Technical Appendix to Accompany

"All Productivity Increases are Not Created Equal"

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Section I of this Technical Appendix provides detailed proofs of the formal conclusions in Bose et al. (2014). Section II provides additional conclusions. The ensuing analyses refer to the following key formulae from Bose et al. (2014).

$$\pi(\beta) \;=\; \phi_L\, x_L\, [\, 1 - q\,]\, [\, p_L V\, (1-\beta) - I\,] + \phi_H\, x_H\, q\, [\, p_H V\, (1-\beta) - I\,]\,. \tag{1}$$

$$W_L\,(\beta) \;=\; \phi_L\, x_L\, [\, 1 - q\,]\, p_L V \beta \;-\; \phi_L\, t_L \int_0^{x_L} x\, dx\,. \tag{2}$$

$$W_H\,(\beta) \;=\; \phi_H\, x_H\, q\, p_L V \beta \;-\; \phi_H\, t_H \int_0^{x_H} x\, dx\,. \tag{3}$$

$$\beta^* \;=\; \frac{\phi_L\, p_L\, [\, 1-q\,]^2\, [\, p_L V - I\,]\, t_H + \phi_H\, p_H\, q^2\, [\, p_H V - I\,]\, t_L}{2\, V\, \left[\, \phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\,\right]}. \tag{4}$$

$$\pi^* \;=\; \frac{\left\{\phi_L\, p_L\, t_H\, [\, 1-q\,]^2\, [\, p_L V - I\,] + \phi_H\, p_H\, t_L\, q^2\, [\, p_H V - I\,]\right\}^2}{4\, t_L\, t_H\, \left[\, \phi_L\, p_L^2\, t_H\, (1-q)^2 + \phi_H\, p_H^2\, t_L\, q^2\,\right]}\,. \tag{5}$$

$$W_L^* \;=\; \frac{\phi_L\, p_L^2\, [\, 1-q\,]^2 \left\{\phi_L\, p_L\, [\, 1-q\,]^2\; t_H\, [\, p_L V - I\,] + \phi_H\, p_H\, q^2\, t_L\, [\, p_H V - I\,]\right\}^2}{8\, t_L\, \left[\, \phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\,\right]^2}\,. \tag{6}$$

$$W_H^* \;=\; \frac{\phi_H\, p_H^2\, q^2 \left\{\phi_L\, p_L\, [\, 1-q\,]^2\; t_H\, [\, p_L V - I\,] + \phi_H\, p_H\, q^2\, t_L\, [\, p_H V - I\,]\right\}^2}{8\, t_H\, \left[\, \phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\,\right]^2}\,. \tag{7}$$

$$W^* \;=\; \frac{3 \left\{\phi_L\, p_L\, [\, 1-q\,]^2\; t_H\, [\, p_L V - I\,] + \phi_H\, p_H\, q^2\, t_L\, [\, p_H V - I\,]\right\}^2}{8\, t_L\, t_H\, \left[\, \phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\,\right]}\,. \tag{8}$$

I. **Detailed Proofs of Formal Conclusions in Bose et al. (2014)**

Proof of Lemma 1.

Following the analysis in the proof of Lemma 2 (below), it is readily verified that the borrower that is indifferent between applying for funding and not applying is located at:

$$x \;=\; \frac{\beta\, V}{t}\, [\, \phi_L\, p_L\, (1-q) + \phi_H\, p_H\, q\,]\,. \tag{9}$$

(9) implies that the lender's (expected) profit when he sets sharing rate β is:

$$\pi_S(\beta) \;=\; \frac{\beta V}{t}\,[\,\phi_L\,p_L\,(1-q)+\phi_H\,p_H\,q\,]\,\{\,\phi_L\,[\,1-q\,]\,[\,p_L\,(1-\beta)\,V-I\,] \\ +\;\phi_H\,q\,[\,p_H\,(1-\beta)\,V-I\,]\,\}. \tag{10}$$

Maximizing $\pi_S(\beta)$ with respect to β provides:

$$\begin{aligned}\beta_S \;&=\; \frac{\phi_L\,[\,1-q\,]\,[\,p_L V-I\,]+\phi_H\,q\,[\,p_H V-I\,]}{2\,V\,[\,\phi_L\,p_L\,(1-q)+\phi_H\,p_H\,q\,]} \\ &=\; \frac{1}{2}-\frac{[\,\phi_L\,(1-q)+\phi_H\,q\,]\,I}{2\,V\,[\,\phi_L\,p_L\,(1-q)+\phi_H\,p_H\,q\,]}\,.\end{aligned} \tag{11}$$

The conclusions in the Lemma follow immediately from (11). ■

Proof of Observation 1.

Substituting (11) from into (10) reveals that the lender's (maximum) profit in the symmetric information setting is:

$$\pi_S \;=\; \frac{1}{4\,t}\,\{\phi_L\,[\,1-q\,]\,[\,p_L V-I\,]+\phi_H\,q\,[\,p_H V-I\,]\}^2\,. \tag{12}$$

The welfare of entrepreneurs in this setting is:

$$\begin{aligned}W_{ES} \;&=\; [\,\phi_L\,(1-q)\,p_L+\phi_H\,q\,p_H\,]\,\beta\,V\,x-\int_0^x t\,x\,dx \\ &=\; \frac{1}{8\,t}\,\{\phi_L\,[\,1-q\,]\,[\,p_L V-I\,]+\phi_H\,q\,[\,p_H V-I\,]\}^2\,.\end{aligned} \tag{13}$$

The equality in (13) follows from (9) and (11). From (12) and (13), welfare in the symmetric information setting is:

$$W_S \;=\; \pi_S+W_{ES} \;=\; \frac{3}{8\,t}\,\{\phi_L\,[\,1-q\,]\,[\,p_L V-I\,]+\phi_H\,q\,[\,p_H V-I\,]\}^2\,. \tag{14}$$

The conclusions in the Observation follow directly from (14). ■

Proof of Lemma 2.

An L entrepreneur's expected payoff from applying for funding is $[1-q]\,p_L\,\beta\,V$. The L entrepreneur located farthest from the lender that will apply for funding is the one for whom this expected payoff equals his transactions cost:

$$[\,1-q\,]\,p_L\,\beta\,V \;=\; t_L\,x_L \quad\Rightarrow\quad x_L \;=\; \beta\,p_L V\left[\frac{1-q}{t_L}\right].$$

The analysis for the type H borrower is analogous, and so is omitted. ■

Proof of Lemma 3.

Substituting from Lemma 2 into (1) reveals that the lender's profit when he sets sharing rate β is:

$$\pi(\beta) \;=\; \left[\frac{\beta\, V}{t_L}\right] \phi_L\, p_L\, [1-q]^2\, [p_L\,(1-\beta)\, V - I] + \left[\frac{\beta\, V}{t_H}\right] \phi_H\, p_H\, q^2\, [p_H\,(1-\beta)\, V - I]\,. \quad (15)$$

Differentiating (15) provides:

$$\begin{aligned}\frac{\partial \pi(\cdot)}{\partial \beta} \;&=\; \frac{V}{t_L}\, \phi_L\, p_L\, [1-q]^2\, [p_L\,(1-2\,\beta)\, V - I] + \frac{V}{t_H}\, \phi_H\, p_H\, q^2\, [p_H\,(1-2\,\beta)\, V - I] \\ &=\; \frac{V}{t_L}\, \phi_L\, p_L\, [1-q]^2\, [p_L V - I] + \frac{V}{t_H}\, \phi_H\, p_H\, q^2\, [p_H V - I] \\ &\qquad\qquad -\; 2\,\beta\, V^2 \left[\phi_L\, p_L^2\, (1-q)^2\, \frac{1}{t_L} + \phi_H\, p_H^2\, q^2\, \frac{1}{t_H}\right]. \quad (16)\end{aligned}$$

It is readily verified that $\pi(\cdot)$ is a strictly concave function of β, that $\left.\frac{\partial \pi(\cdot)}{\partial \beta}\right|_{\beta=1} < 0$, and that $\left.\frac{\partial \pi(\cdot)}{\partial \beta}\right|_{\beta=0} > 0$ when assumption 1 holds. Therefore, (4) follows directly from (16). ■

Proof of Lemma 4.

From (4):

$$1-\beta^* \;=\; \frac{V\left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] + I\left[\phi_L\, p_L\, (1-q)^2\, t_H + \phi_H\, p_H\, q^2\, t_L\right]}{2\, V\left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]}. \quad (17)$$

From (15):

$$\begin{aligned}\pi(\beta) \;&=\; \frac{\beta\, V}{t_L\, t_H}\left\{\phi_L\, p_L\, [1-q]^2\, [p_L\,(1-\beta)\, V - I]\, t_H + \phi_H\, p_H\, q^2\, [p_H\,(1-\beta)\, V - I]\, t_L\right\} \\ &=\; \frac{\beta\, V}{t_L\, t_H}\, [1-\beta]\, V\left[\phi_L\, p_L^2\, (1-q)^2\; t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \\ &\qquad\qquad -\; \frac{\beta\, V}{t_L\, t_H}\left[\phi_L\, p_L\, (1-q)^2\; t_H + \phi_H\, p_H\, q^2\, t_L\right] I. \quad (18)\end{aligned}$$

From (17):

$$\begin{aligned}&[1-\beta^*]\, V\left[\phi_L\, p_L^2\, (1-q)^2\; t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \;= \\ &\quad \frac{1}{2}\, V\left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] + \frac{1}{2}\, I\left[\phi_L\, p_L\, (1-q)^2\, t_H + \phi_H\, p_H\, q^2\, t_L\right]. \quad (19)\end{aligned}$$

(18) and (19) imply:

$$\pi(\beta^*) \;=\; \frac{\beta^*\, V}{2\, t_L\, t_H}\{V\left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]$$

$$- \; I \left[\phi_L \, p_L \, (1-q)^2 \, t_H + \phi_H \, p_H \, q^2 \, t_L\right]\}. \tag{20}$$

(4) and (20) imply that π^* is as specified in (5). ■

Proof of Lemma 5.

From (2) and Lemma 2:

$$\begin{aligned} W_L \; &= \; \phi_L \left\{ [1-q] \, p_L V \, \beta \, x_L - \int_0^{x_L} t_L \, x \, dx \right\} \; = \; \phi_L \left\{ [1-q] \, p_L V \, \beta \, x_L - \frac{1}{2} \, t_L \, x_L^2 \right\} \\ &= \; \phi_L \, \beta \, p_L V \left[\frac{1-q}{t_L} \right] \left\{ [1-q] \, p_L V \, \beta - \frac{1}{2} \, t_L \, \beta \, p_L V \left[\frac{1-q}{t_L} \right] \right\} \\ &= \; \frac{\phi_L}{2 \, t_L} \, [1-q]^2 \, (\beta \, p_L V)^2 \; . \end{aligned} \tag{21}$$

Substituting from (4) into (21) provides (6). (7) is derived in analogous fashion.

(5), (6), and (7) imply that equilibrium total welfare is:

$$\begin{aligned} W^* \; &= \; W_L^* \, + \, W_H^* \, + \, \pi^* \; = \; \frac{\left\{ \phi_L \, p_L \, [1-q]^2 \; t_H \, [p_L V - I] + \phi_H \, p_H \, q^2 \, t_L \, [p_H V - I] \right\}^2}{8 \, t_L \, t_H \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \right]^2} \\ &\qquad \cdot \left\{ \phi_L \, p_L^2 \, t_H \, [1-q]^2 + \phi_H \, p_H^2 \, q^2 \, t_L + 2 \left[\phi_L \, p_L^2 \, t_H \, [1-q]^2 + \phi_H \, p_H^2 \, q^2 \, t_L \right] \right\} \\ &= \; \frac{3 \left\{ \phi_L \, p_L \, [1-q]^2 \; t_H \, [p_L V - I] + \phi_H \, p_H \, q^2 \, t_L \, [p_H V - I] \right\}^2}{8 \, t_L \, t_H \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \right]} \; . \end{aligned}$$

■

Proof of Lemma 6.

It is apparent from (4) that β^* increases as V increases and as I decreases. Furthermore:

$$\begin{aligned} \frac{\partial \beta^*}{\partial t_H} \; &\stackrel{s}{=} \; \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \right] \phi_L \, p_L \, [1-q]^2 \, [p_L V - I] \\ &\qquad - \left\{ \phi_L \, p_L \, [1-q]^2 \, [p_L V - I] \, t_H + \phi_H \, p_H \, q^2 \, [p_H V - I] \, t_L \right\} \phi_L \, p_L^2 \, [1-q]^2 \\ &= \; \phi_H \, p_H \, q^2 \, t_L \, \phi_L \, p_L \, [1-q]^2 \, [p_L V - I] - \phi_H \, p_H \, q^2 \, [p_H V - I] \, t_L \, \phi_L \, p_L^2 \, [1-q]^2 \\ &= \; - \; \phi_L \, p_L \, \phi_H \, p_H \, q^2 \, [1-q]^2 \, [p_H - p_L] \, t_L \, I \; < \; 0. \end{aligned}$$

$$\begin{aligned} \frac{\partial \beta^*}{\partial t_L} \; &\stackrel{s}{=} \; \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \right] \phi_H \, p_H \, q^2 \, [p_H V - I] \\ &\qquad - \left\{ \phi_L \, p_L \, [1-q]^2 \, [p_L V - I] \, t_H + \phi_H \, p_H \, q^2 \, [p_H V - I] \, t_L \right\} \phi_H \, p_H^2 \, q^2 \end{aligned}$$

$$= \phi_L\, p_L^2\, (1-q)^2\, t_H \phi_H\, p_H\, q^2\, [p_H V - I] - \phi_L\, p_L\, [1-q]^2\, [p_L V - I]\, t_H \phi_H\, p_H^2\, q^2$$
$$= \phi_L\, p_L\, \phi_H\, p_H\, q^2\, [1-q]^2\, [p_H - p_L]\, t_H\, I \;>\; 0.$$

$$\frac{\partial \beta^*}{\partial p_H} \overset{s}{=} \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \phi_H\, q^2\, t_L\, [2\, p_H V - I]$$
$$-\; 2\, \phi_H\, p_H\, q^2\, t_L \left\{\phi_L\, p_L\, [1-q]^2\, [p_L V - I]\, t_H + \phi_H\, p_H\, q^2\, [p_H V - I]\, t_L\right\}$$

$$= V\, \phi_H\, p_H\, q^2\, t_L\, [\, 2\, \phi_L\, p_L^2\, (1-q)^2\, t_H + 2\, \phi_H\, p_H^2\, q^2\, t_L$$
$$-\; 2\, \phi_L\, p_L^2\, (1-q)^2\, t_H - 2\, \phi_H\, p_H^2\, q^2\, t_L\,]$$
$$+\; I\, \phi_H\, q^2\, t_L\, \{\, 2\, \phi_L\, p_L\, p_H\, [1-q]^2\; t_H + 2\, \phi_H\, p_H^2\, q^2\, t_L$$
$$-\; \phi_L\, p_L^2\, [1-q]^2\; t_H - \phi_H\, p_H^2\, q^2\, t_L\, \}$$
$$= I\, \phi_H\, q^2\, t_L \left\{\phi_L\, p_L\, [1-q]^2\; t_H\, [2\, p_H - p_L] \,+\, \phi_H\, p_H^2\, q^2\, t_L\right\} \;>\; 0.$$

$$\frac{\partial \beta^*}{\partial \phi_H} \overset{s}{=} \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \left\{p_H\, q^2\, [p_H V - I]\, t_L - p_L\, [1-q]^2\, [p_L V - I]\, t_H\right\}$$
$$-\left[p_H^2\, q^2\, t_L - p_L^2\, (1-q)^2\, t_H\right] \left\{\phi_L\, p_L\, [1-q]^2\, [p_L V - I]\, t_H + \phi_H\, p_H\, q^2\, [p_H V - I]\, t_L\right\}$$
$$= p_H\, q^2\, t_L\, [p_H V - I]\, \{\phi_L\, p_L^2\, [1-q]^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L$$
$$-\; \phi_H\, p_H^2\, q^2\, t_L + p_L^2\, \phi_H\; [1-q]^2\, t_H\}$$
$$-\; p_L\, [1-q]^2\, t_H\, [p_L V - I]\, \{\phi_L\, p_L^2\, [1-q]^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L$$
$$+\; \phi_L\, p_H^2\, q^2\, t_L - \phi_L\, p_L^2\, [1-q]^2\, t_H\}$$
$$= p_H\, q^2\, [1-q]^2\; t_L\, t_H\, [p_H - p_L]\, I \;>\; 0. \quad \blacksquare$$

Proof of Lemma 7.

From (4):

$$\frac{\partial \beta^*}{\partial p_L} \overset{s}{=} \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \phi_L\, [1-q]^2\, t_H\, [2\, p_L V - I]$$
$$-\; 2\, \phi_L\, p_L\, [1-q]^2\; t_H \left\{\phi_L\, p_L\, [1-q]^2\, [p_L V - I]\, t_H + \phi_H\, p_H\, q^2\, [p_H V - I]\, t_L\right\}$$

$$= V\, \phi_L\, [1-q]^2\, t_H\, [2\, \phi_L\, p_L^3\, (1-q)^2\, t_H + 2\, p_L\, \phi_H\, p_H^2\, q^2\, t_L$$
$$-\; 2\, \phi_L\, p_L^3\, (1-q)^2\; t_H - 2\, p_L\, \phi_H\, p_H^2\, q^2\, t_L\,]$$
$$+\; I\, \phi_L\, [1-q]^2\, t_H\, \{\, 2\, \phi_L\, p_L^2\, (1-q)^2\, t_H + 2\, p_L\, \phi_H\, p_H\, q^2\, t_L$$
$$-\; \phi_L\, p_L^2\, (1-q)^2\, t_H - \phi_H\, p_H^2\, q^2\, t_L\, \}$$
$$= I\, \phi_L\, [1-q]^2\, t_H \left\{\phi_L\, p_L^2\, [1-q]^2\, t_H + \phi_H\, p_H\, q^2\, t_L\, [2\, p_L - p_H]\right\}. \tag{22}$$

From (22):

$$\frac{\partial \beta^*}{\partial p_L} \;>\; 0 \quad \Leftrightarrow \quad \phi_L\,[1-q]^2\,t_H\,p_L^2 + 2\,\phi_H\,p_H\,q^2\,t_L\,p_L - \phi_H\,p_H^2\,q^2\,t_L \;>\; 0$$

$$\Leftrightarrow \quad \frac{\phi_L\,[1-q]^2\,t_H}{\phi_H\,q^2\,t_L}\left(\frac{p_L}{p_H}\right)^2 + 2\left(\frac{p_L}{p_H}\right) - 1 \;>\; 0$$

$$\Leftrightarrow \quad \delta_0\,y^2 + 2\,y - 1 \;>\; 0 \;\text{ where } y \;=\; \frac{p_L}{p_H} \;\text{ and }\; \delta_0 \;=\; \frac{\phi_L\,[1-q]^2\,t_H}{\phi_H\,q^2\,t_L}$$

$$\Leftrightarrow \quad y \;>\; \frac{1}{2\,\delta_0}\left[-2+\sqrt{4+4\,\delta_0}\right] \quad \Leftrightarrow \quad y \;>\; \delta_1 \;=\; \frac{\sqrt{1+\delta_0}-1}{\delta_0}. \quad \blacksquare$$

Proof of Proposition 1.

It is apparent from (5), (6), and (7) that $\frac{\partial \pi^*}{\partial V} > 0$, $\frac{\partial W_L^*}{\partial V} > 0$, $\frac{\partial W_H^*}{\partial V} > 0$, $\frac{\partial \pi^*}{\partial I} < 0$, $\frac{\partial W_L^*}{\partial I} < 0$, and $\frac{\partial W_H^*}{\partial I} < 0$. Now define:

$$z \;\equiv\; \phi_L\,p_L\,[1-q]^2\;t_H\,[p_L V - I] + \phi_H\,p_H\,q^2\,t_L\,[p_H V - I] \;>\; 0. \tag{23}$$

The inequality in (23) follows from assumption 1, since $q \geq \frac{1}{2}$. Then from (5):

$$\frac{\partial \pi^*}{\partial p_H} \;\stackrel{s}{=}\; \left[\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right] 2\,z\,\phi_H\,q^2\,t_L\,[p_H V - I] - \; z^2\,2\,\phi_H\,p_H^2\,t_L\,q^2$$

$$\stackrel{s}{=}\; \left[\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right][p_H V - I]$$
$$-\; p_H\;\left\{\phi_L\,p_L\,t_H\,[1-q]^2\,[p_L V - I] + \phi_H\,p_H\,t_L\,q^2\,[p_H V - I]\right\}$$

$$=\; \phi_L\,p_L^2\,t_H\,[1-q]^2\,[p_H V - I] - \phi_L\,p_L\,p_H\,t_H\,[1-q]^2\,[p_L V - I] \;\;>\; 0. \tag{24}$$

The inequality in (24) holds because $p_H V - I \;>\; 0 \;>\; p_L V - I$.

Also from (5):

$$\frac{\partial \pi^*}{\partial t_H} \;\stackrel{s}{=}\; z\,t_H\left[\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right] 2\,\phi_L\,p_L\,[1-q]^2\,[p_L V - I]$$
$$-\; z^2\left\{2\,\phi_L\,p_L^2\,t_H\,[1-q]^2 + \phi_H\,p_H^2\,t_L\,q^2\right\}$$

$$\stackrel{s}{=}\; 2\,\phi_L\,p_L\,[1-q]^2\,t_H\,[p_L V - I]\left[\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right]$$
$$-\left[2\,\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right]$$
$$\cdot\left\{\phi_L\,p_L\,t_H\,[1-q]^2\,[p_L V - I] + \phi_H\,p_H\,t_L\,q^2\,[p_H V - I]\right\}$$

$$=\; \phi_L\,p_L\,[1-q]^2\,t_H\,[p_L V - I]\,\phi_H\,p_H^2\,t_L\,q^2$$
$$-\; \phi_H\,p_H\,t_L\,q^2\,[p_H V - I]\left[2\,\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right] \;\;<\; 0. \tag{25}$$

From (6):

$$\frac{\partial W_L^*}{\partial p_H} \overset{s}{=} \left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]^2\; 2\, z\, \phi_H\, q^2\, t_L\, [2\, p_H V - I]$$
$$- \, z^2\, 2\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]\; 2\, \phi_H\, p_H\, q^2\, t_L$$

$$\overset{s}{=} [2\, p_H V - I]\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]$$
$$- \; 2\, p_H \left\{\phi_L\, p_L\, [1-q]^2\; t_H\, [p_L V - I] + \phi_H\, p_H\, q^2\, t_L\, [p_H V - I]\right\}$$

$$> \; 2\,[p_H V - I]\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] - 2\, \phi_H\, p_H\, q^2\, t_L\, [p_H V - I] \tag{26}$$

$$= \; 2\,[p_H V - I]\, \phi_L\, p_L^2\, [1-q]^2\, t_H \; > \; 0\,.$$

The inequality in (26) holds because $2\, p_H V - I \; > \; 2\,[p_H V - I]$ and $p_L V - I \; < \; 0$.

Also from (6):

$$\frac{\partial W_L^*}{\partial t_H} \overset{s}{=} \left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]^2\; 2\, z\, \phi_L\, p_L\, [1-q]^2\, [p_L V - I]$$
$$- \, z^2\, 2\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]\; \phi_L\, p_L^2\, [1-q]^2$$

$$\overset{s}{=} \left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right][p_L V - I]$$
$$- \; p_L \left\{\phi_L\, p_L\, [1-q]^2\; t_H\, [p_L V - I] + \phi_H\, p_H\, q^2\, t_L\, [p_H V - I]\right\}$$

$$= \; \phi_H\, p_H^2\, q^2\, t_L\, [p_L V - I] - \phi_H\, p_H\, p_L\, q^2\, t_L\, [p_H V - I] \; < \; 0\,. \tag{27}$$

The inequality in (27) holds because $p_H V - I \; > \; 0 \; > \; p_L V - I$.

From (7):

$$\frac{\partial W_H^*}{\partial p_H} \overset{s}{=} \left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]^2 \left\{2\, p_H^2\, z\, \phi_H\, q^2\, t_L\, [2\, p_H V - I] + 2\, p_H\, z^2\right\}$$
$$- \; p_H^2\, z^2\, 2\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] 2\, \phi_H\, p_H\, q^2\, t_L$$

$$\overset{s}{=} 2\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]\left\{p_H\, z + \phi_H\, p_H^2\; q^2\, t_L\, [2\, p_H V - I]\right\}$$
$$- \; 4\, \phi_H\, p_H^3\, q^2\, t_L \left\{\phi_L\, p_L\, [1-q]^2\; t_H\, [p_L V - I] + \phi_H\, p_H\, q^2\, t_L\, [p_H V - I]\right\}$$

$$> \; 4\, \phi_H\, p_H^2\, q^2\, t_L\, [p_H V - I]\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]$$
$$- \; 4\, \phi_H\, p_H^3\, q^2\, t_L \left\{\phi_L\, p_L\, [1-q]^2\; t_H\, [p_L V - I] + \phi_H\, p_H\, q^2\, t_L\, [p_H V - I]\right\} \tag{28}$$

$$> \; 4\, \phi_H\, p_H^2\, q^2\, t_L\, [p_H V - I]\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L - \phi_H\, p_H^2\, q^2\, t_L\right] \tag{29}$$

$$= \; 4\, \phi_H\, p_H^2\, q^2\, t_L\, [p_H V - I]\, \phi_L\, p_L^2\, [1-q]^2\, t_H \; > \; 0\,.$$

The inequality in (28) holds because $2\, p_H V - I \;>\; 2\,[p_H V - I]$ and $z \;>\; 0$. The inequality in (29) holds because $p_L V - I \;<\; 0$.

Also from (7):

$$\begin{aligned}
\frac{\partial W_H^*}{\partial t_H} \;&\overset{s}{=}\; t_H \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]^2\, 2\, z\, \phi_L\, p_L\, [1-q]^2\, [p_L V - I] \\
&\qquad - z^2 \left\{ 2\, t_H \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \phi_L\, p_L^2\, [1-q]^2 \right. \\
&\qquad\qquad \left. +\; \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]^2 \right\} \\
&\overset{s}{=}\; 2\, \phi_L\, p_L\, [1-q]^2\, t_H\, [p_L V - I] \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \\
&\qquad -\; \left[3\, \phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] z \;<\; 0\,.
\end{aligned} \tag{30}$$

The inequality in (30) holds because $z \;>\; 0$ and $p_L V - I \;<\; 0$. ■

Proof of Proposition 2.

From (5):

$$\begin{aligned}
\frac{\partial \pi^*}{\partial \phi_H} \;&\overset{s}{=}\; \left[\phi_L\, p_L^2\, t_H\, (1-q)^2 + \phi_H\, p_H^2\, t_L\, q^2\right] 2\, z \left\{ p_H\, t_L\, q^2\, [p_H V - I] - p_L\, t_H\, [1-q]^2\, [p_L V - I] \right\} \\
&\qquad -\; z^2 \left[p_H^2\, t_L\, q^2 - p_L^2\, t_H\, (1-q)^2\right] \\
&\overset{s}{=}\; 2 \left[\phi_L\, p_L^2\, t_H\, (1-q)^2 + \phi_H\, p_H^2\, t_L\, q^2\right] \left\{ p_H\, t_L\, q^2\, [p_H V - I] - p_L\, t_H\, [1-q]^2\, [p_L V - I] \right\} \\
&\qquad -\; \left[p_H^2\, t_L\, q^2 - p_L^2\, t_H\, (1-q)^2\right] \left\{ \phi_L\, p_L\, t_H\, [1-q]^2\, [p_L V - I] + \phi_H\, p_H\, t_L\, q^2\, [p_H V - I] \right\} \\
&=\; p_H\, t_L\, q^2\, [p_H V - I]\, \{\, 2\, \phi_L\, p_L^2\, t_H\, [1-q]^2 + 2\, \phi_H\, p_H^2\, t_L\, q^2 \\
&\qquad\qquad + \phi_H\, p_L^2\, t_H\, [1-q]^2 - \phi_H\, p_H^2\, t_L\, q^2 \,\} \\
&\qquad - p_L\, t_H\, [1-q]^2\, [p_L V - I]\, \{\, 2\, \phi_L\, p_L^2\, t_H\, [1-q]^2 + 2\, \phi_H\, p_H^2\, t_L\, q^2 \\
&\qquad\qquad -\; \phi_L\, p_L^2\, t_H\, [1-q]^2 + \phi_L\, p_H^2\, t_L\, q^2 \,\} \\
&=\; p_H\, t_L\, q^2\, [p_H V - I] \left\{ [1+\phi_L]\, p_L^2\, t_H\, [1-q]^2 + \phi_H\, p_H^2\, t_L\, q^2 \right\} \\
&\qquad - p_L\, t_H\, [1-q]^2\, [p_L V - I] \left\{ \phi_L\, p_L^2\, t_H\, [1-q]^2 + [1+\phi_H]\, p_H^2\, t_L\, q^2 \right\} \;>\; 0.
\end{aligned} \tag{31}$$

The inequality in (31) holds because $p_H V - I \;>\; 0 \;>\; p_L V - I$.

From (7):

$$\begin{aligned}
\frac{\partial W_H^*}{\partial \phi_H} \;&\overset{s}{=}\; \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]^2 \\
&\qquad \cdot \left\{ \phi_H\, 2\, z \left[p_H\, q^2\, t_L\, (p_H V - I) - p_L\, (1-q)^2\, t_H\, (p_L V - I)\right] +\; z^2 \right\} \\
&\qquad - \phi_H\, z^2\, 2 \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \left[p_H^2\, q^2\, t_L - p_L^2\, (1-q)^2\; t_H\right]
\end{aligned}$$

$$
\begin{aligned}
&\stackrel{s}{=} \left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \left[2\,\phi_H\, p_H\, q^2\, t_L\,(p_H V - I) - 2\,\phi_H\, p_L\,(1-q)^2\, t_H\,(p_L V - I)\right] \\
&\qquad + z \left\{\phi_L\, p_L^2\,[1-q]^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L - 2\,\phi_H\, p_H^2\, q^2\, t_L + 2\,\phi_H\, p_L^2\,[1-q]^2\, t_H\right\} \\
&= \left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \left[2\,\phi_H\, p_H\, q^2\, t_L\,(p_H V - I) - 2\,\phi_H\, p_L\,(1-q)^2\, t_H\,(p_L V - I)\right] \\
&\quad + \left\{[1+\phi_H]\, p_L^2\,[1-q]^2\, t_H - \phi_H\, p_H^2\, q^2\, t_L\right\} \\
&\qquad \cdot \left\{\phi_L\, p_L\,[1-q]^2\; t_H\,[p_L V - I] + \phi_H\, p_H\, q^2\, t_L\,[p_H V - I]\right\} \\
&= \phi_H\, p_H\, q^2\, t_L\,[p_H V - I]\,\{[1+\phi_H]\, p_L^2\,[1-q]^2\, t_H - \phi_H\, p_H^2\, q^2\, t_L \\
&\qquad\qquad + \; 2\,\phi_L\, p_L^2\,(1-q)^2\, t_H + 2\,\phi_H\, p_H^2\, q^2\, t_L\} \\
&\quad - p_L\,[1-q]^2\; t_H\,[p_L V - I]\,\{\; 2\,\phi_L\,\phi_H\, p_L^2\,[1-q]^2\, t_H + 2\,\phi_H^2\, p_H^2\, q^2\, t_L \\
&\qquad\qquad - \;\; \phi_L\,[1+\phi_H]\, p_L^2\,[1-q]^2\, t_H + \phi_L\,\phi_H\, p_H^2\, q^2\, t_L\} \\
&= \phi_H\, p_H\, q^2\, t_L\,[p_H V - I]\left\{[2+\phi_L]\, p_L^2\,[1-q]^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right\} \\
&\quad - p_L\,[1-q]^2\; t_H\,[p_L V - I]\left\{\phi_H\;[2\,\phi_H + \phi_L]\, p_H^2\, q^2\, t_L - \phi_L^2\, p_L^2\,[1-q]^2\, t_H\right\} \\
&= \phi_H\, p_H\, q^2\, t_L\,[p_H V - I]\left\{2\, p_L^2\,[1-q]^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right\} \\
&\quad - p_L\,[1-q]^2\; t_H\,[p_L V - I]\,\phi_H\;[2\,\phi_H + \phi_L]\, p_H^2\, q^2\, t_L \\
&\quad + \phi_L\, p_L^2\,[1-q]^2\, t_H\left\{\phi_L\, p_L\,[1-q]^2\; t_H\,[p_L V - I] + \phi_H\, p_H\, q^2\, t_L\,[p_H V - I]\right\} \;>\; 0. \qquad (32)
\end{aligned}
$$

The inequality in (32) holds because $p_H V - I \;>\; 0 \;>\; p_L V - I$ and because $z \;>\; 0$.

From (6):

$$
\begin{aligned}
\frac{\partial W_L^*}{\partial \phi_H} &\stackrel{s}{=} \left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]^2 \\
&\qquad \cdot \left\{\phi_L\, 2\, z\left[p_H\, q^2\, t_L\,(p_H V - I) - p_L\,(1-q)^2\, t_H\,(p_L V - I)\right] - \, z^2\right\} \\
&\quad - \phi_L\, z^2\, 2\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]\left[p_H^2\, q^2\, t_L - p_L^2\,(1-q)^2\; t_H\right] \\
&\stackrel{s}{=} \left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]\left[2\,\phi_L\, p_H\, q^2\, t_L\,(p_H V - I) - 2\,\phi_L\, p_L\,(1-q)^2\, t_H\,(p_L V - I)\right] \\
&\qquad - z\left\{\phi_L\, p_L^2\,[1-q]^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L + 2\,\phi_L\, p_H^2\, q^2\, t_L - 2\,\phi_L\, p_L^2\,[1-q]^2\, t_H\right\} \\
&= \left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]\left[2\,\phi_L\, p_H\, q^2\, t_L\,(p_H V - I) - 2\,\phi_L\, p_L\,(1-q)^2\, t_H\,(p_L V - I)\right] \\
&\qquad - \left\{[\phi_H + 2\phi_L]\, p_H^2\, q^2\, t_L - \phi_L\, p_L^2\,[1-q]^2\, t_H\right\} \\
&\qquad\quad \cdot\; \left\{\phi_L\, p_L\,[1-q]^2\; t_H\,[p_L V - I] + \phi_H\, p_H\, q^2\, t_L\,[p_H V - I]\right\} \\
&= p_H\, q^2\, t_L\,[p_H V - I]\,\{2\,\phi_L^2\, p_L^2\,[1-q]^2\, t_H \,+\, 2\,\phi_L\,\phi_H\, p_H^2\, q^2\, t_L \\
&\qquad\qquad + \phi_H\,\phi_L\, p_L^2\,[1-q]^2\, t_H - \,\phi_H\,[\phi_H + 2\,\phi_L]\, p_H^2\, q^2\, t_L\}
\end{aligned}
$$

$$- \phi_L\, p_L\, [1-q]^2\; t_H\, [p_L V - I]\, \{[\phi_H + 2\,\phi_L]\, p_H^2\, q^2\, t_L - \phi_L\, p_L^2\, [1-q]^2\; t_H$$
$$+\; 2\,\phi_H\, p_H^2\, q^2\, t_L + 2\,\phi_L\, p_L^2\, [1-q]^2\, t_H\}$$

$$= \; p_H\, q^2\, t_L\, [p_H V - I]\, \left\{\phi_L\, [\phi_H + 2\,\phi_L]\, p_L^2\, [1-q]^2\, t_H - \phi_H^2\, p_H^2\, q^2\, t_L\right\}$$
$$- \phi_L\, p_L\, [1-q]^2\; t_H\, [p_L V - I]\, \left\{[3\,\phi_H + 2\,\phi_L]\, p_H^2\, q^2\, t_L + \phi_L\, p_L^2\, [1-q]^2\; t_H\right\}. \qquad (33)$$

Since $p_H V - I \;>\; 0 \;>\; p_L V - I$, (33) implies:

$$\frac{\partial W_L^*}{\partial \phi_H} \;>\; 0 \;\text{ if }\; \phi_L\, [\phi_H + 2\,\phi_L]\, p_L^2\, [1-q]^2\, t_H - \phi_H^2\, p_H^2\, q^2\, t_L \;>\; 0$$
$$\Leftrightarrow \quad \frac{\phi_L\, [\phi_H + 2\,\phi_L]}{\phi_H^2} = \frac{[1-\phi_H]\, [2-\phi_H]}{\phi_H^2} \;>\; \frac{p_H^2\, q^2\, t_L}{p_L^2\, [1-q]^2\, t_H}. \quad \blacksquare \qquad (34)$$

Proof of Proposition 3.

From (5):

$$\frac{\partial \pi^*}{\partial t_L} \;\overset{s}{=}\; t_L \left[\phi_L\, p_L^2\, t_H\, (1-q)^2 + \phi_H\, p_H^2\, t_L\, q^2\right] z\, 2\,\phi_H\, p_H\, q^2\, [p_H V - I]$$
$$- \; z^2 \left\{t_L\, \phi_H\, p_H^2\, q^2 + \phi_L\, p_L^2\, t_H\, (1-q)^2 + \phi_H\, p_H^2\, t_L\, q^2\right\}$$

$$\overset{s}{=}\; 2\,\phi_H\, p_H\, q^2\, t_L\, [p_H V - I] \left[\phi_L\, p_L^2\, t_H\, (1-q)^2 + \phi_H\, p_H^2\, t_L\, q^2\right]$$
$$- \left\{\phi_L\, p_L\, t_H\, [1-q]^2\, [p_L V - I] + \phi_H\, p_H\, t_L\, q^2\, [p_H V - I]\right\}$$
$$\cdot \left[\phi_L\, p_L^2\, t_H\, (1-q)^2 + 2\,\phi_H\, p_H^2\, t_L\, q^2\right]$$

$$= \; \phi_H\, p_H\, q^2\, t_L\, [p_H V - I]\, \{2\,\phi_L\, p_L^2\, t_H\, (1-q)^2 + 2\,\phi_H\, p_H^2\, t_L\, q^2$$
$$- \; \phi_L\, p_L^2\, t_H\, (1-q)^2 - 2\,\phi_H\, p_H^2\, t_L\, q^2\}$$
$$- \; \phi_L\, p_L\, t_H\, [1-q]^2\, [p_L V - I] \left[\phi_L\, p_L^2\, t_H\, (1-q)^2 + 2\,\phi_H\, p_H^2\, t_L\, q^2\right]$$

$$= \; \phi_H\, p_H\, q^2\, t_L\, [p_H V - I]\, \phi_L\, p_L^2\, t_H\, [1-q]^2$$
$$- \; \phi_L\, p_L\, t_H\, [1-q]^2\, [p_L V - I] \left[\phi_L\, p_L^2\, t_H\, (1-q)^2 + 2\,\phi_H\, p_H^2\, t_L\, q^2\right] \;>\; 0. \qquad (35)$$

The inequality in (35) holds because $p_H V - I \;>\; 0 \;>\; p_L V - I$.

From (7):

$$\frac{\partial W_H^*}{\partial t_L} \;\overset{s}{=}\; \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right]^2\; 2\, z\, \phi_H\, p_H\, q^2\, [p_H V - I]$$
$$- z^2\, 2 \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \phi_H\, p_H^2\, q^2$$

$$\overset{s}{=}\; \left[\phi_L\, p_L^2\, (1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] [p_H V - I]$$

$$- \; p_H \left\{ \phi_L \, p_L \, [1-q]^2 \; t_H \, [p_L V - I] + \phi_H \, p_H \, q^2 \, t_L \, [p_H V - I] \right\}$$

$$= \; \phi_L \, p_L^2 \, [1-q]^2 \, t_H \, [p_H V - I] - \phi_L \, p_L \, p_H \, [1-q]^2 \; t_H \, [p_L V - I] \; > \; 0 \, . \tag{36}$$

The inequality in (36) holds because $p_H V - I \; > \; 0 \; > \; p_L V - I$.

From (8):

$$\frac{\partial W^*}{\partial t_L} \stackrel{s}{=} t_L \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \right] \; 2 \, z \, \phi_H \, p_H \, q^2 \, [p_H V - I]$$
$$- z^2 \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + 2 \, \phi_H \, p_H^2 \, q^2 \, t_L \right]$$

$$\stackrel{s}{=} 2 \, \phi_H \, p_H \, q^2 \, t_L \, [p_H V - I] \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \right]$$
$$- \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + 2 \, \phi_H \, p_H^2 \, q^2 \, t_L \right]$$
$$\cdot \left\{ \phi_L \, p_L \, [1-q]^2 \; t_H \, [p_L V - I] + \phi_H \, p_H \, q^2 \, t_L \, [p_H V - I] \right\}$$

$$= \; \phi_H \, p_H \, q^2 \, t_L \, [p_H V - I] \, \phi_L \, p_L \, [1-q]^2 \; t_H$$
$$- \; \phi_L \, p_L \, [1-q]^2 \; t_H \, [p_L V - I] \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + 2 \, \phi_H \, p_H^2 \, q^2 \, t_L \right] \; > \; 0 \, . \tag{37}$$

The inequality in (37) holds because $p_H V - I \; > \; 0 \; > \; p_L V - I$.

From (6):

$$\frac{\partial W_L^*}{\partial t_L} \stackrel{s}{=} t_L \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \right]^2 \; 2 \, z \, \phi_H \, p_H \, q^2 \, [p_H V - I]$$
$$- z^2 \left\{ 2 \, t_L \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \right] \; \phi_H \, p_H^2 \, q^2 \right.$$
$$\left. + \; \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \right]^2 \right\}$$

$$\stackrel{s}{=} 2 \, \phi_H \, p_H \, q^2 \, t_L \, [p_H V - I] \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \right]$$
$$- \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + 3 \, \phi_H \, p_H^2 \, q^2 \, t_L \right]$$
$$\cdot \left\{ \phi_L \, p_L \, [1-q]^2 \; t_H \, [p_L V - I] + \phi_H \, p_H \, q^2 \, t_L \, [p_H V - I] \right\}$$

$$= \; \phi_H \, p_H \, q^2 \, t_L \, [p_H V - I] \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H - \; \phi_H \, p_H^2 \, q^2 \, t_L \right]$$
$$- \; \phi_L \, p_L \, [1-q]^2 \; t_H \, [p_L V - I] \left[\phi_L \, p_L^2 \, (1-q)^2 \, t_H + 3 \, \phi_H \, p_H^2 \, q^2 \, t_L \right] \equiv \; g(\phi_L) \, . \tag{38}$$

From (38):

$$g(\phi_L) \stackrel{s}{=} p_H \, q^2 \, t_L \, [p_H V - I] \left[\frac{\phi_L}{\phi_H} \, p_L^2 \, (1-q)^2 \, t_H - \; p_H^2 \, q^2 \, t_L \right]$$
$$- \; \frac{\phi_L}{\phi_H} \, p_L \, [1-q]^2 \; t_H \, [p_L V - I] \left[\frac{\phi_L}{\phi_H} \, p_L^2 \, (1-q)^2 \, t_H + 3 \, p_H^2 \, q^2 \, t_L \right]$$

$$= \; p_H \, q^2 \, t_L \, [p_H V - I] \left[r \, p_L^2 \, (1-q)^2 \, t_H - \; p_H^2 \, q^2 \, t_L \right]$$

$$- \ r\ p_L\,[1-q]^2\ t_H\,[p_L V - I]\,\left[\,r\,p_L^2\,(1-q)^2\,t_H + 3\ p_H^2\,q^2\,t_L\right], \tag{39}$$

where $r = \frac{\phi_L}{\phi_H}$. Since $p_L V - I \ < \ 0$, it is apparent from (38) and (39) that:

$$\frac{\partial W_L^*}{\partial t_L} \ > \ 0 \ \text{ if } \ r \ = \ \frac{\phi_L}{\phi_H} \ \geq \ \frac{p_H^2\,q^2\,t_L}{p_L^2\,[1-q]^2\,t_H}. \quad \blacksquare$$

Proof of Proposition 4.

Lemma A1. $$\frac{\partial \pi^*}{\partial p_L} \ = \ \frac{N_1\,D_1}{2\,t_L\left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right]^2} \ \stackrel{s}{=} \ N_1\,D_1 \tag{40}$$

where

$$N_1 \ \equiv \ \phi_L\,[1-q]^2\left\{\phi_H\,p_H\,q^2\,t_L\,[p_H V - I] + \phi_L\,p_L\,[1-q]^2\,t_H\,[p_L V - I]\right\} \tag{41}$$

and

$$D_1 \ \equiv \ \phi_L\,p_L^3\,[1-q]^2\,t_H\,V + \phi_H\,p_H\,q^2\,t_L\,[p_L\,p_H V - I\,(p_H - p_L)]\ . \tag{42}$$

Proof. Differentiating (5) provides:

$$\begin{aligned}
\frac{\partial \pi^*}{\partial p_L} \ = \ & \frac{1}{16\,t_L^2\,t_H^2\left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right]^2}\{4\,t_L\,t_H\left[\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right] \\
& \cdot\ 2\left\{V\left[\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right] - I\left[\phi_L\,p_L\,t_H\,(1-q)^2 + \phi_H\,p_H\,t_L\,q^2\right]\right\} \\
& \cdot\ \left[2\,V\,\phi_L\,p_L\,t_H\,(1-q)^2 - I\,\phi_L\,t_H\,(1-q)^2\right] \\
& - 8\,t_L\,t_H^2\,\phi_L\,p_L\,[1-q]^2\,\{V\left[\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right] \\
& \qquad - \ I\left[\phi_L\,p_L\,t_H\,(1-q)^2 + \phi_H\,p_H\,t_L\,q^2\right]\}^2\} \\
= \ & \frac{V\left[\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right] - I\left[\phi_L\,p_L\,t_H\,(1-q)^2 + \phi_H\,p_H\,t_L\,q^2\right]}{2\,t_L\,t_H\left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right]^2} \\
& \cdot\ \{\left[\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right]\left[2\,V\,\phi_L\,p_L\,t_H\,(1-q)^2 - I\,\phi_L\,t_H\,(1-q)^2\right] \\
& - \ \phi_L\,p_L\,t_H\,[1-q]^2\,\{V\left[\phi_L\,p_L^2\,t_H\,(1-q)^2 + \phi_H\,p_H^2\,t_L\,q^2\right] \\
& \qquad - \ I\left[\phi_L\,p_L\,t_H\,(1-q)^2 + \phi_H\,p_H\,t_L\,q^2\right]\}\} \\
= \ & \frac{\phi_H\,p_H\,q^2\,t_L\,[p_H V - I] + \phi_L\,p_L\,[1-q]^2\,t_H\,[p_L V - I]}{2\,t_L\,t_H\left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right]^2} \\
& \cdot\ \{2\,V\,\phi_L^2\,p_L^3\,t_H^2\,[1-q]^4 + 2\,V\,\phi_L\,p_L\,\phi_H\,p_H^2\,t_L\,t_H\,q^2\,[1-q]^2 \\
& - \ I\,\phi_L^2\,p_L^2\,t_H^2\,[1-q]^4 - I\,\phi_L\,\phi_H\,p_H^2\,t_L\,t_H\,q^2\,[1-q]^2 \\
& - V\,\phi_L^2\,p_L^3\,t_H^2\,[1-q]^4 - \ V\,\phi_L\,p_L\,\phi_H\,p_H^2\,t_L\,t_H\,q^2\,[1-q]^2
\end{aligned}$$

$$+ I\,\phi_L^2\,p_L^2\,t_H^2\,[1-q]^4 + I\,\phi_L\,p_L\,\phi_H\,p_H\,t_L\,t_H\,q^2\,[1-q]^2\}$$

$$= \frac{N_1/\left[\phi_L\,(1-q)^2\right]}{2\,t_L\,t_H\left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right]^2}$$
$$\cdot\,\{V\,\phi_L^2\,p_L^3\,t_H^2\,[1-q]^4 + V\,\phi_L\,p_L\,\phi_H\,p_H^2\,t_L\,t_H\,q^2\,[1-q]^2$$
$$- I\,\phi_L\,\phi_H\,p_H\,t_L\,t_H\,q^2\,[1-q]^2\,[p_H - p_L]\}$$

$$= \frac{N_1/\left[\phi_L\,(1-q)^2\right]}{2\,t_L\,t_H\left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right]^2}$$
$$\cdot\,\{\phi_L\,t_H\,[1-q]^2\,\{V\,\phi_L\,p_L^3\,t_H\,[1-q]^2 + V\,p_L\,\phi_H\,p_H^2\,t_L\,q^2 - I\,\phi_H\,p_H\,t_L\,q^2\,[p_H - p_L]\}$$

$$= \frac{N_1\,D_1}{2\,t_L\left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right]^2}\,. \quad \square$$

Lemma A2. D_1 is monotonically increasing in p_L. Furthermore, there exists a $\widehat{p}_L \in (0, p_H)$ such that $D_1 < 0$ for $p_L \in (0, \widehat{p}_L)$ and $D_1 > 0$ for $p_L \in (\widehat{p}_L, p_H)$.

Proof. From (42):

$$\frac{\partial D_1}{\partial p_L} = 3\,\phi_L\,p_L^2\,[1-q]^2\,t_H\,V + \phi_H\,p_H\,q^2\,t_L\,[\,p_H V + I] > 0\,. \tag{43}$$

Furthermore, from (42):

$$D_1|_{p_L=0} = -\,\phi_H\,p_H^2\,q^2\,t_L\,I < 0\,, \quad \text{and}$$
$$D_1|_{p_L=p_H} = \phi_L\,p_H^3\,[1-q]^2\,t_H\,V + \phi_H\,p_H^3\,q^2\,t_L\,V > 0\,. \quad \square$$

Lemma A3. There exists a $\widetilde{p}_L \in (0, p_H)$ such that $\frac{\partial \pi^*}{\partial p_L} < 0$ for $p_L \in (0, \widetilde{p}_L)$ and $\frac{\partial \pi^*}{\partial p_L} > 0$ for $p_L \in (\widetilde{p}_L, p_H)$.

Proof. The proof follows immediately from Lemmas A1 and A2 since $N_1 > 0$ for all $p_L \in (0, p_H)$. This conclusion follows from assumption 1, since $q \geq \frac{1}{2}$. $\square$

From (7):

$$\frac{\partial W_H^*}{\partial p_L} \overset{s}{=} \left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right]^2\,2\,z\,\phi_L\,[1-q]^2\,t_H\,[2\,p_L V - I]$$
$$-\,z^2\,2\left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right]\,2\,\phi_L\,p_L\,[1-q]^2\,t_H$$

$$\overset{s}{=} [2\,p_L V - I]\left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right] - 2\,p_L\,z \tag{44}$$

$$= [2\,p_L V - I]\left[\phi_L\,p_L^2\,(1-q)^2\,t_H + \phi_H\,p_H^2\,q^2\,t_L\right]$$
$$-\,2\,p_L\left\{\phi_H\,p_H\,q^2\,t_L\,[p_H V - I] + \phi_L\,p_L\,[1-q]^2\,t_H\,[p_L V - I]\right\}$$

$$
\begin{aligned}
&= \phi_L\, p_L^2\, [1-q]^2\, t_H \{2\, p_L V - I - 2\,[p_L V - I]\} \\
&\qquad + \phi_H\, p_H\, q^2\, t_L \{p_H\,[2\, p_L V - I] - 2\, p_L\,[p_H V - I]\} \\
&= \phi_L\, p_L^2\, [1-q]^2\, t_H\, I - \phi_H\, p_H\, q^2\, t_L\,[p_H - 2\, p_L]\, I\,. \qquad (45)
\end{aligned}
$$

(44) reveals that $\frac{dW_H^*}{dp_L} < 0$ if $p_L \leq \frac{I}{2V}$ (since $z > 0$). (45) reveals that $\frac{dW_H^*}{dp_L} > 0$ if $p_L > \frac{1}{2}\, p_H$

From (8):

$$
\begin{aligned}
\frac{\partial W^*}{\partial p_L} &\overset{s}{=} \left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] 2\, z\, \phi_L\, [1-q]^2\; t_H\,[2\, p_L V - I] \\
&\qquad - z^2\, 2\, \phi_L\, p_L\,[1-q]^2\; t_H \\
&\overset{s}{=} [2\, p_L V - I]\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] - p_L\, z \qquad (46) \\
&= [2\, p_L V - I]\left[\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L\right] \\
&\qquad - \; p_L \left\{\phi_L\, p_L\,[1-q]^2\; t_H\,[p_L V - I] + \phi_H\, p_H\, q^2\, t_L\,[p_H V - I]\right\} \qquad (47) \\
&= \phi_H\, p_H\, q^2\, t_L \{p_H\,[2\, p_L V - I] - p_L\,[p_H V - I]\} \\
&\qquad + \phi_L\, p_L^2\,[1-q]^2\, t_H \{2\, p_L V - I - [p_L V - I]\} \\
&= \phi_H\, p_H\, q^2\, t_L\,[p_L\, p_H V - (p_H - p_L)\, I] + \phi_L\, p_L^2\,[1-q]^2\, t_H\, p_L V\,. \qquad (48)
\end{aligned}
$$

Since $z > 0$, (46) implies that $\frac{\partial W^*}{\partial p_L} < 0$ if $p_L < \frac{I}{2V}$. Also, (48) implies that $\frac{\partial W^*}{\partial p_L} > 0$ if $p_L\, p_H V > [p_H - p_L]\, I \;\; \Leftrightarrow \;\; \frac{p_L\, p_H}{p_H - p_L} > \frac{I}{V}$.

From (6):

$$
\sqrt{W_L^*} \;=\; \left[\frac{[1-q]\,\sqrt{\phi_L}}{\sqrt{8\, t_L}}\right] \omega\,(p_L)\;, \qquad (49)
$$

where:

$$
\omega\,(p_L) \;\equiv\; \frac{p_L \left\{\phi_L\, p_L\,[1-q]^2\; t_H\,[p_L V - I] + \phi_H\, p_H\, q^2\, t_L\,[p_H V - I]\right\}}{\phi_L\, p_L^2\,[1-q]^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L}\,. \qquad (50)
$$

(49) and (50) imply:

$$
\frac{\partial W_L^*}{\partial p_L} \;\gtreqless\; 0 \quad \text{as} \quad \frac{\partial \omega\,(p_L)}{\partial p_L} \;\gtreqless\; 0\,. \qquad (51)
$$

From (50):

$$
\begin{aligned}
\omega\,(p_L) &= p_L \left[V - \left(\frac{\phi_L\, p_L\,[1-q]^2\; t_H + \phi_H\, p_H\, q^2\, t_L}{\phi_L\, p_L^2\,(1-q)^2\, t_H + \phi_H\, p_H^2\, q^2\, t_L}\right) I\right] \\
&= p_L \left[V - \left(\frac{\left[\frac{\phi_L\,[1-q]^2\, t_H}{\phi_H\, p_H\, q^2\, t_L}\right] p_L + \left[\frac{\phi_H\, p_H\, q^2\, t_L}{\phi_H\, p_H\, q^2\, t_L}\right]}{\left[\frac{\phi_L\,(1-q)^2 t_H}{\phi_H\, p_H\, q^2\, t_L}\right] p_L^2 + \left[\frac{\phi_H\, p_H^2\, q^2\, t_L}{\phi_H\, p_H\, q^2\, t_L}\right]}\right) I\right]\,. \qquad (52)
\end{aligned}
$$

Let $A \equiv \frac{\phi_L\,[1-q]^2 t_H}{\phi_H\; p_H\, q^2\, t_L}$. Then, (52) implies:

$$\omega\left(p_L\right) \;=\; p_L\left[V - \left(\frac{A\,p_L + 1}{A\,p_L^2 + p_H}\right) I\right] \;=\; V\,p_L - \left[\frac{A\,p_L^2 + p_L}{A\,p_L^2 + p_H}\right] I$$

$$= \; V\,p_L - \left[\frac{A\,p_L^2 + p_H + p_L - p_H}{A\,p_L^2 + p_H}\right] I \;=\; V\,p_L - \left[1 + \frac{p_L - p_H}{A\,p_L^2 + p_H}\right] I. \tag{53}$$

(53) implies:

$$\frac{\partial\omega\left(p_L\right)}{\partial p_L} \;=\; V - \left[\frac{A\,p_L^2 + p_H - \left(p_L - p_H\right)\left(2\,A\,p_L\right)}{\left(A\,p_L^2 + p_H\right)^2}\right] I$$

$$= \; V - \left[\frac{-\,A\,p_L^2 + p_H\left(1 + 2\,A\,p_L\right)}{\left(A\,p_L^2 + p_H\right)^2}\right] I$$

$$= \; \frac{\left[A\,p_L^2 + p_H\right]^2 V - \left[-\,A\,p_L^2 + p_H\left(1 + 2\,A\,p_L\right)\right] I}{\left(A\,p_L^2 + p_H\right)^2} \;>\; 0 \tag{54}$$

if $\quad \left[A\,p_L^2 + p_H\right]^2 V + \left[A\,p_L^2 - p_H\left(1 + 2\,A\,p_L\right)\right] I$

$$= \; A^2\,p_L^4\,V + 2\,A\,p_L^2\,p_H V + p_H^2 V + A\,p_L^2\,I \;-\; p_H\,I \;-\; 2\,A\,p_L\,p_H\,I$$

$$= \; A^2\,p_L^4 V + A\,p_L^2\,I + p_H^2 V - p_H\,I + 2\,A\,p_L^2\,p_H V - 2\,A\,p_L\,p_H\,I$$

$$= \; A^2\,p_L^4\,V + A\,p_L^2\,I + p_H\left[p_H V - I + 2\,A\,p_L\left(p_L V - I\right)\right] \;>\; 0. \tag{55}$$

Let

$$h\left(p_L\right) \;=\; p_H V - I + 2\,A\,p_L\left[p_L V - I\right]$$

$$= \; p_H V - I + 2\,p_L\left[p_L V - I\right]\left[\frac{\phi_L\left(1-q\right)^2 t_H}{\phi_H\,p_H\,q^2\,t_L}\right]$$

$$\stackrel{s}{=} \; \phi_H\,p_H\,q^2\,t_L\left[p_H V - I\right] + 2\,\phi_L\,p_L\left[1-q\right]^2 t_H\left[p_L V - I\right]\,. \tag{56}$$

(51), (54), (55), and (56) imply that $\frac{\partial W_L^*}{\partial p_L} > 0$ when q is sufficiently close to 1.

The systematic losses identified in the Proposition are illustrated in Table 1 in the text. The data in the table were derived using *Mathematica*. ■

Proof of Corollary 1.

From assumption 1:

$$\phi_H\,p_H\left[p_H V - I\right] t_L + \phi_L\,p_L\left[p_L V - I\right] t_H \;>\; 0\,.$$

Therefore, $h\left(p_L\right) > 0$ from (56), and so $\frac{\partial W_L^*}{\partial p_L} > 0$ from (51), (54), and (55) if:

$$q^2 \;\geq\; 2\left[1-q\right]^2 \quad\Leftrightarrow\quad q \;\geq\; \frac{\sqrt{2}}{1+\sqrt{2}} \;=\; 0.58579\,. \quad ■$$

Proof of Corollary 2.

The data in Table 2 provide a proof of the corollary. ■

Proof of Proposition 5.

As noted in the text, the proposition is an immediate corollary of Proposition 4 since the relevant increase in p_L can be made arbitrarily large relative to the increase in p_H. ■

Proof of Proposition 6.

From (5), the lender's profit in this setting when she implements screening accuracy q is:

$$\Pi(q, C) = \pi^*(q) - C(q), \tag{57}$$

where:

$$\pi^*(q) = \frac{\left[\phi_H \, p_H \, q^2 \, t_L \, (p_H V - I) + \phi_L \, p_L \, (1-q)^2 \, t_H \, (p_L V - I)\right]^2}{4 \, t_L \, t_H \left[\phi_H \, p_H^2 \, q^2 t_L + \phi_L \, p_L^2 \, (1-q)^2 \, t_H\right]} \,. \tag{58}$$

Let $q^* = \arg\max_q \Pi(q, C)$. Also let $\Pi^* = \Pi(q^*, C)$. Then:

$$\frac{d\Pi^*}{dp_L} = \left.\frac{\partial \Pi(\cdot)}{\partial p_L}\right|_{q \, = \, q^*} + \left\{ \left.\frac{\partial \Pi(\cdot)}{\partial q}\right|_{q \, = \, q^*} \right\} \frac{\partial q^*}{\partial p_L} = \left.\frac{\partial \Pi(\cdot)}{\partial p_L}\right|_{q \, = \, q^*} . \tag{59}$$

The last equality in (59) reflects the envelope theorem. (57) and (59) imply that $\frac{d\Pi^*}{dp_L} \overset{s}{=} \frac{\partial \pi^*}{\partial p_L}$.

From (8), total welfare in this setting when the lender implements screening accuracy q at cost $C(q)$ is:

$$\widehat{W}(q, C) = \frac{3\left[\phi_H \, p_H \, q^2 \, t_L \, (p_H V - I) + \phi_L \, p_L \, (1-q)^2 \, t_H \, (p_L V - I)\right]^2}{8 \, t_H \, t_L \left[\phi_H \, p_H^2 \, q^2 t_L + \phi_L \, p_L^2 \, (1-q)^2 \, t_H\right]} - C(q) \,. \tag{60}$$

(57), (58), and (60) imply:

$$\widehat{W}(q, C) = \frac{3}{2}\pi^*(q) - C(q) = \frac{3}{2}\left[\pi^*(q) - \frac{2}{3}C(q)\right]$$

$$= \frac{3}{2}\left[\pi^*(q) - \widetilde{C}(q)\right] = \frac{3}{2}\Pi(q, \widetilde{C}), \quad \text{where } \widetilde{C}(q) = \frac{2}{3}C(q) \,. \tag{61}$$

(61) implies:

$$\left[\frac{2}{3}\right] \frac{d\widehat{W}(q, C)}{dp_L} = \frac{d\Pi(q, \widetilde{C})}{dp_L} = \left.\frac{\partial \Pi(q, \widetilde{C})}{\partial p_L}\right|_{q \, = \, q^*} = \left.\frac{\partial \pi^*(q)}{\partial p_L}\right|_{q \, = \, q^*}$$

$$= \frac{2}{3} \left.\frac{\partial \widehat{W}(q, C)}{\partial p_L}\right|_{q \, = \, q^*} = \frac{2}{3}\frac{\partial W^*}{\partial p_L} \,. \tag{62}$$

The second equality in (62) reflects the envelope theorem. (62) implies that $\frac{d\widehat{W}(q,C)}{dp_L} \overset{s}{=} \frac{\partial W^*}{\partial p_L}$. Therefore, the conclusion in the proposition follows from Proposition 4. ■

II. Additional Conclusions.

Let Π^0 denote the level of maximum expected profit the lender can secure when she: (i) pays 0 to each entrepreneur whose project either fails or generates the unfavorable signal; and (ii) makes the same strictly positive payment to each entrepreneur whose project succeeds.

Conclusion 1. *The lender cannot secure a level of expected profit strictly above* Π^0 *by introducing two distinct payment pairs,* (S_H, F_H) *and* (S_L, F_L)*, where* S_i *is the payment the lender delivers to the entrepreneur when his project succeeds after he reports his project to have success probability* p_i*, and where* F_i *is the corresponding payment when the project fails.*

Proof. It is convenient to focus on the setting where the lender always finances some projects in equilibrium and where the i entrepreneur can ensure project failure by delivering no effort, whereas he can secure success with probability $p_i > 0$ by delivering an infinitesimally small level of effort (with associated infinitesimally small cost). Therefore, if the lender promises a strictly positive payment to the i entrepreneur, she will set $S_i \geq F_i$.

Let $w(p_j \,|\, p_i, x)$ denote expected welfare of the entrepreneur at location x with success probability p_i under the (S_j, F_j) contract in this "screening setting." Because the lender finances an entrepreneur's project if and only if she observes the favorable signal about the project:

$$\begin{aligned} w(p_L \,|\, p_L, x) &= [1-q]\,[p_L\, S_L + (1-p_L)\, F_L] - t_L\, x\,; \\ w(p_H \,|\, p_L, x) &= [1-q]\,[p_L\, S_H + (1-p_L)\, F_H] - t_L\, x\,; \\ w(p_H \,|\, p_H, x) &= q\,[p_H\, S_H + (1-p_H)\, F_H] - t_H\, x\,; \text{ and} \\ w(p_L \,|\, p_H, x) &= q\,[p_H\, S_L + (1-p_H)\, F_L] - t_H\, x\,. \end{aligned} \tag{63}$$

(63) implies that when he reports his project quality truthfully, the location of the i entrepreneur ($i \in \{L, H\}$) farthest from the lender that applies for funding is:

$$x_L = \frac{1-q}{t_L}\,[p_L\, S_L + (1-p_L)\, F_L] \quad \text{and} \quad x_H = \frac{q}{t_H}\,[p_H\, S_H + (1-p_H)\, F_H]\,. \tag{64}$$

(63) also implies that to ensure truthful reporting of project quality (which is without loss of generality), it must be the case that:

$$w(p_L \,|\, p_L, x) \geq w(p_H \,|\, p_L, x) \;\Leftrightarrow\; p_L\, S_L + [1-p_L]\, F_L \geq p_L\, S_H + [1-p_L]\, F_H\,; \text{ and} \tag{65}$$

$$w(p_H \,|\, p_H, x) \geq w(p_L \,|\, p_H, x) \;\Leftrightarrow\; p_H\, S_H + [1-p_H]\, F_H \geq p_H\, S_L + [1-p_H]\, F_L\,. \tag{66}$$

Because a successful project generates payoff V and an unsuccessful project generates payoff 0, the lender's expected profit when the entrepreneurs report their project quality truthfully is:

$$\phi_L \, x_L \, [\, 1 - q \,] \, [\, p_L \, (V - S_L) - (1 - p_L) \, F_L - I \,] \\ + \; \phi_H \, x_H \, q \, [\, p_H \, (V - S_H) - (1 - p_H) \, F_H - I \,] \, . \qquad (67)$$

(64) and (67) imply that the lender's problem, [LP], is:

$$Maximize \quad \frac{\phi_L}{t_L} [\, 1 - q \,]^2 \, [\, p_L \, S_L + (1 - p_L) \, F_L \,] \, [\, p_L \, (V - S_L) - (1 - p_L) \, F_L - I \,] \\ + \; \frac{\phi_H}{t_H} \, q^2 \, [\, p_H \, S_H + (1 - p_H) \, F_H \,] \, [\, p_H \, (V - S_H) - (1 - p_H) \, F_H - I \,]$$

subject to (65), (66), $S_L \geq 0$, $F_L \geq 0$, $S_H \geq 0$, and $F_H \geq 0$.

Let λ_{LH} and λ_{HL} denote the Lagrange multipliers associated with constraints (65) and (66), respectively. Then the necessary conditions for a solution to [LP] include:

$$L_{F_L} \;\equiv\; \frac{\phi_L}{t_L} [\, 1 - q \,]^2 \, [\, 1 - p_L \,] \, [\, p_L V - I - 2 \, (p_L \, S_L + [\, 1 - p_L \,] \, F_L) \,] \\ + \; \lambda_{LH} \, [\, 1 - p_L \,] - \lambda_{HL} \, [\, 1 - p_H \,] \;\leq\; 0 \, ; \qquad [\, L_{F_L} \,] \, F_L \;=\; 0. \qquad (68)$$

$$L_{S_L} \;\equiv\; \frac{\phi_L}{t_L} [\, 1 - q \,]^2 \; p_L \, [\, p_L V - I - 2 \, (p_L \, S_L + [\, 1 - p_L \,] \, F_L) \,] \\ + \; \lambda_{LH} \, p_L - \lambda_{HL} \, p_H \;\leq\; 0 \, ; \qquad [\, L_{S_L} \,] \, S_L \;=\; 0. \qquad (69)$$

$$L_{F_H} \;\equiv\; \frac{\phi_H}{t_H} \, q^2 \, [\, 1 - p_H \,] \, [\, p_H V - I - 2 \, (p_H \, S_H + [\, 1 - p_H \,] \, F_H) \,] \\ - \; \lambda_{LH} \, [\, 1 - p_L \,] + \lambda_{HL} \, [\, 1 - p_H \,] \;\leq\; 0 \, ; \qquad [\, L_{F_H} \,] \, F_H \;=\; 0. \qquad (70)$$

$$L_{S_H} \;\equiv\; \frac{\phi_H}{t_H} \, q^2 \, p_H \, [\, p_H V - I - 2 \, (p_H \, S_H + [\, 1 - p_H \,] \, F_H) \,] \\ - \; \lambda_{LH} \, p_L + \lambda_{HL} \, p_H \;\leq\; 0 \, ; \qquad [\, L_{S_H} \,] \, S_H \;=\; 0. \qquad (71)$$

Result 1. $\lambda_{LH} > 0$ and so $p_L \, S_L + [1 - p_L] \, F_L \;=\; p_L \, S_H + [1 - p_L] \, F_H$.

<u>Proof</u>. Suppose $\lambda_{LH} = 0$. Then since $p_L V - I < 0$ and $p_L \, S_L + [\, 1 - p_L \,] \, F_L \geq 0$, (68) implies that $F_L = 0$ and (69) implies that $S_L = 0$. Because the lender always funds some projects in equilibrium, it must be the case that $F_H > 0$ and/or $S_H > 0$. But then:

$$0 \;=\; p_L \, S_L + [\, 1 - p_L \,] \, F_L \;<\; p_L \, S_H + [\, 1 - p_L \,] \, F_H \, ,$$

which violates (65). ■

Result 2. $F_H = 0$.

Proof. Suppose $F_H > 0$. Then $S_H > 0$ since $S_H \geq F_H$. Therefore, from (70) and (71):

$$\frac{\phi_H}{t_H} q^2 [p_H V - I - 2(p_H S_H + [1 - p_H] F_H)] = \frac{\lambda_{LH} [1 - p_L] - \lambda_{HL} [1 - p_H]}{1 - p_H}$$

$$= \frac{\lambda_{LH} p_L - \lambda_{HL} p_H}{p_H}$$

$$\Rightarrow \lambda_{LH} p_H [1 - p_L] = \lambda_{LH} p_L [1 - p_H] \quad \Rightarrow \quad \lambda_{LH} = 0$$

since $p_H [1 - p_L] > p_L [1 - p_H]$. But this contradicts Result 1. ■

Result 3. $S_H > 0$.

Proof. If $S_H = 0$, then $F_H = S_H = 0$, from Result 2. Consequently, $F_L > 0$ and/or $S_L > 0$, given the maintained assumption that the lender finances some projects in equilibrium. But these payments violate (66). ■

Result 4. If $F_L > 0$, then $S_H = \beta^* V$, as defined in (4).

Proof. Suppose $F_L > 0$. Then $S_L > 0$ since $S_L \geq F_L$. Therefore, from (68) and (69):

$$\frac{\phi_L}{t_L} [1 - q]^2 [p_L V - I - 2(p_L S_L + [1 - p_L] F_L)] = \frac{\lambda_{HL} [1 - p_H] - \lambda_{LH} [1 - p_L]}{1 - p_L}$$

$$= \frac{\lambda_{HL} p_H - \lambda_{LH} p_L}{p_L} \tag{72}$$

$$\Rightarrow \lambda_{HL} p_L [1 - p_H] = \lambda_{HL} p_H [1 - p_L] \quad \Rightarrow \quad \lambda_{HL} = 0$$

since $p_L [1 - p_H] < p_H [1 - p_L]$. Therefore, from (72):

$$\frac{\phi_L}{t_L} [1 - q]^2 [p_L V - I - 2(p_L S_L + [1 - p_L] F_L)] = -\lambda_{LH} .$$

Furthermore, since $S_H > 0$ from Result 3, (71) implies:

$$\frac{\phi_H}{t_H} q^2 [p_H V - I - 2(p_H S_H + [1 - p_H] F_H)] = \lambda_{LH} \left[\frac{p_L}{p_H} \right] . \tag{73}$$

(72) and (73) along with Results 1 and 2 imply:

$$\frac{\phi_H}{t_H} q^2 [p_H V - I - 2 p_H S_H] + \frac{\phi_L}{t_L} [1 - q]^2 [p_L V - I - 2 p_L S_H] \left[\frac{p_L}{p_H} \right] = 0$$

$$\Rightarrow \quad \phi_H \, p_H \, q^2 \, [\, p_H V - I \,] \, t_L + \phi_L \, p_L \, [\, 1 - q \,]^2 \, [\, p_L V - I \,] \, t_H$$

$$= \; 2 \, S_H \left[\, \phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \, \right]$$

$$\Rightarrow \quad S_H \; = \; \frac{\phi_L \, p_L \, [\, 1 - q \,]^2 \, [\, p_L V - I \,] \, t_H + \phi_H \, p_H \, q^2 \, [\, p_H V - I \,] \, t_L}{2 \left[\, \phi_L \, p_L^2 \, (1-q)^2 \, t_H + \phi_H \, p_H^2 \, q^2 \, t_L \, \right]} \; = \; \beta^* V. \quad \blacksquare$$

Result 5. The single payment pair $(S_H, F_H) = (S_L, F_L) = (\beta^* V, 0)$ is a solution to [LP].

Proof. Results 2 and 4 imply that $(S_H, F_H) = (\beta^* V, 0)$ at the solution to [LP]. It remains to show that the lender cannot secure a strict increase in expected profit by introducing a distinct (S_L, F_L) payment pair that satisfies Result 1.

Observe from (64) that x_L will be unchanged by the introduction of a distinct payment pair that satisfies Result 1. Furthermore, the lender's expected profit from financing a low quality project under any distinct payment pair that satisfies Result 1 is:

$$p_L \, [\, V - S_L \,] - [\, 1 - p_L \,] \, F_L - I \; = \; p_L V - [\, p_L \, S_L + (1 - p_L) \, F_L \,] - I \; = \; p_L \, [\, V - S_H \,] - I$$

which is precisely the lender's expected profit from financing a low quality project under the $(S_H, 0)$ payment pair.

Therefore, the lender cannot secure a strict increase in expected profit by introducing a distinct (S_L, F_L) payment pair that satisfies Result 1. $\blacksquare$

The Setting where Locations are Observable

Definition. The full-information outcome is the outcome the lender would implement if she could observe each entrepreneur's location and the quality of his project.

In the full-information outcome: (i) the lender only funds the projects of H entrepreneurs; (ii) an H entrepreneur whose project is funded secures no rent; and (iii) the lender funds all projects with an expected payoff in excess of relevant investment and transaction costs.

Conclusion 2. *Suppose the location of each entrepreneur is observable. Further suppose $t_L \geq t_H$. Then the lender can secure the full-information outcome.*

Proof. Let x^* be defined by the equality $p_H V = I + t_H \, x^*$. Suppose the lender offers to an entrepreneur at location $x \leq x^*$: (i) $\beta_x V$ if his project succeeds; and (ii) 0 if his project fails, where $\beta_x = \frac{t_H \, x}{p_H \, V}$. An H entrepreneur at location $x \leq x^*$ who applies for funding under this contract secures expected profit:

$$p_H \, \beta_x V - t_H \, x \; = \; p_H \left[\frac{t_H \, x}{p_H \, V} \right] V - t_H \, x \; = \; 0 \, .$$

An L entrepreneur at location $x \leq x^*$ will not apply for funding because his expected profit under this contract is:

$$p_L\,\beta_x V - t_L\,x \;=\; p_L\left[\frac{t_H\,x}{p_H\,V}\right]V - t_L\,x \;<\; p_H\left[\frac{t_H\,x}{p_H\,V}\right]V - t_H\,x \;=\; 0\,.$$

The lender's expected profit on the project she funds for the entrepreneur at location x is:

$$p_H\,[\,1-\beta_x\,]\,V - I \;=\; p_H V\left[\frac{p_H\,V - t_H\,x}{p_H\,V}\right] - I \;=\; p_H\,V - t_H\,x - I \;=\; t_H\,[\,x^* - x\,]\,. \quad (74)$$

(74) implies that the lender will fund all projects with an expected payoff in excess of relevant investment and transaction costs (i.e., she will fund the projects of all H entrepreneurs located at $x \le x^*$). ■

The Setting where Each Entrepreneur has Wealth $w > 0$

Consider the setting where the lender: (i) funds a project if and only if the project generates a favorable signal; and (ii) pays an entrepreneur who applies for funding T^0 if his project is not funded, T^S if his funded project succeeds, and T^F if his funded project fails.

Definition. An H entrepreneur's gross expected payoff if he applies for funding is $q\,[\,p_H\,T^S + (1-p_H)\,T^F\,] + [\,1-q\,]\,T^0$. An L entrepreneur's corresponding gross expected payoff is $[\,1-q\,]\left[\,p_L\,T^S + (1-p_L)\,T^F\,\right] + q\,T^0$.

Conclusion 3. *Among all lending arrangements that ensure at least gross expected payoff $\widehat{\pi} > 0$ for an H entrepreneur, the arrangement that minimizes the gross expected payoff for an L entrepreneur has $T^F = T^0 = -\,w$.*

Proof. The lending arrangement that minimizes the gross expected payoff for an L entrepreneur while ensuring at least gross expected payoff $\widehat{\pi} > 0$ for an H entrepreneur is the solution to the following problem, [P]:

$$\underset{T^S,\,T^F,\,T^0}{Maximize} \quad -\left\{\,[\,1-q\,]\left[\,p_L\,T^S + (1-p_L)\,T^F\,\right] + q\,T^0\,\right\}$$

subject to:

$$q\left[\,p_H\,T^S + (1-p_H)\,T^F\,\right] + [\,1-q\,]\,T^0 \;\ge\; \widehat{\pi}\,; \quad (75)$$

$$T^S \;\ge\; -\,w\,; \quad T^F \;\ge\; -\,w\,; \;\text{ and }\; T^0 \;\ge\; -\,w\,. \quad (76)$$

Let λ denote the Lagrange multiplier associated with constraint (75) and let λ^S, λ^F, and λ^0, respectively, denote the Lagrange multipliers associated with the three constraints in (76). The necessary conditions for a solution to [P] include:

$$T^S: \quad -\,p_L\,[\,1-q\,] + \lambda\,p_H\,q + \lambda^S \;=\; 0\,. \quad (77)$$

$$T^F: \quad -\,[\,1-p_L\,]\,[\,1-q\,] + \lambda\,[\,1-p_H\,]\,q + \lambda^F \;=\; 0\,. \quad (78)$$

$$T^0: \quad -\,q + \lambda\,[\,1-q\,] + \lambda^0 \;=\; 0\,. \quad (79)$$

Adding (77) - (79) provides:

$$\lambda + \lambda^0 + \lambda^S + \lambda^F \;=\; 1 \quad \Rightarrow \quad \lambda,\ \lambda^0,\ \lambda^S,\ \lambda^F \;\in\; [\,0, 1\,]\,. \tag{80}$$

Suppose $\lambda = 0$. Then (77), (78), and (79) imply:

$$\lambda^S \;=\; p_L\,[\,1-q\,] \;>\; 0,\;\; \lambda^F \;=\; [\,1-p_L\,]\,[\,1-q\,] \;>\; 0,\;\text{ and }\;\lambda^0 \;=\; q \;>\; 0\,.$$

Consequently, $T^S = T^F = T^0 = -\,w$, which violates (75) since $\widehat{\pi} > 0$. Therefore, $\lambda > 0$.

Suppose $\lambda^0 = 0$. Then from (79), $\lambda = \frac{q}{1-q} > 1$ (since $q > \frac{1}{2}$), which violates (80). Therefore, $\lambda^0 > 0$, and so $T^0 = -\,w$.

From (77) and (78):

$$\lambda\left[\frac{p_H}{p_L}\right] q + \frac{\lambda^S}{p_L} \;=\; 1-q \;=\; \lambda\left[\frac{1-p_H}{1-p_L}\right] q + \frac{\lambda^F}{1-p_L}$$

$$\Rightarrow\quad \lambda\left[\frac{p_H}{p_L} - \frac{1-p_H}{1-p_L}\right] q \;=\; \frac{\lambda^F}{1-p_L} - \frac{\lambda^S}{p_L} \quad\Rightarrow\quad \lambda^F \;>\; 0. \tag{81}$$

The last inequality in (81) holds because $\frac{p_H}{p_L} > \frac{1-p_H}{1-p_L}$, $q > 0$, and $\lambda > 0$. The last inequality in (81) implies that $T^F = -\,w$. ■

<u>Definition</u>. The setting with observable project quality is the (hypothetical) setting in which the lender can observe perfectly the quality (i.e., the success probability) of each entrepreneur's project.

Conclusion 4. *In the setting with observable project quality, the lender secures expected profit* $\frac{\phi_H}{4\,t_H}\,[\,p_H V - I\,]^2$. *She does so by delivering to each H entrepreneur that applies for funding a gross expected payoff of* $\frac{1}{2}\,[\,p_H V - I\,]$. *This payoff induces all H entrepreneurs in the interval* $\left[\,0,\; \frac{p_H V - I}{2\,t_H}\,\right]$ *to apply for funding.*

<u>Proof</u>. Let π_H denote the gross expected payoff the lender provides to an H entrepreneur that applies for funding. An H entrepreneur at location x will apply for funding as long as $\pi_H - t_H\,x \geq 0$. Therefore, H entrepreneurs in the $[\,0,\, x_H(\pi_H)]$ interval will apply for funding, where $x_H(\pi_H) = \frac{\pi_H}{t_H}$.

The lender's expected profit when she promises gross expected payoff π_H to each H entrepreneur that applies for funding is:

$$\Pi(\pi_H) \;=\; \phi_H\, x_H(\pi_H)\,[\,p_H V - I - \pi_H\,] \;=\; \frac{\phi_H\,\pi_H}{t_H}\,[\,p_H V - I - \pi_H\,]\,. \tag{82}$$

The value of π_H that maximizes $\Pi(\pi_H)$ is determined by:

$$\Pi'(\pi_H) \;=\; 0 \;\Leftrightarrow\; -\,\pi_H + p_H V - I - \pi_H \;=\; 0 \;\Leftrightarrow\; \pi_H^* \;=\; \frac{1}{2}\,[\,p_H V - I\,]\,. \tag{83}$$

(83) implies:

$$x_H(\pi_H^*) \;=\; \frac{1}{2\,t_H}\,[\,p_H V - I\,]\,. \tag{84}$$

(82), (83), and (84) provide:

$$\Pi(\pi_H^*) \;=\; \left[\frac{\phi_H}{t_H}\right]\frac{1}{2}\,[\,p_H V - I\,]\left[p_H V - I - \frac{1}{2}\,(p_H V - I)\right] \;=\; \frac{\phi_H}{4\,t_H}\,[\,p_H V - I\,]^2\,. \quad \blacksquare$$

Conclusion 5. *Suppose* $w \geq \frac{p_L[1-q][p_H V - I]}{2\,[\,q\,p_H - p_L(1-q)\,]}$. *Then the lender can secure expected profit* $\frac{\phi_H}{4\,t_H}\,[\,p_H V - I\,]^2$, *the same expected profit she secures in the setting with observable project quality.*

Proof. Conclusion 4 implies that the lender can secure expected profit $\frac{\phi_H}{4\,t_H}\,[\,p_H V - I\,]^2$ if she can ensure that no L entrepreneur applies for funding when the lender offers expected gross profit $\frac{1}{2}\,[\,p_H V - I\,]$ to each H entrepreneur that applies for funding. From Conclusion 3, if the lender delivers this expected gross profit to H entrepreneurs in the form that minimizes the expected gross payoff of L entrepreneurs:

$$\begin{aligned}
\frac{1}{2}\,[\,p_H\,V - I\,] \;&=\; q\,\big[\,p_H\,T^S - (1-p_H)\,w\,\big] - [\,1-q\,]\,w \\
\Rightarrow\quad q\,p_H\,T^S \;&=\; \frac{1}{2}\,[\,p_H\,V - I\,] + w\,[\,1 - q + q\,(1-p_H)] \\
\Rightarrow\quad T^S \;&=\; \frac{1}{q\,p_H}\left\{\frac{1}{2}\,[\,p_H\,V - I\,] + w\,[\,1 - q\,p_H]\right\}.
\end{aligned} \tag{85}$$

The expected profit of the L entrepreneur located at $x = 0$ under these financing terms is:

$$\begin{aligned}
&[\,1-q\,]\,\big[\,p_L\,T^S - (1-p_L)\,w\,\big] - q\,w \;\leq\; 0 \\
\Leftrightarrow\quad &[\,1-q\,]\,p_L\,T^S \;\leq\; w\,[\,q + (1-q)\,(1-p_L)\,] \;=\; w\,[\,1 - p_L\,(1-q)\,] \\
\Leftrightarrow\quad &T^S \;\leq\; w\left[\frac{1 - p_L\,(1-q)}{p_L\,(1-q)}\right].
\end{aligned} \tag{86}$$

(85) and (86) imply that no L entrepreneur will apply for funding under these financing terms if:

$$\begin{aligned}
&\frac{1}{2\,q\,p_H}\,[\,p_H\,V - I\,] + w\left[\frac{1 - q\,p_H}{q\,p_H}\right] \;\leq\; w\left[\frac{1 - p_L\,(1-q)}{p_L\,(1-q)}\right] \\
\Leftrightarrow\quad &\frac{1}{2\,q\,p_H}\,[\,p_H\,V - I\,] \;\leq\; w\left[\frac{1 - p_L\,(1-q)}{p_L\,(1-q)} - \frac{1 - q\,p_H}{q\,p_H}\right] \\
\Leftrightarrow\quad &\frac{1}{2}\,p_L\,[\,1-q\,]\,[\,p_H\,V - I\,] \;\leq\; w\,\{\,q\,p_H\,[\,1 - p_L\,(1-q)\,] - p_L\,[\,1-q\,]\,[\,1 - q\,p_H\,]\,\}
\end{aligned}$$

$$\Leftrightarrow \quad \frac{1}{2} p_L [1-q][p_H V - I] \;\le\; w\{q p_H - q[1-q] p_L p_H - p_L[1-q] + q[1-q] p_L p_H\}$$

$$\Leftrightarrow \quad \frac{1}{2} p_L [1-q][p_H V - I] \;\le\; w[q p_H - p_L(1-q)]$$

$$\Leftrightarrow \quad w \;\ge\; \frac{p_L[1-q][p_H V - I]}{2[q p_H - p_L(1-q)]}. \quad \blacksquare$$

The Setting with Lender Competition

For simplicity, we consider a symmetric setting with two lenders in which each entrepreneur applies to at most one lender for funding. Lender 1 is located at $\frac{3}{8}$ and lender 2 is located at $\frac{5}{8}$. The two lenders have the same screening accuracy (q), and all entrepreneurs face the same transaction cost (so $t_L = t_H = t$).

We focus on settings in which, in equilibrium: (i) all H entrepreneurs in $[\frac{3}{8}, \frac{5}{8}]$ apply for funding, but all L entrepreneurs do not; and (ii) some L entrepreneurs and some H entrepreneurs in $(0, \frac{3}{8})$ and in $(\frac{5}{8}, 1)$ do not apply for funding. To do so, we assume the entrepreneurs' transaction cost is intermediate in magnitude, i.e.:

$$maximum\,\{t_1, t_2\} \;<\; t \;<\; t_3 \tag{87}$$

where:

$$t_1 \;=\; \frac{8 p_H q\left[3\phi_H p_H q^2 (p_H V - I) + 4\phi_L p_L (1-q)^2 (p_L V - I)\right]}{24\phi_L p_L^2 [1-q]^2 + 17\phi_H p_H^2 q^2},$$

$$t_2 \;=\; \frac{8 p_L q\left[3\phi_H p_H q^2 (p_H V - I) + 4\phi_L p_L (1-q)^2 (p_L V - I)\right]}{8\phi_L p_L^2 [1-q]^2 + 5\phi_H p_H^2 q^2 + 2\phi_H p_H p_L q^2}, \text{ and}$$

$$t_3 \;=\; \frac{8 p_H q\left[3\phi_H p_H q^2 (p_H V - I) + 4\phi_L p_L (1-q)^2 (p_L V - I)\right]}{8\phi_L p_L^2 [1-q]^2 + 7\phi_H p_H^2 q^2}.$$

Let β_i denote the sharing rate offered by lender $i \in \{1, 2\}$. Also let $\underline{x}_{L01} \le \frac{3}{8}$ and $\overline{x}_{L01} \ge \frac{3}{8}$ denote the locations of the L entrepreneurs who are indifferent between applying to lender 1 and not applying for funding. It is readily verified that:

$$\underline{x}_{L01} \;=\; \frac{3}{8} - \frac{[1-q] p_L V \beta_1}{t} \quad \text{and} \quad \overline{x}_{L01} \;=\; \frac{3}{8} + \frac{[1-q] p_L V \beta_1}{t}. \tag{88}$$

Let $\underline{x}_{H01} \le \frac{3}{8}$ denote the location of the H entrepreneur who is indifferent between applying to lender 1 and not applying for funding. Also let $\widehat{x}_H \in (\frac{3}{8}, \frac{5}{8})$ denote the location of the H entrepreneur who is indifferent between applying to lenders 1 and 2 for funding. It is readily verified that:

$$\underline{x}_{H01} \;=\; \frac{3}{8} - \frac{q p_H V \beta_1}{t} \quad \text{and} \quad \widehat{x}_H \;=\; \frac{1}{2} + \frac{q p_H V [\beta_1 - \beta_2]}{2t}. \tag{89}$$

The profit of lender 1, given sharing rates β_1 and β_2, is:

$$\begin{aligned}
\pi_1(\beta_1,\beta_2) &= \phi_L\,[\,1-q\,]\,[\,p_L V\,(1-\beta_1)-I\,]\,[\,\overline{x}_{L01}-\underline{x}_{L01}\,] \\
&\qquad\qquad + \ \phi_H\, q\,[\,p_H\, V\,(1-\beta_1)-I\,]\,[\,\widehat{x}_H-\underline{x}_{H01}] \\
&= \phi_L\,[\,1-q\,]\,[\,p_L V\,(1-\beta_1)-I\,]\,\frac{2\,[\,1-q\,]\,p_L V\beta_1}{t} \\
&\qquad + \ \phi_H\, q\,[\,p_H\, V\,(1-\beta_1)-I\,]\left[\frac{1}{8}+\frac{p_H\, V q\,(3\,\beta_1-\beta_2)}{2\,t}\right]. \qquad (90)
\end{aligned}$$

The last equality in (90) reflects (88) and (89).

(90) implies that firm 1's profit given sharing rates β_1 and β_2 can be written as

$$\pi_1(\beta_1,\beta_2) \ = \ a_1+a_2\,\beta_1+a_3\,\beta_1^2+a_4\,\beta_1\,\beta_2+a_5\,\beta_2\,, \qquad (91)$$

where:

$$\begin{aligned}
a_1 &= \frac{\phi_H\, q\,[\,p_H\, V-I\,]}{8}\,; \\
a_2 &= \frac{2\,[\,1-q\,]^2\,\phi_L\, p_L V\,[\,p_L V-I\,]}{t}+\frac{3\,q^2\phi_H\, p_H\, V\,[\,p_H V-I\,]}{2\,t}-\frac{\phi_H\, q\, p_H\, V}{8}\,; \\
a_3 &= -\left[\frac{2\,\phi_L\, p_L^2\,(1-q)^2}{t}+\frac{3\,\phi_H\, p_H^2\, q^2}{2\,t}\right]V^2\,; \\
a_4 &= \frac{\phi_H\, p_H^2\, q^2\, V^2}{2\,t}\,; \quad \text{and} \quad a_5 \ = \ -\,\frac{\phi_H\, p_H\, V q^2\,[\,p_H\, V-I\,]}{2\,t}\,. \qquad (92)
\end{aligned}$$

Let $\underline{x}_{L02} \leq \frac{5}{8}$ and $\overline{x}_{L02} \geq \frac{5}{8}$ denote the locations of the L entrepreneurs who are indifferent between applying to lender 2 and not applying for funding. Also let $\overline{x}_{H02} \geq \frac{5}{8}$ denote the location of the H entrepreneur who is indifferent between applying to lender 2 and not applying for funding. It is readily verified that:

$$\begin{aligned}
\underline{x}_{L02} &= \frac{5}{8}-\frac{[\,1-q\,]\,p_L V\beta_2}{t}\,; \quad \overline{x}_{L02} \ = \ \frac{5}{8}+\frac{[\,1-q\,]\,p_L V\beta_2}{t}\,; \quad \text{and} \\
&\qquad\qquad \overline{x}_{H02} \ = \ \frac{5}{8}+\frac{q\, p_H\, V\beta_2}{t}\,. \qquad (93)
\end{aligned}$$

The profit of lender 2, given sharing rates β_1 and β_2, is:

$$\begin{aligned}
\pi_2(\beta_1,\beta_2) &= \phi_L\,[\,1-q\,]\,[\,p_L V\,(1-\beta_2)-I\,]\,[\,\overline{x}_{L02}-\underline{x}_{L02}\,] \\
&\qquad\qquad + \ \phi_H\, q\,[\,p_H\, V\,(1-\beta_2)-I\,]\,[\,\overline{x}_{H02}-\widehat{x}_H\,] \\
&= \phi_L\,[\,1-q\,]\,[\,p_L\, V\,(1-\beta_2)-I\,]\ \frac{2\,[\,1-q\,]\,p_L V\,\beta_2}{t}
\end{aligned}$$

$$+\ \phi_H\, q\,[\,p_H\, V\,(1-\beta_2) - I\,]\left[\frac{1}{8} + \frac{p_H\, V q\,(3\,\beta_2 - \beta_1)}{2\,t}\right]. \tag{94}$$

The last equality in (94) reflects (89) and (93).

(94) implies that lender 2's profit given sharing rates β_1 and β_2 can be rewritten as:

$$\pi_2(\beta_1, \beta_2) \ = \ b_1 + b_2\,\beta_2 + b_3\,\beta_2^2 + b_4\,\beta_1\,\beta_2 + b_5\,\beta_1\,. \tag{95}$$

Differentiating (91) and (95) provides:

$$\frac{\partial \pi_1}{\partial \beta_1} \ = \ a_2 + 2\,a_3\,\beta_1 + a_4\,\beta_2 \ = \ 0 \quad \text{and} \quad \frac{\partial \pi_2}{\partial \beta_2} \ = \ b_2 + 2\,b_3\,\beta_2 + b_4\,\beta_1 \ = \ 0$$

$$\Rightarrow \begin{bmatrix} 2a_3 & a_4 \\ b_4 & 2b_3 \end{bmatrix}\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \ = \ -\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \ \Rightarrow \ \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \ = \ -\begin{bmatrix} 2a_3 & a_4 \\ b_4 & 2b_3 \end{bmatrix}^{-1}\begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

$$= \ -\frac{1}{4\,a_3\,b_3 - a_4\,b_4}\begin{bmatrix} 2\,b_3 & -\,a_4 \\ -\,b_4 & 2\,a_3 \end{bmatrix}\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \ = \ -\frac{1}{4\,a_3\,b_3 - a_4\,b_4}\begin{bmatrix} 2\,b_3\,a_2 - a_4\,b_2 \\ -\,b_4\,a_2 + 2\,a_3\,b_2 \end{bmatrix}$$

$$\Rightarrow \quad \beta_1^* \ = \ \frac{a_4\,b_2 - 2\,b_3\,a_2}{4\,a_3\,b_3 - a_4\,b_4} \quad \text{and} \quad \beta_2^* \ = \ \frac{b_4\,a_2 - 2\,a_3\,b_2}{4\,a_3\,b_3 - a_4\,b_4}\,. \tag{96}$$

The symmetry in (90) and (94) and in (91) and (95) ensures that $a_i = b_i$ for $i = 1, ..., 5$. Therefore, from (96):

$$\beta_1^* \ = \ \beta_2^* \ = \ \frac{a_4\,a_2 - 2\,a_3\,a_2}{4\,a_3^2 - a_4^2} \ = \ \frac{a_2\,[\,a_4 - 2\,a_3\,]}{[\,2\,a_3 + a_4\,]\,[\,2\,a_3 - a_4\,]} \ = \ -\frac{a_2}{2\,a_3 + a_4}\,. \tag{97}$$

From (92):

$$2\,a_3 + a_4 \ = \ \frac{\phi_H\, p_H^2\, q^2\, V^2}{2\,t} - 2\left[\frac{2\,\phi_L\, p_L^2\,(1-q)^2}{t} + \frac{3\,\phi_H\, p_H^2\, q^2}{2\,t}\right]V^2$$

$$= \ -\frac{1}{2\,t}\left[\,8\,\phi_L\, p_L^2\,(1-q)^2 + 5\,\phi_H\, p_H^2\, q^2\,\right]V^2 \ < \ 0\,. \tag{98}$$

(97) and (98) provide:

$$\beta_1^* \ = \ \beta_2^* \ = \ \frac{2\,t\,a_2}{\left[\,8\,\phi_L\, p_L^2\,(1-q)^2 + 5\,\phi_H\, p_H^2\, q^2\,\right]V^2}\,. \tag{99}$$

From (92):

$$a_2 \ = \ \frac{1}{8\,t}\left\{\,16\,[\,1-q\,]^2\,\phi_L\, p_L V\,[\,p_L V - I\,] + 12\,q^2 \phi_H\, p_H V\,[\,p_H\, V - I\,] - \phi_H\, q\, p_H\, V\, t\,\right\}$$

$$= \ \frac{1}{8\,t}\left\{\,16\,[\,1-q\,]^2\,\phi_L\, p_L\,[\,p_L V - I\,] + \phi_H\, q\, p_H\,[\,12\,(p_H\, V - I)\, q - t\,]\right\}V\,. \tag{100}$$

(99) and (100) imply that the equilibrium sharing rates are:

$$\beta_1^* \;=\; \beta_2^* \;=\; \frac{16\,[\,1-q\,]^2\,\phi_L\,p_L\,[\,p_L V - I\,] + \phi_H\,q\,p_H\,[\,12\,(p_H\,V - I)\,q - t\,]}{4\left[\,8\,\phi_L\,p_L^2\,(1-q)^2 + 5\,\phi_H\,p_H^2\,q^2\,\right]V} \;\equiv\; \beta^*\,. \tag{101}$$

Conclusion 6. $\left.\frac{\partial \beta^*}{\partial p_L}\right|_{p_L=0} < 0$ *in the setting with lender competition.*

Proof. From (101):

$$\frac{\partial \beta^*}{\partial p_L} \overset{s}{=} \left[\,8\,\phi_L\,p_L^2\,(1-q)^2 + 5\,\phi_H\,p_H^2\,q^2\,\right]\left[\,16\,(1-q)^2\,\phi_L\,(\,2\,p_L V - I\,)\,\right]$$

$$- \left[\,16\,\phi_L\,p_L\,(1-q)^2\;\phi_L\,p_L\,\right]\left[\,16\,(1-q)^2\,\phi_L\,p_L\,(p_L V - I) + \phi_H\,q\,p_H\,(12\,[\,p_H\,V - I\,]\,q - t\,)\,\right]$$

$$= \left[\,2\,p_L V - I\,\right]\left[\,8\,\phi_L\,p_L^2\,(1-q)^2 + 5\,\phi_H\,p_H^2\,q^2\,\right]V$$

$$-\; p_L\left[\,16\,(1-q)^2\,\phi_L\,p_L\,(p_L V - I) + \phi_H\,q\,p_H\,(12\,[\,p_H\,V - I\,]\,q - t)\,\right].$$

$$\Rightarrow \quad \left.\frac{\partial \beta^*}{\partial p_L}\right|_{p_L=0} \;=\; -\,5\,I\,\phi_H\,p_H^2\,q^2\,V \;<\; 0\,. \quad \blacksquare$$

Conclusion 7. $\frac{\partial W_L^*}{\partial p_L} > 0$ *for* p_L *sufficiently close to* 0 *in the setting with lender competition.*

Proof. The equilibrium aggregate welfare of L entrepreneurs who secure funding from lender 1 is:

$$W_{L1}^* \;=\; \phi_L\,[\,\overline{x}_{L01}^* - \underline{x}_{L01}^*\,]\,[\,1-q\,]\,p_L V \beta^* - \phi_L\,t\left[\int_{\underline{x}_{L01}^*}^{\frac{3}{8}}\left(\frac{3}{8} - \xi\right)d\xi + \int_{\frac{3}{8}}^{\overline{x}_{L01}^*}\left(\xi - \frac{3}{8}\right)d\xi\right]$$

$$= \phi_L\left[\frac{2\,(1-q)\,p_L V \beta^*}{t}\right][\,1-q\,]\,p_L V \beta^* - \phi_L\,t\left[\frac{1}{2}\left(\frac{3}{8} - \underline{x}_{L01}^*\right)^2 + \frac{1}{2}\left(\overline{x}_{L01}^* - \frac{3}{8}\right)^2\right]$$

$$= \frac{2\,\phi_L\,[\,(1-q)\,p_L V \beta^*]^2}{t} - \frac{\phi_L\,t}{2}\left[\frac{2\,[\,(1-q)\,p_L V \beta^*\,]^2}{t^2}\right] \;=\; \frac{\phi_L\,[\,(1-q)\,p_L V \beta^*\,]^2}{t}\,. \tag{102}$$

The second equality in (102) reflects (88).

(102) implies that, due to the symmetry in the problem, the equilibrium aggregate welfare of all L entrepreneurs is:

$$W_L^* \;=\; 2\,W_{L1}^* \;=\; \frac{2\,\phi_L\,[\,(1-q)\,p_L V \beta^*\,]^2}{t}\,. \tag{103}$$

(103) implies that W_L^* is an increasing function of p_L if $p_L\,\beta^*$ is an increasing function of p_L. Note that $p_L\,\beta^*$ is an increasing function of p_L if $log\,(p_L\,\beta^*)$ is an increasing function of p_L. Also:

$$\log(p_L\,\beta^*) \;=\; \log(p_L) + \log(\beta^*) \quad\Rightarrow\quad \frac{\partial \log(p_L\,\beta^*)}{\partial p_L} \;=\; \frac{1}{p_L} + \frac{1}{\beta^*}\left[\frac{\partial \beta^*}{\partial p_L}\right]. \tag{104}$$

(101) implies that β^* can be expressed as:

$$\beta^* \;=\; \frac{\alpha\, p_L\,[\,p_L V - I\,] + \gamma}{c\, p_L^2 + d}\,,$$

where α, γ, c, and d are positive terms that do not vary with p_L. Therefore:

$$\begin{aligned}
\frac{\partial \beta^*}{\partial p_L} \;&=\; \frac{[\,c\,p_L^2 + d\,]\,\alpha\,[\,2\,p_L V - I\,] - [\,\alpha\, p_L\,(p_L V - I) + \gamma\,]\,2\,c\,p_L}{[\,c\,p_L^2 + d\,]^2} \\
&=\; \frac{-\,2\,\gamma\,c\,p_L + \alpha\,[\,d\,(2\,p_L V - I) + c\,p_L^2\, I\,]}{[\,c\,p_L^2 + d\,]^2}
\end{aligned}$$

$$\Rightarrow\; \frac{1}{\beta^*}\left[\frac{\partial \beta^*}{\partial p_L}\right] \;=\; \frac{-\,2\,\gamma\,c\,p_L + \alpha\,[\,d\,(2\,p_L V - I) + c\,p_L^2\, I\,]}{[\,c\,p_L^2 + d\,]\,[\,\alpha\, p_L\,(p_L V - I) + \gamma\,]} \;\to\; -\,\frac{\alpha\, I}{\gamma} \;\text{ as }\; p_L \;\to\; 0\,. \tag{105}$$

(103), (104), and (105) imply:

$$\frac{\partial \log(p_L\,\beta^*)}{\partial p_L} \;=\; \frac{1}{p_L} + \frac{1}{\beta^*}\left[\frac{\partial \beta^*}{\partial p_L}\right] \;\to\; \infty \;\text{ as }\; p_L \;\to\; 0\,,$$

and so $\frac{\partial W_L^*}{\partial p_L} > 0$ for p_L sufficiently close to 0. ■

Conclusion 8. $\left.\frac{\partial \pi_1^*}{\partial p_L}\right|_{p_L = 0} < 0$ *in the setting with lender competition.*

<u>Proof</u>. From (91) and (97):

$$\begin{aligned}
\pi_1^* \;&=\; a_1 + a_2\left[-\,\frac{a_2}{2\,a_3 + a_4}\right] + a_3\left[-\,\frac{a_2}{2a_3 + a_4}\right]^2 + a_4\,(\beta^*)^2 + a_5\,\beta^* \\
&=\; a_1 - \frac{a_2^2}{2\,a_3 + a_4} + \frac{a_3\,a_2^2}{(2\,a_3 + a_4)^2} + a_4\,(\beta^*)^2 + a_5\,\beta^* \\
&=\; a_1 - \frac{a_2^2}{2\,a_3 + a_4}\left[1 - \frac{a_3}{2\,a_3 + a_4}\right] + a_4\,(\beta^*)^2 + a_5\,\beta^* \\
&=\; a_1 - \frac{a_2^2\,[\,a_3 + a_4\,]}{[\,2\,a_3 + a_4\,]^2} + a_4\,(\beta^*)^2 + a_5\,\beta^* \\
&=\; a_1 - [\,a_3 + a_4\,]\left[\frac{a_2}{2\,a_3 + a_4}\right]^2 + a_4\,(\beta^*)^2 + a_5\,\beta^* \\
&=\; a_1 - [\,a_3 + a_4\,]\,(\beta^*)^2 + a_4\,(\beta^*)^2 + a_5\,\beta^* \;=\; a_1 - a_3\,(\beta^*)^2 + a_5\,\beta^*\,.
\end{aligned} \tag{106}$$

Since $\left.\frac{\partial a_3}{\partial p_L}\right|_{p_L=0} = \frac{\partial a_1}{\partial p_L} = \frac{\partial a_5}{\partial p_L} = 0$ from (92), (106) implies:

$$\left.\frac{\partial \pi_1^*}{\partial p_L}\right|_{p_L=0} = -\,2\,a_3 \left.\frac{\partial \beta^*}{\partial p_L}\right|_{p_L=0} + a_5 \left.\frac{\partial \beta^*}{\partial p_L}\right|_{p_L=0} = \left.\frac{\partial \beta^*}{\partial p_L}\right|_{p_L=0} \left[\left.(a_5 - 2\,a_3)\right|_{p_L=0} \right]. \quad (107)$$

From (92):

$$a_5 - 2\,a_3 = 2\left[\frac{2\,\phi_L\,p_L^2\,(1-q)^2}{t} + \frac{3\,\phi_H\,p_H^2\,q^2}{2\,t} \right] V^2 - \frac{\phi_H\,p_H\,V q^2\,[\,p_H V - I\,]}{2\,t}$$

$$\Rightarrow \quad \left.(a_5 - 2\,a_3)\right|_{p_L=0} = \frac{6\,\phi_H\,p_H^2\,q^2\,V^2}{2\,t} - \frac{\phi_H\,p_H\,V q^2\,[\,p_H V - I\,]}{2\,t}$$

$$= \frac{\phi_H\,p_H\,V q^2}{2\,t}\,[\,6\,p_H\,V - (p_H\,V - I)\,] = \frac{\phi_H\,p_H\,V q^2}{2\,t}\,[\,5\,p_H\,V + I\,] > 0\,. \quad (108)$$

(107), (108), and Conclusion 6 imply $\left.\frac{\partial \pi_1^*}{\partial p_L}\right|_{p_L=0} < 0$. ■

Conclusion 9. $\left.\frac{\partial W_H^*}{\partial p_L}\right|_{p_L=0} < 0$ *in the setting with lender competition.*

Proof. The equilibrium aggregate welfare of H entrepreneurs who secure funding from lender 1 is:

$$W_{H1}^* = \phi_H\,[\,\widehat{x}_H^* - \underline{x}_{H01}^*\,]\,q\,p_H\,V\beta^* - \phi_H\,t\left[\int_{\underline{x}_{H01}^*}^{\frac{3}{8}} \left(\frac{3}{8} - \xi\right) d\xi + \int_{\frac{3}{8}}^{\widehat{x}_H^*} \left(\xi - \frac{3}{8}\right) d\xi \right]$$

$$= \phi_H\left[\frac{1}{8} + \frac{p_H\,V q\,\beta^*}{t} \right] q\,p_H\,V\beta^* - \phi_H\,t\left[\frac{1}{2}\left(\frac{p_H\,V q\,\beta^*}{t}\right)^2 + \frac{1}{2}\left(\frac{1}{8}\right)^2 \right]$$

$$= \frac{1}{8}\,\phi_H\,q\,p_H\,V\beta^* + \frac{\phi_H\,[\,p_H\,V q\,\beta^*\,]^2}{t} - \frac{1}{2}\frac{\phi_H\,[\,p_H\,V q\,\beta^*\,]^2}{t} - \frac{\phi_H\,t}{128}$$

$$= \frac{1}{8}\,\phi_H\,q\,p_H\,V\beta^* + \frac{1}{2}\,\frac{\phi_H\,[\,p_H\,V q\,\beta^*\,]^2}{t} - \frac{\phi_H\,t}{128}\,. \quad (109)$$

The second equality in (109) reflects (89) and the fact that $\widehat{x}_H^* = \frac{1}{2}$.

(109) and the symmetry in the problem imply that W_H^*, the equilibrium aggregate welfare of all H entrepreneurs, is:

$$W_H^* = 2\,W_{H1}^* = \frac{1}{4}\,\phi_H\,q\,p_H\,V\beta^* + \frac{\phi_H\,[\,p_H\,V q\,\beta^*\,]^2}{t} - \frac{\phi_H\,t}{64}$$

$$\Rightarrow \quad \frac{\partial W_H^*}{\partial p_L} = \frac{1}{4}\,\phi_H\,q\,p_H\,V\left[\frac{\partial \beta^*}{\partial p_L}\right] + \frac{2\,\phi_H\,(p_H\,V q)^2\,\beta^*}{t}\left[\frac{\partial \beta^*}{\partial p_L}\right]. \quad (110)$$

(110) implies that $\left.\frac{\partial W_H^*}{\partial p_L}\right|_{p_L=0} < 0$ because $\left.\frac{\partial \beta^*}{\partial p_L}\right|_{p_L=0} < 0$ from Conclusion 6. ■

Conclusion 10. $\frac{\partial W^*}{\partial p_L} < 0$ *for* p_L *sufficiently close to* 0 *in the setting with lender competition.*

Proof. Since $W^* = \pi_1^* + \pi_2^* + W_L^* + W_H^*$:

$$\frac{\partial W^*}{\partial p_L} = \frac{\partial \pi_1^*}{\partial p_L} + \frac{\partial \pi_2^*}{\partial p_L} + \frac{\partial W_L^*}{\partial p_L} + \frac{\partial W_H^*}{\partial p_L}. \tag{111}$$

Differentiating (103) provides:

$$\frac{\partial W_L^*}{\partial p_L} = -\frac{\phi_L\, p_L\, [1-q]^2\, [B]\, [\widetilde{B}]}{4\left[8\,\phi_L\, p_L^2\, (1-q)^2 + 5\,\phi_H\, p_H^2\, q^2\right]^3 t} \tag{112}$$

where

$$B \equiv 16\,\phi_L\, p_L\, [1-q]^2\, [p_L V - I] + \phi_H\, p_H\, q\, [12\,(p_H V - I)\, q - t], \text{ and} \tag{113}$$

$$\begin{aligned}\widetilde{B} \equiv\; & -128\,\phi_L^2\, p_L^4\, [1-q]^4\, V - 5\,\phi_H^2\, p_H^3\, q^3\, [12\,(p_H V - I)\, q - t] \\ & + 8\,\phi_H\, \phi_L\, p_H\, p_L\, [1-q]^2\, q\, [4\, I\, (5 p_H - 3 p_L)\, q - p_L\, (t + 18\, p_H\, q\, V)].\end{aligned}$$

It is apparent from (112) that $\left.\frac{\partial W_L^*}{\partial p_L}\right|_{p_L=0} = 0$. Furthermore, $\left.\frac{\partial W_H^*}{\partial p_L}\right|_{p_L=0} < 0$, from Conclusion 9. In addition, Conclusion 8 and the symmetry in the problem ensure that $\left.\frac{\partial \pi_1^*}{\partial p_L}\right|_{p_L=0} < 0$ and $\left.\frac{\partial \pi_2^*}{\partial p_L}\right|_{p_L=0} < 0$. Therefore, (111) implies that $\left.\frac{\partial W^*}{\partial p_L}\right|_{p_L=0} < 0$. Consequently, $\frac{\partial W^*}{\partial p_L} < 0$ for all p_L sufficiently close to 0 since $\frac{\partial W^*}{\partial p_L}$ is a continuous function of p_L. ■

Conclusions 7 and 10 imply that in order to identify conditions under which an increase in p_L generates losses for lenders, L entrepreneurs, and H entrepreneurs alike, settings in which p_L is bounded above 0 must be considered.

Observe from (112) that W_L^* declines as p_L increases if $B > 0$ and $\widetilde{B} > 0$. (113) implies that $B > 0$ when the inequality in (87) holds. Furthermore, it is readily verified that $\widetilde{B} > 0$ when:

$$V < \frac{\phi_H\, p_H\, q\left[5\,\phi_H\, p_H^2\, q^2\, (12\, I\, q + t) + 8\,\phi_L\, p_L\, (1-q)^2\, (4\, I\, [5\, p_H - 3\, p_L]\, q - p_L\, t)\right]}{4\left[32\,\phi_L^2\, p_L^4\, (1-q)^4 + 36\,\phi_H\, \phi_L\, p_H^2\, p_L^2\, (1-q)^2\, q^2 + 15\,\phi_H^2\, p_H^4\, q^4\right]}. \tag{114}$$

Table A1 illustrates the systematic losses that can arise as p_L increases in a setting where $V = 30$, $I = 18$, $p_H = 0.8$, $\phi_H = 0.2$, $\phi_L = 0.8$, $t = 4$, and $q = 0.6$. As p_L increases between 0.06 and 0.07 in this setting, the profit of each lender, the welfare of H entrepreneurs, and the welfare of L entrepreneurs all decline.[1]

p_L	β^*	x^*_{L1}	x^*_{L2}	x^*_{H1}	x^*_{H2}	$W^{*\dagger}_L$	W^*_H	π^*	W^*
0.03	0.0977	0.3662	0.3838	0.0232	0.5	4.9503	0.1699	0.1946	0.3651
0.04	0.0857	0.3647	0.3853	0.0664	0.5	6.7724	0.1582	0.1734	0.3323
0.05	0.0742	0.3639	0.3861	0.1080	0.5	7.9230	0.1441	0.1585	0.3034
0.06	0.0631	0.3636	0.3864	0.1478	0.5	8.2567	0.1279	0.1493	0.2781
0.07	0.0525	0.3640	0.3860	0.1859	0.5	7.7860	0.1102	0.1451	0.2561
0.08	0.0425	0.3648	0.3852	0.2221	0.5	6.6457	0.0911	0.1454	0.2371

Table A1. Effects of a Change in p_L in the Setting with Lender Competition.

† For expositional clarity, the entries in this column represent $W^*_L \times 10^4$.

References

Bose, Arup, Debashis Pal, and David Sappington, "All Productivity Increases are Not Created Equal," University of Florida mimeo, 2014.

[1] $p_L \geq 0.03$ in the setting of Table A1 to ensure $x^*_{H1} > 0$. $p_L \leq 0.08$ in this setting to ensure the H entrepreneur at $\frac{1}{2}$ anticipates non-negative profit from applying for funding from lender 1.