ a + 6u + c_v - \xi \right] - 2d \left[ f_C + \frac{k}{D_R} - f_R \right] \alpha_1 . \quad (67)
\]

Assumption 1 ensures:

\[
2a - u - c_v - \xi > a + 6u + c_v - \xi > a + 3u + c_v - \xi . \quad (68)
\]
(67) and (68) imply:
\[ \tilde{G}(0) > \left\{ \frac{1}{3} \left[ \bar{d} + d \right] - \left[ \frac{k}{D_R} + 1 - f_R \right] \left[ \bar{d} - d \right] - 2d \left[ 1 - f_C \right] \right\} \left[ 2a - u - c_v - c \right] \\
- 2d \left[ f_C + \frac{k}{D_R} - f_R \right] \alpha_1. \] 
(69)

Assumption 1 implies that \( \tilde{G}(0) > 0 \) (and so \( G(0) > 0 \), from (65)) if:
\[ \frac{1}{3} \left[ \bar{d} + d \right] - \left[ \frac{k}{D_R} + 1 - f_R \right] \left[ \bar{d} - d \right] - 2d \left[ 1 - f_C \right] > 0. \] 
(70)

It remains to demonstrate that \( G(\bar{w}) > 0 \). Observe that:
\[ G(\bar{w}) > d \left\{ \frac{2}{9b} \left[ \frac{7a - 7u - 2c - 5c_v}{4} \right] D_R \\
- \left[ 2k \left( r - \frac{1}{2} \right) + D_R (1 - f_R) \right] \frac{4}{9b} \left[ q - q \right] [c_H - c_L] \right\} \\
+ [1 - f_C] d \left[ \frac{2D_R}{3b} \right] \frac{2}{3b} \left[ q - q \right] [c_H - c_L]. \] 
(71)

Since PC binds by assumption:
\[ \frac{2}{9b} \left[ \frac{7a - 7u - 2c - 5c_v}{4} \right] D_R \\
> \left[ 2k \left( r - \frac{1}{2} \right) + D_R (1 - f_R) \right] \frac{4}{9b} \left[ q - q \right] [c_H - c_L]. \] 
(72)

(71) and (72) imply \( G(\bar{w}) > 0 \). Finally, since \( \bar{d} + d > 2d \), (70) holds if:
\[ \left[ \frac{k}{D_R} + f_C - f_R \right] \left[ \frac{\bar{d} - d}{d} \right] < 2 \left[ f_C - \frac{2}{3} \right]. \] 

Proof of Proposition 5.
\[ \frac{dW^*}{d \bar{d}} = \frac{\partial L}{\partial \bar{d}} = \mu D_C > 0. \] 

Proof of Proposition 6.
\[ \frac{dW^*}{d \bar{d}} = \frac{\partial L}{\partial \bar{d}} = D_C \left[ 1 - f_C - (\lambda + 2\mu) + \mu \right]. \] 
(73)
Since \( \lambda + 2 \mu = \frac{2k}{D_R} \left[ r - \frac{1}{2} \right] + 1 - f_R \) from (56), (73) implies:

\[
\frac{dW^*}{d\bar{d}} = D_C \left[ f_R - f_C - \frac{2k}{D_R} \left( r - \frac{1}{2} \right) + \mu \right] > D_C \left[ f_R - f_C - \frac{k}{D_R} + \mu \right].
\] (74)

(74) implies:

\[\text{If } \frac{dW^*}{d\bar{d}} < 0, \text{ then it must be the case that } f_C \geq f_R - \frac{k}{D_R}.\] (75)

It can be verified that:

\[3 \lambda \left[ a + 3 u + c_v - 2 c - 4 w \right] + 4 \mu \left[ \bar{q} - q \right] \left[ c_H - c_L \right] = 2 a - w - u - c - c_v,\]

and \( \lambda = \frac{2k}{D_R} \left[ r - \frac{1}{2} \right] + 1 - f_R - 2 \mu \)

\[\Rightarrow 3 \left[ \frac{2k}{D_R} \left( r - \frac{1}{2} \right) + 1 - f_R - 2 \mu \right] \left[ a + 3 u + c_v - 2 c - 4 w \right] + 4 \mu \left[ \bar{q} - q \right] \left[ c_H - c_L \right] = 2 a - w - u - c - c_v\]

\[\Rightarrow \mu = \frac{3 \left[ \frac{2k}{D_R} \left( r - \frac{1}{2} \right) + 1 - f_R \right] \left[ a + 3 u + c_v - 2 c - 4 w \right] - \left[ 2 a - w - u - c - c_v \right]}{6 \left[ a + 3 u + c_v - 2 c - 4 w - \frac{2}{3} \left( \bar{q} - q \right) \left( c_H - c_L \right) \right]}.
\] (76)

It can be verified that the numerator in (76) is positive when IC binds. Therefore, since \( \mu > 0 \), the denominator in (76) is also positive. Consequently, (74) and (76) imply:

\[
\frac{dW^*}{d\bar{d}} < 0 \iff f_R - f_C - \frac{2k}{D_R} \left( r - \frac{1}{2} \right) + \mu < 0 \iff \gamma(w) < 0
\] (77)

where

\[
\gamma(w) \equiv 6 \left[ f_R - f_C - \frac{2k}{D_R} \left( r - \frac{1}{2} \right) \right] \left[ a + 3 u + c_v - 2 c - 4 w - \frac{2}{3} \left( \bar{q} - q \right) \left( c_H - c_L \right) \right]
\]

\[+ 3 \left[ \frac{2k}{D_R} \left( r - \frac{1}{2} \right) + 1 - f_R \right] \left[ a + 3 u + c_v - 2 c - 4 w \right] - \left[ 2 a - w - u - c - c_v \right]
\]

\[\Rightarrow \gamma(0) = [3 \varphi - 2] a + [9 \varphi + 1] u + [3 \varphi + 1] c_v - [6 \varphi - 1] c - 6 \left[ f_R - f_C - \frac{2k}{D_R} \left( r - \frac{1}{2} \right) \right] \left[ \frac{2}{3} \left( \bar{q} - q \right) \left( c_H - c_L \right) \right],
\] (79)
where $\varphi \equiv 2 \left[ f_R - f_C - \frac{2k}{D_R} \left( r - \frac{1}{2} \right) \right] + \left[ \frac{2k}{D_R} \left( r - \frac{1}{2} \right) + 1 - f_R \right].$ \hfill (80)

(79) and (80) imply:
\[
\Rightarrow \gamma(0) < 0 \iff 3\varphi \left[ a + 3u + c_v - 2\xi \right] - \left[ 2a - u - c_v - \xi \right] < 6 \left[ f_R - f_C - \frac{2k}{D_R} \left( r - \frac{1}{2} \right) \right] \left[ \frac{2}{3} (\bar{q} - q) (c_H - c_L) \right]. \hfill (81)
\]

Since $r \in \left[ \frac{1}{2}, 1 \right]$, (81) implies:
\[
\gamma(0) < 0 \text{ if } f_R - 2f_C + \frac{1}{3} < 0 \text{ and } a \geq a_4, \text{ where } a_4 \text{ is defined in (18).} \hfill (82)
\]

From Lemma 3, $w < \tilde{w} = \frac{1}{4} \left[ a + 3u + c_v - 2\xi \right]$ at the solution to [RP]. From (78):
\[
\gamma(\tilde{w}) < -4 \left[ f_R - f_C - \frac{k}{D_R} \right] \left[ \bar{q} - q \right] \left[ c_H - c_L \right] - \frac{1}{4} \left[ 7a - 7u - 5c_v - 2\xi \right] < 0. \hfill (83)
\]

The last inequality in (83) reflects Assumption 1.

Observe from (78) that $\gamma(w)$ is linear in $w$. Therefore, since $w < \tilde{w}$ at the solution to [RP], $\gamma(w) < 0$ for all relevant $w$ if $\gamma(0) < 0$ and $\gamma(\tilde{w}) < 0$. Consequently, the Conclusion follows from (75), (82), and (83). \qed

**Proof of Proposition 7.**

(74) implies:
\[
\frac{dW^*}{d\theta} = D_C \left[ 1 - f_C - \left( \frac{2k}{D_R} \left[ r - \frac{1}{2} \right] + 1 - f_R \right) + \mu \right] > D_C \left[ f_R - f_C - \frac{k}{D_R} + \mu \right] > 0 \text{ when } f_R - f_C - \frac{k}{D_R} > 0. \hfill \blacksquare
\]
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