All Entrepreneurial Productivity Increases
are Not Created Equal

by

Arup Bose,* Debashis Pal,** and David E. M. Sappington***

Abstract

We examine the impact of productivity increases in a setting where wealth-constrained entrepreneurs are privately informed about whether their project will succeed with high ($p_H$) or low ($p_L$) probability. Not surprisingly, many productivity increases (e.g., an increase in $p_H$) generate gains for entrepreneurs and/or venture capitalists. However, some productivity increases (e.g., an increase in $p_L$) can generate widespread losses. Furthermore, entrepreneurs with low productivity projects can benefit more from policies that increase the productivity of high quality projects than from policies that increase their own productivity. Therefore, the design of policy to enhance welfare in the entrepreneurial sector can entail important subtleties.

Key Words: productivity; training; entrepreneurs.

JEL Codes: D60, D82, J24.

* Indian Statistical Institute, 203 B. T. Road, Kolkata 700108 India (bosearu@gmail.com).

** Department of Economics, University of Cincinnati, Cincinnati, Ohio 45221 USA (debashis.pal@uc.edu).

*** Department of Economics, University of Florida, PO Box 117140, Gainesville, FL 32611 USA (sapping@ufl.edu). Tel: 1-352-392-3904; Fax: 1-352-392-2111. (Corresponding Author)

We thank Phillip Koellinger and conference participants at Northwestern University and HEC (Paris) for helpful comments.
1 Introduction.

The entrepreneurial sector is viewed as a key source of potential economic growth in economies throughout the world. The Bright China Foundation\(^1\), Enterprise Asia\(^2\), and the Entrepreneurship Development Institute of India\(^3\) all promote the entrepreneurial sector in Asia. The Technical Education, Vocational and Entrepreneurship (TEVET) Authority of the Republic of Zambia\(^4\) is among the entities with a corresponding mission in Africa. These entities and programs pursue a variety of activities that include enhancing the skills of potential and actual entrepreneurs, training educators to better develop entrepreneurial skills, providing information about opportunities in the entrepreneurial sector, and facilitating contacts between established and aspiring entrepreneurs.

These activities might be broadly construed as attempts to increase productivity in the entrepreneurial sector. The purpose of this research is to examine how several types of productivity increases affect performance in a simple, stylized model of the entrepreneurial sector where entrepreneurs face severe wealth constraints, as is often the case in many developing countries. The productivity increases that we analyze can arise from many different sources, including reduced input prices, more efficient capital markets, and enhanced entrepreneurial skills generated by training programs.

It would be natural to presume that any productivity increase in the entrepreneurial sector would increase the welfare of some or all sector participants. Indeed, all of the productivity increases that we analyze generate gains for all sector participants in the presence of symmetric information about the potential returns from each entrepreneur’s project. However, different conclusions can emerge when entrepreneurs have limited wealth and are privately informed about the quality of their projects. In such settings, some productivity increases can reduce the expected welfare of the lender and both types of entrepreneurs in

\(^1\)http://www.bcf.org.cn/english/xmzs-ghgcycyxd.asp.
\(^2\)http://www.enterpriseasia.org/programme.
\(^3\)http://www.ediindia.org/Index.asp.
\(^4\)http://www.teveta.org.zm.
Some entrepreneurs in our model have access to high quality projects that are relatively likely to succeed. Other entrepreneurs only have access to low quality projects that are less likely to succeed. The difference in project success probabilities may reflect innate differences in project characteristics and/or differences in entrepreneurial skills. Entrepreneurs in our model have no wealth, and so must borrow the funds required to implement their projects from a financier or venture capitalist that we refer to as “the lender.” Although each entrepreneur knows the quality of his project, the lender only observes an imperfect signal about project quality. The lender funds projects that generate a favorable signal and declines to fund projects that generate an unfavorable signal.

This streamlined representation of the entrepreneurial sector admits a straightforward analysis of the effects of different types of productivity increases. We consider increases in the revenue generated by successful projects, which might reflect enhanced product quality or improved marketing channels, for example. We also analyze the effects of reduced costs of funding and operating projects, which could stem from more efficient capital markets or lower costs of raw materials. In addition, we consider increases in the success rates of entrepreneurial projects, which could reflect the impact of training programs, for instance. In practice, some training programs are designed to improve basic skills, and so tend to be of primary benefit to entrepreneurs with relatively limited skills. Other training programs focus on developing more advanced capabilities and so tend to be of particular benefit to more highly skilled entrepreneurs.\(^5\)

We find that different types of productivity increases can produce distinct outcomes. To

\(^5\)Abramovsky et al. (2011, pp. 154-5) describe the “Train to Gain” program in the UK which provides free training to individuals “who lack basic literacy, numeracy, or language skills.” The TEVET Authority of the Republic of Zambia offers a broad range of training programs that are designed to assist individuals with diverse backgrounds and skill levels. Some of the Authority’s programs target individuals with only nine years of education, whereas other programs are designed to enhance the performance of individuals who have earned craft certificates or even PhD’s (http://www.teveta.org.zm/index.php?option=com_content&view=article&id=110&Itemid=115). Heckman (1998), Krueger and Rouse (1998), Heckman et al. (1999), and Leuven and Oosterbeek (2004) provide descriptions and assessments of additional training programs.
illustrate this conclusion, first consider the effect of an advanced training program that serves primarily to increase the probability that high quality projects succeed. This productivity increase induces the lender to increase the payment she promises to approved entrepreneurs whose projects ultimately succeed. The higher payment attracts more entrepreneurs, particularly those with high quality projects, and thereby increases the lender’s profit. The higher payment also increases the expected welfare of both entrepreneurs with high quality projects and entrepreneurs with low quality projects.

A different conclusion can emerge when the productivity increase arises from a more basic training program that serves primarily to increase the probability that low quality projects succeed. This increased success probability renders entrepreneurs with low quality projects more anxious to have their projects funded. To deter excessive applications for funding, the lender will sometimes reduce the payment she promises to an entrepreneur whose project succeeds. The reduced payment reduces the welfare of entrepreneurs. The increased proportion of entrepreneurs with low quality projects that apply for funding also reduces the lender’s profit. Thus, productivity increases can generate widespread losses.

Our finding that productivity gains can generate systematic losses in the presence of asymmetric information is an illustration of the general theory of the second best (Lipsey and Lancaster, 1956-7). In this regard, our analysis is related to studies that show how an agent can enhance his well-being by reducing his capability. For example, Gelman and Salop (1983) demonstrate how a producer can profit by limiting its ability to expand output. Gupta et al. (1994, 1995) explain how a supplier can benefit from choosing inefficient locations or transportation costs. Fletcher and Slutsky (2010) show how a political candidate can benefit from a less favorable reputation.

In many of these studies, an agent benefits as his ability declines because his diminished capability induces a less aggressive response from a direct rival (e.g., a competing producer or an opposing political candidate). In our model, enhanced productivity can harm an en-

Joskow and Kahn’s (2002) observation that an electricity supplier can benefit by withholding electricity from the market can be viewed as a variation on this theme.
trepreneur because of the ensuing reaction of the supplier of a complementary input (the lender) rather than a rival. The aforementioned studies typically demonstrate how a reduction in one party’s productivity can enable him to gain at the expense of another party. Our analysis differs by demonstrating that productivity gains can reduce the expected welfare of all groups in the economy.

Ghatak et al. (2007) examine a related general equilibrium model in which borrowers decide whether to become entrepreneurs or work for a constant (endogenously determined) wage. The authors demonstrate that all workers can benefit from a small tax on entrepreneurship. The benefit arises because the tax induces workers with limited talent to leave the entrepreneurial sector, which increases the equilibrium payoff to entrepreneurs.

Our finding that productivity gains can generate systematic losses for entrepreneurs and lenders is also related to insights about the detrimental role of “lemons” in product markets (e.g., Akerlof, 1979). Lemons are variants of a product (e.g., pre-owned automobiles) with unobservable defects that are known to the seller but not to potential buyers. An increased presence of lemons can reduce the equilibrium price of the product and thereby reduce the profits of sellers. Buyers can also suffer if their loss from the restricted supply of high quality products outweighs their gain from a lower product price. Because productivity gains can increase the supply of low quality projects (lemons) in our model, it may not be entirely surprising that the gains can harm entrepreneurs and/or the lender. However, our analysis emphasizes the subtleties of the relevant considerations. We demonstrate that the same increase in productivity can produce welfare gains in some settings and welfare losses in other settings. We also show that even systematic increases in the productivity of both low

---

7 Anant et al. (1995) show that a producer may gain by adopting an inefficient production technology that induces the government to impose a less stringent ad valorem tax.
8 Bose et al. (2011) analyze a moral hazard setting in which a principal and agent can both gain as the agent’s production costs increase.
9 Ghatak et al. (2007) consider competition among lenders that drives lender profit to zero. Our focus on a monopoly lender allows us to analyze the impact of productivity changes on both lender profit and borrower welfare.
10 Similarly, de Meza and Webb (2000) demonstrate that a payment to workers who choose not to participate in the entrepreneurial sector can enhance aggregate welfare.
quality and high quality projects can sometimes impose losses on all groups in the economy. Similarly, systematic reductions in the transaction costs that entrepreneurs experience can reduce their welfare and the lender’s profit.

Our analysis proceeds as follows. Section 2 describes the key elements of our model. Section 3 analyzes a benchmark setting in which all parties share the same information *ex ante* about the quality of each entrepreneur’s project. Section 4 presents our main findings, emphasizing the conditions under which productivity increases generate systematic losses. Section 5 discusses extensions of our analysis, considers alternative assumptions (including different information structures, positive entrepreneurial wealth, and competition among lenders), and provides concluding observations.

2 Elements of the Model.

We consider a setting where entrepreneurs have no wealth and so rely on a lender (e.g., a venture capitalist) to finance their projects.\(^\text{11}\) The sole lender has imperfect information about the likely payoff from the project of any particular entrepreneur.\(^\text{12}\) The lender employs the available screening technology to determine whether to finance the projects of entrepreneurs who apply for funding.

For simplicity, we assume that each entrepreneur has either a high quality project or a low quality project. A high quality project has a relatively high probability of success \((p_H \in (0, 1))\). A low quality project has a relatively low probability of success \((p_L \in (0, p_H))\). A project generates payoff \(V > 0\) when it succeeds and 0 when it fails. Each project requires a fixed investment of \(I > 0\). A high quality project generates positive expected net surplus, i.e., \(p_H V > I\). A low quality project generates negative expected net surplus, i.e.,

\(^{11}\) The assumption that entrepreneurs have no wealth may be a reasonable caricature of reality in developing countries with very low per capita income. More generally, entrepreneurs often have little wealth relative to the large investment required to operationalize their projects at an economic scale. The concluding section discusses some of the additional considerations that arise when entrepreneurs have some wealth at the start of their interaction with the lender.

\(^{12}\) The concluding section demonstrates that our key qualitative conclusions extend to settings with multiple lenders.
Each risk neutral entrepreneur knows the quality of his project. The risk neutral lender does not share this knowledge. Initially, the lender knows only that the fraction $\phi_H \in (0, 1)$ of entrepreneurs have high quality projects and the complementary fraction $\phi_L = 1 - \phi_H$ have low quality projects. The lender eventually observes an informative signal $(s \in \{p_L, p_H\})$ about the quality (i.e., the success probability) of the project of each entrepreneur that applies for funding. The signal reveals the true project quality with probability $q \in (\frac{1}{2}, 1)$ and reports the incorrect project quality with probability $1 - q$. $q$ is sufficiently large that the lender optimally funds projects for which she observes the favorable signal and declines to fund projects for which she observes the unfavorable signal.

Each entrepreneur experiences a transaction cost when applying for funding. This cost could reflect the (opportunity cost of the) time required to develop a compelling project description and associated business plan, for example. Variation in total transaction costs among entrepreneurs is captured in standard Hotelling fashion. Entrepreneurs with low quality projects (“L entrepreneurs”) and entrepreneurs with high quality projects (“H entrepreneurs”) are both distributed uniformly on the line segment $[0, 1]$. The lender is located at 0. The total number of entrepreneurs is normalized to 1. An L entrepreneur located at point $x$ incurs transaction cost $t_L x$ in applying for funding. The corresponding cost of the H entrepreneur is $t_H x$. Each entrepreneur’s location (and thus the total transaction cost he incurs when he applies for funding) is known only to the entrepreneur. We will denote by $x_i \in \{x_L, x_H\}$ the location of the $i \in \{L, H\}$ entrepreneur farthest from the lender that

---

13 For expositional ease, the ensuing discussion will often refer to “expected net surplus” as “net surplus” and to “expected surplus” as “surplus.”

14 $q$ is assumed to exceed $\frac{1}{2}$ to ensure the signal is truly informative about project quality, and not simply pure noise. $q$ is assumed to be less than 1 to avoid unrealistic settings in which the lender can ascertain perfectly the quality of each lender’s project.

15 We abstract from any cost of producing this informative signal, and take the screening accuracy ($q$) to be exogenous. As explained in more detail in the concluding section, our primary findings persist in settings where $q$ is endogenous and the lender must incur higher costs in order to discern project qualities with greater accuracy.

16 The transaction cost that an entrepreneur faces could also represent the value of his outside opportunity, as in Ghatak et al. (2007) and Inci (2013), for example.
applies for funding.

The timing in the model is as follows. First, each entrepreneur privately learns his location and the quality of his project. Second, the lender announces the sharing rate for approved projects.\textsuperscript{17} Third, entrepreneurs decide whether to seek funding for their project from the lender. Fourth, the lender assesses the project quality of each entrepreneur that applies for funding. The lender funds the projects that produce a favorable signal and declines to fund projects that produce an unfavorable signal.\textsuperscript{18} Finally, the outcome of each funded project is observed publicly. When an entrepreneur’s project succeeds, he receives $\beta V > 0$ and the lender receives $[1 - \beta] V > 0$. An entrepreneur whose approved project fails receives a payoff of 0.\textsuperscript{19}

An entrepreneur will apply for funding if and only if his expected return from doing so exceeds his transaction costs. Thus, an $L$ entrepreneur located at $x$ will apply for funding if and only if $\beta p_L V [1 - q] \geq t_L x$. An $H$ entrepreneur located at $x$ will apply for funding if and only if $\beta p_H V q \geq t_H x$.

We assume that the unit transaction cost for $H$ entrepreneurs ($t_H$) is not too much higher than the corresponding cost for $L$ entrepreneurs ($t_L$) relative to the incremental surplus generated by a high quality project and the proportion of high quality projects in the population. This assumption avoids the uninteresting outcome in which the lender sets $\beta = 0$ and thereby avoids funding any projects. Formally, Assumption 1 is assumed to hold throughout the ensuing analysis:

\textsuperscript{17}We assume that the lender can make a binding commitment to deliver the promised sharing rate. In doing so, we abstract from the possibility that the lender might try to expropriate entrepreneurs by reducing $\beta$ after they apply for funding. Reputation concerns can promote such commitment, for example.

\textsuperscript{18}Entrepreneurs whose request for funding is denied, like entrepreneurs who do not seek funding, earn 0 (in the wage sector of the economy, for example).

\textsuperscript{19}In principle, the lender could offer two distinct payment pairs, $(S_H, F_H)$ and $(S_L, F_L)$, where $S_i$ is the payment the lender delivers to the entrepreneur when his project succeeds after he reports his project to have success probability $p_i$, and where $F_i$ is the corresponding payment when the project fails. The lender could design these payment pairs so that $i$ entrepreneurs select the $(S_i, F_i)$ payment pair (for $i \in \{L, H\}$). Bose et al. (2014) show that the lender cannot secure a strictly higher level of expected profit by introducing two distinct contracts of this form in the setting under consideration.
Assumption 1. $^{20} \phi_H p_H q^2 t_L [p_H V - I] + \phi_L p_L t_H [1 - q]^2 [p_L V - I] > 0.$ \hspace{1cm} (1)

The lender’s (expected) profit in this setting when she sets sharing rate $\beta$ is:

$$\pi(\beta) = \phi_L x_L [1 - q] [p_L V (1 - \beta) - I] + \phi_H x_H q [p_H V (1 - \beta) - I].$$ \hspace{1cm} (2)

The first term to the right of the equality in (2) reflects the profit the lender anticipates from $L$ entrepreneurs. This profit is the product of the number of $L$ entrepreneurs that apply for funding ($\phi_L x_L$), the probability that a low quality project is funded ($1 - q$), and the difference between the lender’s expected payoff from a low quality project ($p_L V [1 - \beta]$) and the cost of financing the project ($I$). The last term in (2) reflects the corresponding profit the lender anticipates from $H$ entrepreneurs.

The ensuing analysis examines the effects of productivity increases on the lender’s profit and on the welfare of entrepreneurs. The (expected) welfare of $L$ entrepreneurs, given sharing rate $\beta$, is:

$$W_L(\beta) = \phi_L x_L [1 - q] p_L V \beta - \phi_L t_L \int_0^{x_L} x \, dx.$$ \hspace{1cm} (3)

The first term to the right of the equality in (3) is the product of the number of entrepreneurs whose projects are funded ($\phi_L x_L [1 - q]$) and an $L$ entrepreneur’s expected payoff from a funded project ($p_L V \beta$). The second term to the right of the equality in (3) reflects the aggregate transaction costs of $L$ entrepreneurs that apply for funding. The welfare of an entrepreneur who does not apply for funding is normalized to 0.

The corresponding welfare of $H$ entrepreneurs, given sharing rate $\beta$, is:

$$W_H(\beta) = \phi_H x_H q p_H V \beta - \phi_H t_H \int_0^{x_H} x \, dx.$$ \hspace{1cm} (4)

Section 4 analyzes the impacts of different types of productivity increases in this setting.

First, though, section 3 considers the impacts in a benchmark setting.

$^{20}$ As demonstrated in the proof of Lemma 3, Assumption 1 ensures that the lender’s profit is strictly increasing in $\beta$ at $\beta = 0$ for all $q \in (\frac{1}{2}, 1)$. Assumption 1 does not preclude the possibility that $H$ entrepreneurs have the same or lower unit transactions costs than $L$ entrepreneurs.

$^{21}$ As Lemma 2 (below) indicates, $x_L$, like $x_H$, is an endogenous variable that varies with $\beta$. 

8
3 Setting with Symmetric Project Quality Information.

In this section, we briefly consider the effects of productivity increases in a benchmark setting where each entrepreneur and the lender initially share the same imperfect knowledge of the quality of the entrepreneur’s project.\(^{22}\) It is common knowledge that an entrepreneur has a high quality project with probability \(\phi_H\) and a low quality project with probability \(\phi_L\) in this setting with symmetric project quality information. Each entrepreneur, who is privately informed about his location, decides whether to apply for funding and pursue his project before learning the quality of his project. All parties also know that an informative signal about the quality of each entrepreneur’s project will be produced if the entrepreneur applies for funding. A high quality project generates a favorable signal (and so is funded) with probability \(q \in (\frac{1}{2}, 1)\) and an unfavorable signal (and so is not funded) with probability \(1 - q\). A low quality project generates a favorable signal (and so is funded) with probability \(1 - q\) and an unfavorable signal (and so is not funded) with probability \(q\). Because the entrepreneurs are identical \textit{ex ante} (except for their locations) in this setting, they are all assumed to face the same expected unit transaction cost, \(t^e\).\(^{23}\)

Lemma 1 explains how the various productivity increases under consideration affect the profit maximizing sharing rate.\(^{24}\)

\textbf{Lemma 1.} The lender increases the sharing rate (\(\beta\)) in the setting with symmetric project quality information as \(V\), \(p_L\), \(p_H\), or \(\phi_H\) increases, and as \(I\) decreases.

The conclusions reported in Lemma 1 reflect standard considerations. The expected net

\(^{22}\)If the lender shared each entrepreneur’s perfect knowledge of his project quality, she would only allow entrepreneurs with high quality projects to apply for funding. If, in addition, the lender could observe each entrepreneur’s location and could offer location-specific sharing rates, she would finance the high quality projects of all entrepreneurs located in \([0, x^*_H]\), where \(x^*_H = \frac{1}{t^H}[p_H V - I]\). Observe that \(x^*_H\) is the location of the entrepreneur whose high quality project generates an expected payoff \((p_H V)\) equal to the sum of the associated investment cost \((I)\) and transaction cost \((t^H x^*_H)\).

\(^{23}\)We also assume that \(\phi_H q [p_H V - I] + \phi_L (1 - q) [p_L V - I] > 0\) in the setting with symmetric project quality information to ensure the lender optimally sets \(\beta > 0\).

\(^{24}\)The proof of Lemma 1 and the proofs of all other formal conclusions are presented in the Appendix. Bose et al. (2014) provides more detailed proofs of selected conclusions.
surplus from each entrepreneur’s project increases as $V$, $p_L$, $p_H$, and/or $\phi_H$ increase and as $I$ decreases. Consequently, the lender increases the sharing rate in order to induce more entrepreneurs to apply for funding.\textsuperscript{25}

The lender’s optimal response to productivity increases in this setting produces gains for the lender and both groups of entrepreneurs, as Observation 1 reports.

**Observation 1.** The lender’s expected profit and the entrepreneurs’ expected welfare both increase in the setting with symmetric project quality information: (i) as $V$, $p_L$, $p_H$, or $\phi_H$ increases; and (ii) as $I$ or $t^e$ declines.

The lender benefits from the increased potential profit generated by the various productivity increases considered in Observation 1. The entrepreneurs benefit from the increased sharing rate and, when relevant, their reduced transaction costs or the increased expected payoff from their projects.

The conclusions reported in Observation 1 are not surprising. It is natural to expect productivity increases to generate gains that are shared by the participants in the entrepreneurial sector of an economy. What may be more surprising is that productivity gains can impose losses on the lender and on both $L$ entrepreneurs and $H$ entrepreneurs in arguably more realistic settings where entrepreneurs initially have private information about the quality of their projects.

4 Main Findings.

We now return to the setting of primary interest in which each entrepreneur is privately informed about both his location and the quality of his project. We begin by identifying the

\textsuperscript{25}As the proof of Lemma 1 demonstrates, the profit maximizing sharing rate does not vary with $t^e$ in the setting with symmetric project quality information. As $t^e$ declines, a given increase in $\beta$ induces more entrepreneurs to apply for funding, which encourages the lender to increase $\beta$. However, a reduction in $t^e$ renders entrepreneurs more willing to apply for funding even when $\beta$ is low, which leads the lender to reduce $\beta$. These two effects are offsetting in the setting with symmetric project quality information. Although the value of $t^e$ does not affect the lender’s preferred sharing rate ($\beta$), it does affect the lender’s profit, as Observation 1 (below) reports.
locations of the $L$ entrepreneur ($x_L$) and the $H$ entrepreneur ($x_H$) farthest from the lender that apply for funding, given sharing rate $\beta$.

**Lemma 2.** $x_L = \beta p_L V \left[ \frac{1-q}{t_L} \right]$ and $x_H = \beta p_H V \left[ \frac{q}{t_H} \right]$.

Lemma 2 reflects the obvious conclusion that more entrepreneurs will apply for funding as the expected payoff from their project ($p_i V$) increases, as the sharing rate ($\beta$) increases, as the probability that their project will be funded increases, and as their transaction costs ($t_i$) decline.

Lemma 2 helps to characterize the equilibrium (profit maximizing) sharing rate ($\beta^*$), the lender’s equilibrium profit ($\pi^* = \pi(\beta^*)$), the equilibrium welfare of $L$ entrepreneurs ($W_L^* = W_L(\beta^*)$), the equilibrium welfare of $H$ entrepreneurs ($W_H^* = W_H(\beta^*)$), and equilibrium total welfare ($W^* = \pi^* + W_L^* + W_H^*$). These measures are specified in Lemmas 3–5.

**Lemma 3.** The equilibrium sharing rate is:

$$\beta^* = \frac{\phi_L p_L [1-q]^2 [p_L V - I] t_H + \phi_H p_H q^2 [p_H V - I] t_L}{2 V \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]}.$$  

(5)

**Lemma 4.** The lender’s equilibrium profit is:

$$\pi^* = \frac{\left( \phi_L p_L [1-q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] \right)^2}{4 t_L t_H \left[ \phi_L p_L^2 t_H (1-q)^2 + \phi_H p_H^2 t_L q^2 \right]}.$$  

(6)

**Lemma 5.** The equilibrium welfare of $L$ entrepreneurs, the equilibrium welfare of $H$ entrepreneurs, and equilibrium total welfare are, respectively:

$$W_L^* = \frac{\phi_L p_L^2 [1-q]^2 \left\{ \phi_L p_L [1-q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] \right\}^2}{8 t_L \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2};$$  

(7)

$$W_H^* = \frac{\phi_H p_H^2 q^2 \left\{ \phi_L p_L [1-q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] \right\}^2}{8 t_H \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2};$$  

and

(8)
\[ W^* = \frac{3 \{ \phi_L p_L [1 - q]^2 t_H (p_L V - I) + \phi_H p_H q^2 t_L (p_H V - I) \}^2}{8 t_L t_H \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]} . \] (9)

Now consider how the lender adjusts the equilibrium sharing rate in response to various productivity increases.

**Lemma 6.** The equilibrium sharing rate \((\beta^*)\) increases: (i) as \(V, p_H, \phi_H, \) or \(t_L\) increases; and (ii) as \(I\) or \(t_H\) decreases.

The conclusions in Lemma 6 reflect the following five considerations. First, as \(V\) increases and/or \(I\) declines, each project generates more surplus. Consequently, the lender increases the sharing rate \((\beta)\) in order to induce more entrepreneurs to apply for funding. Second, as \(p_H\) increases, a given increase in \(\beta\) attracts more \(H\) entrepreneurs.\(^{26}\) Therefore, the lender increases \(\beta\) as \(p_H\) increases in order to attract more high quality projects, which offer greater potential profit as \(p_H\) increases.

Third, as the proportion of \(H\) entrepreneurs in the population \((\phi_H)\) increases (or as the proportion of \(L\) entrepreneurs in the population \((\phi_L)\) declines), a given increase in \(\beta\) attracts a higher proportion of high quality projects. Consequently, the lender’s profit increases more rapidly as \(\beta\) increases, which leads the lender to set a higher sharing rate. Fourth, when \(L\) entrepreneurs incur higher transaction costs (i.e., as \(t_L\) increases), they find it more burdensome to apply for funding. Consequently, the lender can increase \(\beta\) to attract more high quality projects without fear of attracting too many low quality projects. Fifth, when \(H\) entrepreneurs incur smaller transaction costs (so \(t_H\) declines), an increase in \(\beta\) becomes more effective at inducing them to apply for funding. Consequently, the lender increases \(\beta\) to attract the profitable high quality projects.\(^{27}\)

\(^{26}\)Formally, from Lemma 2, \(\frac{\partial^2 x_H}{\partial p_H^2} = \frac{V q}{t_H} > 0.\)

\(^{27}\)Lemma 6 identifies the impact of an isolated change in \(t_L\) or an isolated change in \(t_H\). In practice, a change in basic skills or bureaucratic procedures, for example, can affect the transaction costs that all entrepreneurs experience. Formally, \(t_L\) and \(t_H\) might both change simultaneously. The concluding section considers this possibility. It also considers the possibility that \(p_L\) and \(p_H\) might both change simultaneously.
The impact of an increase in $p_L$ on the profit-maximizing sharing rate is less straightforward, as Lemma 7 suggests.

**Lemma 7.** As the success probability of a low quality project ($p_L$) increases, the equilibrium sharing rate ($\beta^*$) declines if $p_L \leq \tilde{p}_L$ whereas $\beta^*$ increases if $p_L > \tilde{p}_L$, where $\frac{\tilde{p}_L}{p_H} = \delta_1 = \sqrt{\frac{\delta_0 + 1}{\delta_0}}$ and $\delta_0 = \frac{\phi_L[1-q]^2 t_H}{\phi_H q^2 t_L}$.

An increase in $p_L$ has two countervailing effects. First, it renders the decision of $L$ entrepreneurs to seek funding more sensitive to the prevailing sharing rate, $\beta$.

$^{28}$This effect leads the lender to reduce $\beta$ in order to discourage $L$ entrepreneurs from applying for funding. Second, the lender incurs a smaller loss on each low quality project she finances. This effect leads the lender to increase $\beta$ (in order to attract more $H$ entrepreneurs). The second effect is of greater importance when $p_L$ is relatively large, and so the lender finances relatively many low quality projects.$^{29}$

The conclusions in Lemmas 6 – 7 help to explain the effects of productivity increases on equilibrium profit and welfare. Proposition 1 identifies productivity increases that generate gains for the lender and for both groups of entrepreneurs.

**Proposition 1.** The lender’s profit ($\pi^*$) and the welfare of both $L$ and $H$ entrepreneurs ($W_L^*$ and $W_H^*$) increase: (i) as $V$ or $p_H$ increases; and (ii) as $I$ or $t_H$ declines.

The widespread gains identified in Proposition 1 reflect the following four considerations. First, an increase in $V$ increases the lender’s profit by increasing the expected payoff from each entrepreneur’s project. The increased profit induces the lender to increase $\beta$ in order to attract more entrepreneurs, which increases their welfare. Second, a reduction in $I$ increases the lender’s expected profit by reducing her investment costs. In response to the increased profitability of each project, the lender increases $\beta$ in order to attract more entrepreneurs,$^{29}$

$^{28}$Formally, from Lemma 2, $\frac{\partial^2 \pi_L}{\partial p_L^2} = \frac{V'[1-q]}{t_L} > 0$.

$^{29}$Notice from Lemma 2 that $\frac{x_L}{x_H} = \frac{p_L[1-q] t_H}{p_H q t_L}$, which increases as $p_L$ increases.
which increases their welfare.

Third, as \( t_H \) declines, a given increase in \( \beta \) induces more \( H \) entrepreneurs to apply for funding.\(^{30}\) Consequently, the lender increases \( \beta \) in order to attract more high quality projects. The lender’s profit increases as she finances a higher proportion of profitable projects, and all entrepreneurs benefit from the higher sharing rate. Fourth, an increase in \( p_H \) increases the profit the lender secures from each high quality project she finances. An increase in \( p_H \) also increases the sensitivity of the decision of \( H \) entrepreneurs to apply for funding to the prevailing sharing rate.\(^{31}\) Consequently, the lender increases \( \beta \) in order to attract more \( H \) entrepreneurs, which increases the welfare of all entrepreneurs.

Proposition 1 demonstrates that several forms of productivity gains serve to increase the welfare of all groups in the economy. Proposition 2 considers the impact of one additional form of productivity gain – an increase in the proportion of high quality projects in the population (\( \phi_H \)).\(^{32}\) The proposition refers to the following assumption.

**Assumption 2.** \( \frac{[1-\phi_H][2-\phi_H]}{(\phi_H)^2} > \frac{p_H^2 q^2 t_L}{p_L^2 [1-q]^2 t_H} \).

**Proposition 2.** The lender’s profit (\( \pi^* \)) and the welfare of \( H \) entrepreneurs (\( W_H^* \)) increase as the proportion of \( H \) entrepreneurs (\( \phi_H \)) increases. The welfare of \( L \) entrepreneurs (\( W_L^* \)) also increases if Assumptions 1 and 2 hold.

An increase in the proportion of high quality projects increases the lender’s profit directly. It also induces the lender to increase \( \beta \) in order to attract more \( H \) entrepreneurs. The increase in \( \beta \) increases the welfare of \( H \) entrepreneurs. It can also increase the aggregate welfare of (the now smaller group of) \( L \) entrepreneurs under many plausible settings, including those in which Assumption 2 holds. Assumption 2 will be satisfied, for example, when the transaction

\(^{30}\)Formally, from Lemma 2, \( -\frac{\partial^2 x_H}{\partial \phi_H \partial \beta} = \frac{p_H V q}{(t_H)^2} > 0 \).

\(^{31}\)Formally, from Lemma 2, \( \frac{\partial^2 x_H}{\partial p_H \partial \beta} = \frac{q V}{t_H} > 0 \).

\(^{32}\)An increase in \( \phi_H \) might arise from a training program that transforms some low-skilled entrepreneurs into high-skilled entrepreneurs, for example.
costs of $L$ entrepreneurs are relatively small.\textsuperscript{33} When $t_L$ is relatively small, the increase in the sharing rate induces many $L$ entrepreneurs to apply for funding. This source of increased participation for $L$ entrepreneurs can outweigh the reduced participation that arises from the decline in the proportion of $L$ entrepreneurs in the population.\textsuperscript{34}

The discussion to this point has focused on productivity increases that generate gains for the lender and for both groups of entrepreneurs. In contrast, Proposition 3 identifies a productivity increase that can impose losses on the lender and on both groups of entrepreneurs. The proposition refers to the following assumption.

Assumption 3. $\frac{\phi_L}{\phi_H} > \frac{p_H^2 q^2 t_L}{p_L^2 [1-q]^2 t_H}$.

**Proposition 3.** The lender’s profit ($\pi^*$), the welfare of $H$ entrepreneurs ($W^*_H$), and total welfare ($W^*$) all decline as the transaction costs of $L$ entrepreneurs ($t_L$) decline. The welfare of $L$ entrepreneurs ($W^*_L$) also declines as $t_L$ declines if Assumptions 1 and 3 hold.\textsuperscript{35}

When the transaction costs that $L$ entrepreneurs experience decline, the lender reduces $\beta$ to avoid attracting too many low quality projects. This reduction in $\beta$ reduces the welfare of the $H$ entrepreneurs. In contrast, the reduction in their transaction costs can increase the aggregate welfare of $L$ entrepreneurs as a group. However, their welfare can decline in plausible settings, including those in which $L$ entrepreneurs experience substantially smaller transaction costs than $H$ entrepreneurs. When $t_L$ is small, an $L$ entrepreneur’s decision about whether to seek financing is relatively sensitive to changes in $\beta$. Consequently, the reduction in $\beta$ induces many $L$ entrepreneurs not to apply for funding, which reduces their

\textsuperscript{33}A small value of $t_L/t_H$ could arise, for example, when $L$ entrepreneurs have a relatively small opportunity cost of applying for funding.

\textsuperscript{34} $W^*_L$ can increase as $\phi_H$ increases even when $t_L/t_H$ is not small. (Equation (32) in the Appendix provides a less stringent sufficient condition for $\frac{\partial W^*_L}{\partial \phi_H} > 0$.) For example, it is readily verified that $W^*_L$ increase as $\phi_H$ increases between 0.40 and 0.45 when $V = 40$, $I = 20$, $p_H = 0.6$, $p_L = 0.25$, $q = 0.7$, and $t_L = t_H = 4$.

\textsuperscript{35}Observe that when Assumptions 1 and 3 both hold, the expected increase in surplus from implementing the project of an $H$ entrepreneur ($p_H V - I$) must exceed the expected reduction in surplus from implementing the project of an $L$ entrepreneur ($\mid p_L V - I \mid$).
Proposition 3 provides an example of a particular form of productivity increase that can reduce the lender’s profit and the aggregate welfare of both \( L \) entrepreneurs and \( H \) entrepreneurs. Proposition 4 provides an additional example of a productivity increase that can produce systematic gains or losses, depending upon the environment in which the productivity increase arises.

**Proposition 4.** The lender’s profit \((\pi^*)\), the welfare of \( H \) entrepreneurs \((W_H^*)\), and aggregate welfare \((W^*)\) all decline as the success probability of a low quality project \((p_L)\) increases when \( p_L \) is sufficiently small. The welfare of \( L \) entrepreneurs \((W_L^*)\) also can decline, and so an increase in \( p_L \) can generate losses for all groups in the economy. In contrast, \( \pi^* \), \( W_H^* \), and \( W_L^* \) all increase as \( p_L \) increases when \( p_L \) and \( q \) are sufficiently large.

When \( p_L \) is small, relatively few \( L \) entrepreneurs apply for funding. (Recall Lemma 2.) Because she does not fund the projects of many \( L \) entrepreneurs, the lender does not secure much of an increase in profit as the success probability of low quality projects increases. Furthermore, the increase in \( p_L \) renders the number of \( L \) entrepreneurs who apply for funding more sensitive to \( \beta \). To discourage the \( L \) entrepreneurs from applying for funding, the lender reduces the sharing rate, \( \beta \). (Recall Lemma 7.) The reduction in \( \beta^* \) reduces the welfare of the \( H \) entrepreneurs \((W_H^*)\). The increased relative participation of \( L \) entrepreneurs reduces the lender’s profit \((\pi^*)\). The reductions in \( W_H^* \) and \( \pi^* \) cause aggregate welfare \((W^*)\) to decline, even when the welfare of \( L \) entrepreneurs \((W_L^*)\) increases. The reduction in \( \beta^* \) induced by an increase in \( p_L \) can be so pronounced as to diminish the aggregate welfare of \( L \) entrepreneurs as a group.

---

\(^{36}\) \( W_L^* \) can decline as \( t_L \) declines even when \( t_L/t_H \) is not small. (As the proof of Proposition 3 reveals, \( \frac{\partial W_L^*}{\partial p_L} > 0 \) when the function \( g(\cdot) \) defined in equation (33) in the Appendix is positive.) For example, it is readily verified that \( W_L^* \) decreases as \( t_L \) declines between 6.0 and 4.0 when \( V = 40 \), \( I = 20 \), \( p_H = 0.6 \), \( p_L = 0.25 \), \( q = 0.7 \), \( \phi_L = .7 \), and \( t_H = 4 \).

\(^{37}\) As the proof of Proposition 4 reveals, \( \frac{\partial W^*}{\partial p_L} < 0 \) if \( \left[ A p_L^2 + p_H \right]^2 V < \left[ p_H (1 + 2 A p_L) - A p_L^2 \right] I \), where \( A = \frac{\phi_L \left[ 1 - q^2 t_H \right]}{\phi_H p_H q^2 t_L} \).
This outcome is illustrated in Table 1 in the setting where a successful project generates $V = 40$, each project requires investment $I = 20$, sixty percent of entrepreneurs have low quality projects (so $\phi_L = 0.6$ and $\phi_H = 0.4$), $L$ and $H$ entrepreneurs experience the same transaction costs (so $t_L = t_H = 4$), high quality projects succeed with probability $p_H = 0.6$, and the screening accuracy is $q = 0.55$. The table presents the changes in the equilibrium sharing rate $(\beta^*)$, the lender’s profit $(\pi^*)$, the welfare of entrepreneurs ($W_L^*$ and $W_H^*$), and total welfare ($W^*$) that arise as $p_L$ increases from 0.07 to 0.11. Here and more generally, an increase in $p_L$ is most likely to generate systematic losses for the lender and both groups of entrepreneurs when $p_L$ is moderately small. Table 1 reveals that these losses can be substantial. To illustrate, as $p_L$ increases by 10%, from 0.10 to 0.11, aggregate welfare declines by more than 27%. This decline reflects a reduction of (approximately) 27% in the lender’s profit, 28% in the welfare of $H$ entrepreneurs, and 13% in the welfare of $L$ entrepreneurs.

<table>
<thead>
<tr>
<th>$p_L$</th>
<th>$\beta^*$</th>
<th>$\pi^*$</th>
<th>$W_L^*$</th>
<th>$W_H^*$</th>
<th>$W^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07</td>
<td>40.7974</td>
<td>29.3974</td>
<td>0.1982</td>
<td>14.5005</td>
<td>44.0961</td>
</tr>
<tr>
<td>0.08</td>
<td>35.8341</td>
<td>22.7733</td>
<td>0.1997</td>
<td>11.187</td>
<td>34.1600</td>
</tr>
<tr>
<td>0.09</td>
<td>31.1674</td>
<td>17.3082</td>
<td>0.1912</td>
<td>8.4629</td>
<td>25.9623</td>
</tr>
<tr>
<td>0.10</td>
<td>26.8007</td>
<td>12.8643</td>
<td>0.1745</td>
<td>6.2576</td>
<td>19.2965</td>
</tr>
<tr>
<td>0.11</td>
<td>22.7364</td>
<td>9.3113</td>
<td>0.1512</td>
<td>4.5036</td>
<td>13.9669</td>
</tr>
</tbody>
</table>

**Table 1. Systematic Losses from an Increase in $p_L$.**

In contrast, when $p_L$ is large, relatively many $L$ entrepreneurs apply for and receive funding if the lender’s screening accuracy ($q$) is sufficiently limited. In this case, an increase

---

38This is approximately the range of $p_L$ realizations in which the lender secures more profit by financing only projects that generate a positive signal than by financing the projects of all entrepreneurs that apply for funding. For expositional ease, the entries in all columns but the first in Table 1 represent 1,000 times the actual relevant value.

39When $p_L$ is close to 0, the lender funds the projects of very few $L$ entrepreneurs and so does not reduce $\beta^*$ much, if at all, as $p_L$ increases. Consequently, an increase in $p_L$ serves primarily to increase the welfare of $L$ entrepreneurs as more of them apply for funding. Formally, as shown in equation (22) in the proof of Lemma 5, $W_L^* = \frac{\phi_L |1-q|^2(\beta p_L V)}{2t_L}$. Consequently, $\frac{dW_L^*}{dp_L} = \beta^* + p_L \frac{d\beta^*}{dp_L} > 0$ for $p_L$ sufficiently close to 0.
in \( p_L \) reduces the loss the lender incurs on each low quality project she funds. Consequently, the lender becomes less focused on avoiding low quality projects, and so increases \( \beta^* \) in order to attract more high quality projects. The increase in \( \beta^* \) increases \( W_L^* \) and \( W_H^* \). Therefore, gains for all groups in the economy can arise as \( p_L \) increases in this setting with asymmetric information, just as they arise in the benchmark setting with symmetric project quality information.

When the lender’s ability to identify project quality (\( q \)) is sufficiently pronounced, the lender can simply decline to finance what he considers to be low quality projects rather than discourage \( L \) entrepreneurs from applying for funding by reducing \( \beta \). Consequently, the lender does not reduce \( \beta \) much, if at all, as \( p_L \) increases when \( q \) is relatively large. Therefore, \( L \) entrepreneurs benefit from an increase in \( p_L \) when the lender has sufficient ability to distinguish between low quality projects and high quality projects. As Corollary 1 indicates, this ability need not be particularly pronounced.

**Corollary 1.** The welfare of \( L \) entrepreneurs (\( W_L^* \)) increases as the success probability of a low quality project (\( p_L \)) increases if \( q \geq \frac{\sqrt{3}}{1+\sqrt{2}} \approx 0.586 \).

One further implication of Propositions 1 and 4 merits brief mention.

**Corollary 2.** An increase in \( p_H \) can increase the welfare of \( L \) entrepreneurs by more than a commensurate increase in \( p_L \).

Corollary 2 implies that \( L \) entrepreneurs as a group can benefit more from a training program (or technological change) that enhances the performance of \( H \) entrepreneurs than from a program that improves their own performance. This more pronounced gain for \( L \) entrepreneurs arises from the larger increase in the sharing rate generated by the increase in \( p_H \). Table 2 illustrates the magnitudes of the relevant effects in the setting where \( V = 50 \), \( I = 20 \), \( \phi_H = \phi_L = 0.5 \), \( q = 0.65 \), and \( t_H = t_L = 2.5 \).

Table 2A reveals that when \( p_H = 0.50 \), the welfare of \( L \) entrepreneurs (\( W_L^* \)) increases by 65\%, from .0040 to .0066, as \( p_L \) increases by five percentage points (or by 33\%), from .15 to
.20. Table 2B demonstrates that when $p_L = 0.15$, $W^*_L$ increases by more than 130%, from .0040 to .0093, as $p_H$ increases by five percentage points (or by 10%), from .50 to .55.

<table>
<thead>
<tr>
<th>$p_L$</th>
<th>$p_H$</th>
<th>$\beta^*$</th>
<th>$\pi^*$</th>
<th>$W^*_L$</th>
<th>$W^*_H$</th>
<th>$W^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.5</td>
<td>0.0763</td>
<td>0.3152</td>
<td>0.0040</td>
<td>0.1536</td>
<td>0.4728</td>
</tr>
<tr>
<td>0.16</td>
<td>0.5</td>
<td>0.0755</td>
<td>0.3099</td>
<td>0.0045</td>
<td>0.1505</td>
<td>0.4649</td>
</tr>
<tr>
<td>0.17</td>
<td>0.5</td>
<td>0.0748</td>
<td>0.3055</td>
<td>0.0050</td>
<td>0.1478</td>
<td>0.4583</td>
</tr>
<tr>
<td>0.18</td>
<td>0.5</td>
<td>0.0742</td>
<td>0.3021</td>
<td>0.0055</td>
<td>0.1456</td>
<td>0.4531</td>
</tr>
<tr>
<td>0.19</td>
<td>0.5</td>
<td>0.0738</td>
<td>0.2995</td>
<td>0.0060</td>
<td>0.1437</td>
<td>0.4492</td>
</tr>
<tr>
<td>0.20</td>
<td>0.5</td>
<td>0.0734</td>
<td>0.2977</td>
<td>0.0066</td>
<td>0.1423</td>
<td>0.4466</td>
</tr>
</tbody>
</table>

Table 2A. The Effects of an Increase in $p_L$.

<table>
<thead>
<tr>
<th>$p_L$</th>
<th>$p_H$</th>
<th>$\beta^*$</th>
<th>$\pi^*$</th>
<th>$W^*_L$</th>
<th>$W^*_H$</th>
<th>$W^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.50</td>
<td>0.0763</td>
<td>0.3152</td>
<td>0.0040</td>
<td>0.1536</td>
<td>0.4728</td>
</tr>
<tr>
<td>0.15</td>
<td>0.51</td>
<td>0.0848</td>
<td>0.4052</td>
<td>0.0050</td>
<td>0.1976</td>
<td>0.6078</td>
</tr>
<tr>
<td>0.15</td>
<td>0.52</td>
<td>0.0930</td>
<td>0.5063</td>
<td>0.0060</td>
<td>0.2472</td>
<td>0.7595</td>
</tr>
<tr>
<td>0.15</td>
<td>0.53</td>
<td>0.1009</td>
<td>0.6187</td>
<td>0.0070</td>
<td>0.3023</td>
<td>0.9280</td>
</tr>
<tr>
<td>0.15</td>
<td>0.54</td>
<td>0.1086</td>
<td>0.7422</td>
<td>0.0081</td>
<td>0.3630</td>
<td>1.1133</td>
</tr>
<tr>
<td>0.15</td>
<td>0.55</td>
<td>0.1159</td>
<td>0.8768</td>
<td>0.0093</td>
<td>0.4291</td>
<td>1.3152</td>
</tr>
</tbody>
</table>

Table 2B. The Effects of an Increase in $p_H$.

5 Extensions and Conclusions.

Our simple model of the entrepreneurial sector illustrates that the design of policy to enhance performance in the entrepreneurial sector can entail some subtleties. Our findings indicate, for example, that although it would be natural to expect any policy that enhances industry productivity to increase the welfare of some groups of entrepreneurs and/or venture
capitalists, this is not necessarily the case. To the contrary, some productivity increases can impose losses on all groups in the economy. (Recall Propositions 3 and 4.) Our findings also suggest that if a primary social objective is to increase the welfare of less capable entrepreneurs, this objective may be better pursued in some instances by enhancing the productivity of more capable entrepreneurs than by enhancing the productivity of the less capable entrepreneurs. (Recall Corollary 2.)

Our findings further imply that policies that marginally increase the success probability of entrepreneurial projects with limited potential for success can be counterproductive when there are relatively few of these projects in the population. (Recall Proposition 4.) The predominant effect of such policies (e.g., training programs that disproportionately attract entrepreneurs with limited skills and fail to substantially enhance their skills) can be to induce lenders to implement lending terms that harm all groups of entrepreneurs and thereby reduce welfare. Policies that primarily reduce the personal costs that entrepreneurs with limited prospects for success incur when applying for funding also can be counterproductive. Such policies are particularly likely to reduce welfare when the expected surplus from a profitable project substantially exceeds the expected loss from an unprofitable project.

For simplicity, the analysis to this point has only considered isolated changes in the probability that a low quality project succeeds or that a high quality project succeeds. In practice, technological change or a general education or training program can affect simultaneously the probability that different types of entrepreneurial projects succeed. Proposition 5 reports that even simultaneous increases in the success probabilities of all projects can reduce both the lender’s profit and the welfare of all entrepreneurs.

\[^{40}\text{In contrast, policies that systematically transform unprofitable projects into profitable projects can increase welfare by inducing lenders to implement more attractive lending terms.}\]

\[^{41}\text{Such policies might include educational programs that emphasize basic presentation and communication skills, for example.}\]

\[^{42}\text{Recall Proposition 3 and footnote \#36.}\]
Proposition 5. *Simultaneous increases in the success probabilities of all projects can reduce both the lender’s profit and the welfare of L entrepreneurs and H entrepreneurs (i.e., \( \pi^*, W_L^*, \) and \( W_H^* \) can all decline as \( p_L \) and \( p_H \) both increase).*

Proposition 5 is an immediate corollary of Proposition 4. As long as the increase in \( p_L \) is sufficiently pronounced relative to the increase in \( p_H \), the conclusions of Proposition 4 will prevail.

The analysis to this point has also considered in isolation changes in the transaction costs of \( L \) entrepreneurs and \( H \) entrepreneurs. In practice, changes in basic skills or bureaucratic procedures, for example, can affect the transaction costs of all entrepreneurs. Proposition 6 considers this possibility formally. The proposition reports that a simultaneous reduction in \( t_L \) and \( t_H \) (due, say, to a reduction in the bureaucratic procedures required to secure financing) can harm both the lender and the entrepreneurs.

Proposition 6. *A simultaneous reduction in the transaction costs experienced by \( L \) and \( H \) entrepreneurs can reduce both the lender’s profit and the welfare of \( L \) entrepreneurs and \( H \) entrepreneurs. This is the case even if all entrepreneurs experience the same reduction in their unit transaction cost.*

The conclusion in Proposition 6 is illustrated in Table 3. 43 In the setting considered in the table, the lender reduces the equilibrium sharing rate (\( \beta^* \)) as \( t_L \) and \( t_H \) decline by equal amounts. The reduction in \( \beta^* \) reduces the welfare of both groups of entrepreneurs (i.e., \( W_L^* \) and \( W_H^* \) both decline), despite the systematic reduction in each entrepreneur’s unit transaction cost. In this setting with a large proportion of \( L \) entrepreneurs (\( \phi_L = 0.9 \)), the reduction in the transaction cost experienced by all entrepreneurs reduces the lender’s profit (i.e., \( \pi^* \) declines). Thus, just as an isolated reduction in \( t_L \) can impose losses on all groups in the economy (recall Proposition 3), so too can a symmetric reduction in the unit transaction cost.

---

43 For expositional clarity, the entries recorded in the last four columns of Table 3 are 1,000 times their actual values.
cost of all entrepreneurs.\textsuperscript{44}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$t_L$ & $t_H$ & $\beta^*$ & $W_L^*$ & $W_H^*$ & $\pi^*$ & $W^*$ \\
\hline
3.0 & 4.0 & 0.0192 & 0.1420 & 1.7041 & 3.6923 & 5.5385 \\
2.9 & 3.9 & 0.0187 & 0.1392 & 1.6564 & 3.5913 & 5.3869 \\
2.8 & 3.8 & 0.0182 & 0.1359 & 1.6024 & 3.4766 & 5.2149 \\
2.7 & 3.7 & 0.0176 & 0.1320 & 1.5415 & 3.3470 & 5.0205 \\
2.6 & 3.6 & 0.0170 & 0.1275 & 1.4730 & 3.2010 & 4.8015 \\
2.5 & 3.5 & 0.0163 & 0.1222 & 1.3964 & 3.0371 & 4.5557 \\
\hline
\end{tabular}
\caption{The Effects of Simultaneous Reductions in $t_H$ and $t_L$.}
\end{table}

(\(V = 40, I = 20, q = 0.8, \phi_L = 0.9, p_H = 0.6, p_L = 0.2\))

Of course, sophisticated agents typically will not support programs that impose losses on all groups in the economy. Consequently, absent strong political pressure by those who are paid to implement projects, projects that generate widespread losses should not be expected to arise unless the equilibrium impacts of the projects are not fully anticipated.

Given our focus on changes in entrepreneurial productivity, we have not considered the impact of a change in the lender’s screening accuracy (\(q\)), which might be viewed as a component of the lender’s productivity. Recall that an \(L\) entrepreneur is unable to secure funding for his project when the lender assesses the project’s quality accurately. Consequently, one might suspect that the aggregate welfare of \(L\) entrepreneurs (\(W_L^*\)) will decline as the lender’s screening accuracy (\(q\)) increases. However, this is not always the case. It can be shown that the lender increases the sharing rate (\(\beta^*\)) as \(q\) increases. Furthermore, the increase in \(\beta^*\) can be sufficient to generate an increase in \(W_L^*\), despite the reduced probability that the project

\textsuperscript{44}Using Lemmas 4 and 5, it can be shown that symmetric proportional declines in $t_L$ and $t_H$ increase $\pi^*$, $W_L^*$, and $W_H^*$. Formally, if $t_H = \alpha_H t$ and $t_L = \alpha_L t$, then $\frac{d\pi^*}{dt} < 0$, $\frac{dW_L^*}{dt} < 0$, and $\frac{dW_H^*}{dt} < 0$. This conclusion and Proposition 6 together imply that the welfare effects of reduced transaction costs can vary with the precise nature of the reduced costs. The conclusion in Proposition 6 reflects in part the fact that the symmetric reduction in $t_L$ and $t_H$ constitutes a larger proportionate reduction in the unit transaction cost of \(L\) entrepreneurs because $t_L < t_H$. The smaller transaction cost for \(L\) entrepreneurs might reflect lower opportunity costs of applying for financing, for example.
of a given $L$ entrepreneur is funded. This conclusion reinforces the basic message that asymmetric information about likely entrepreneurial performance can introduce important subtleties into an assessment of the welfare implications of improved productivity in the entrepreneurial sector.

In practice, a lender’s ability to discern project quality ($q$) may be endogenous. As Proposition 7 indicates, many of our key qualitative conclusions continue to hold when $q$ is endogenous. The proposition refers to the setting with endogenous screening, where $C(q)$ is the lender’s cost of implementing screening accuracy $q$. $C(\cdot)$ is an increasing, convex function of $q$ that gives rise to a profit function for the lender that is strictly concave in $q$ with an interior optimum ($q^* \in \left(\frac{1}{2}, 1\right)$).

**Proposition 7.** The lender’s profit ($\pi^*$) and total welfare ($W^*$) both decline as $p_L$ increases in the setting with endogenous screening when $p_L$ is sufficiently small.

The preceding findings reflect in part the maintained assumption that entrepreneurs initially have no wealth. When entrepreneurs have wealth, the lender can require each entrepreneur to post his entire wealth as a bond and to forfeit this bond if his project either fails or generates the unfavorable signal. Such financing terms help to deter $L$ entrepreneurs from applying for funding, and can thereby limit the lender’s tendency to reduce welfare by reducing the reward she offers for project success when $p_L$ increases or $t_L$ decreases. In fact, if entrepreneurs have sufficient wealth, the lender can design reward structures that induce only $H$ entrepreneurs to apply for funding. In this case, changes in $p_L$ or $t_L$ will

---

45 This will be the case, for example, when $V = 40$, $I = 20$, $\phi_L = 0.6$, $\phi_H = 0.4$, $t_L = t_H = 4$, $p_L = 0.1$, and $p_H = 0.65$. In this setting, $W_L^*$ increases as $q$ increases when $q \in [0.58, 0.62]$.

46 Sufficient conditions are: (i) $C'(q) > 0$ and $C''(q) > 0$ for all $q \in \left(\frac{1}{2}, 1\right)$; (ii) $C\left(\frac{1}{2}\right) = 0$; (iii) $\lim_{q \to \frac{1}{2}} C'(q) = 0$; and (iv) $\lim_{q \to 1} C'(1) = \infty$.

47 Bose et al. (2014) demonstrate that among all financing arrangements that provide a specified payoff for $H$ entrepreneurs, this arrangement minimizes the expected payoff of $L$ entrepreneurs. See Manove et al. (2001) and Jimenez et al. (2006), for example, for additional analyses of the role of forfeitable bonds in financing contracts.

48 Bose et al. (2014) show that the lender can secure this ideal outcome if entrepreneurs have wealth $w \geq \frac{p_L(1-q)\cdot p_H V - I}{2(qp_H - p_L(1-q))}$.
not affect the lender’s profit or the welfare of entrepreneurs. Thus, the widespread losses from productivity increases identified above are a consequence of asymmetric knowledge of project quality and limited entrepreneurial wealth.\textsuperscript{49} Consequently, the potential losses are likely to be of particular concern in developing countries where potential entrepreneurs often have limited wealth.

These losses also reflect the entrepreneurs’ private knowledge of their transaction costs. If these costs were common knowledge, the lender could specify cost-contingent (i.e., location-specific) financing terms that eliminate the rents of all \( H \) entrepreneurs. These financing terms will be unprofitable for \( L \) entrepreneurs as long as their transaction costs are not too much smaller than the transaction costs of \( H \) entrepreneurs. In this case, the lender can secure the same expected profit that she secures when she is fully informed about both the location of each entrepreneur and the quality of his project. She can do so even if entrepreneurs have no wealth.\textsuperscript{50}

For simplicity, we have focused on a setting with a single lender. Our key qualitative conclusions persist when lenders compete to finance entrepreneurs’ projects. It is readily shown, for example, that an increase in \( p_L \) can reduce the lender’s profit and the aggregate welfare of both \( L \) entrepreneurs and \( H \) entrepreneurs when entrepreneurs decide which of two lenders to approach for funding. An increase in \( p_L \) can induce both lenders to reduce their equilibrium sharing rates. The lower sharing rates reduce the welfare of entrepreneurs directly. They can also reduce the profit of both lenders when the diminished sharing rates reduce the number of \( H \) entrepreneurs who apply for funding.\textsuperscript{51}

\textsuperscript{49}Ghatak et al. (2007) demonstrate that wealthy talented workers may oppose a tax on entrepreneurship that they would support if they had no wealth.

\textsuperscript{50}Bose et al. (2014) provide a formal proof of this conclusion. In practice, a lender is unlikely to know precisely the full cost that an individual entrepreneur faces in applying for funding. This cost reflects not only the time that an individual must devote to preparing an acceptable funding request, but also the opportunity cost of this time and the cost of securing any additional resources that the individual employs to formulate and implement his request.

\textsuperscript{51}If the two lenders are located on the boundaries of the \([0, 1]\) interval and if model parameters are such that all \( H \) entrepreneurs always apply for funding, then a reduction in sharing rates will not reduce the number of \( H \) entrepreneurs who apply for funding. Consequently, lender profit will not decline as \( p_L \) increases when the welfare of \( L \) entrepreneurs declines as \( p_L \) increases. However, an increase in \( p_L \) can generate
Additional extensions of our simple model, including scalable projects with richer payoff structures and entrepreneurial moral hazard, await further research.\textsuperscript{52} These extensions may introduce additional conclusions of interest. However, the extensions seem unlikely to alter the observation that the impacts of productivity increases in the entrepreneurial sector can be subtle and varied, and so policies that are clearly welfare-enhancing in the absence of asymmetric information merit close scrutiny in its presence, particularly in settings where entrepreneurs have very limited wealth.\textsuperscript{53}

\textsuperscript{52}In addition, entrepreneurs might be able to monitor and assist one another, and so lenders might optimally link one entrepreneur’s payoff to the realized performance of other entrepreneurs (e.g., Ghatak and Guinnane, 1999; Chowdhury, 2005). Dynamic models in which the mix of low and high quality projects in the population varies endogenously over time (e.g., Jovanovic, 1982) might also be explored. Furthermore, the impact of competitive lending sectors (e.g., Jaimovich, 2011; Inci, 2013) might be analyzed.

\textsuperscript{53}Similarly, policies that are counterproductive in the absence of asymmetric information may enhance welfare in its presence. These policies include the tax on entrepreneurship suggested by Ghatak et al. (2007) and the subsidy for not participating in the entrepreneurial sector suggested by de Meza and Webb (2000).
Appendix

Proof of Lemma 1.

Much as in the analysis in the proof of Lemma 2 (below), it is readily verified that the borrower that is indifferent between applying for funding and not applying is located at:

\[ x = \frac{\beta V}{t_c} \left[ \phi_L p_L (1 - q) + \phi_H p_H q \right]. \]  

(10)

Using (10), the lender’s (expected) profit when he sets sharing rate \( \beta \) is:

\[ \pi_S(\beta) = \frac{\beta V}{t_c} \left[ \phi_L p_L (1 - q) + \phi_H p_H q \right] \{ \phi_L [1 - q] [p_L (1 - \beta) V - I] + \phi_H q [p_H (1 - \beta) V - I] \}. \]  

(11)

It is readily verified that \( \pi_S(\cdot) \) is a concave function of \( \beta \) that is maximized at:

\[ \beta_S = \frac{\phi_L [1 - q] [p_L V - I] + \phi_H q [p_H V - I]}{2 V [\phi_L p_L (1 - q) + \phi_H p_H q]} = \frac{1}{2} - \frac{[\phi_L (1 - q) + \phi_H q] I}{2 V [\phi_L p_L (1 - q) + \phi_H p_H q]}. \]  

(12)

The conclusions in the Lemma follow immediately from (12).

Proof of Observation 1.

Substituting (12) into (11) reveals that the lender’s (maximum) profit in the setting with symmetric project quality information is:

\[ \pi_S = \frac{1}{4 t} \{ \phi_L [1 - q] [p_L V - I] + \phi_H q [p_H V - I] \}^2. \]  

(13)

The welfare of entrepreneurs in this setting is:

\[ W_{ES} = \left[ \phi_L (1 - q) p_L + \phi_H q p_H \right] \beta V x - \int_0^x t x \, dx \]

\[ = \frac{1}{8 t} \{ \phi_L [1 - q] [p_L V - I] + \phi_H q [p_H V - I] \}^2. \]  

(14)

The equality in (14) follows from (10) and (12). From (13) and (14), welfare in the setting with symmetric project quality information is:

\[ W_S = \pi_S + W_{ES} = \frac{3}{8 t} \{ \phi_L [1 - q] [p_L V - I] + \phi_H q [p_H V - I] \}^2. \]  

(15)

The conclusions in the Observation follow directly from (15).

Proof of Lemma 2.

An \( L \) entrepreneur’s expected payoff from applying for funding is \([1 - q] p_L \beta V\). The \( L \) entrepreneur located farthest from the lender that will apply for funding is the one for whom this expected payoff equals his transaction cost:
\[ [1-q] p_L \beta V = t_L x_L \Rightarrow x_L = \beta p_L V \left[ \frac{1-q}{t_L} \right]. \]

The analysis for the type \( H \) borrower is analogous, and so is omitted. ■

**Proof of Lemma 3.**

Substituting from Lemma 2 into (2) reveals that the lender’s profit when he sets sharing rate \( \beta \) is:

\[ \pi(\beta) = \left[ \frac{\beta V}{t_L} \right] \phi_L p_L [1-q]^2 [p_L (1-\beta) V - I] + \left[ \frac{\beta V}{t_H} \right] \phi_H p_H q^2 [p_H (1-\beta) V - I]. \]  

(16)

Differentiating (16) provides:

\[ \frac{\partial \pi(\cdot)}{\partial \beta} = \frac{V}{t_L} \phi_L p_L [1-q]^2 [p_L V - I] + \frac{V}{t_H} \phi_H p_H q^2 [p_H V - I] \]

\[ - 2 \beta V^2 \left[ \phi_L p_L^2 (1-q)^2 \frac{1}{t_L} + \phi_H p_H^2 q^2 \frac{1}{t_H} \right]. \]  

(17)

It is readily verified that \( \pi(\cdot) \) is a strictly concave function of \( \beta \), that \( \frac{\partial \pi(\cdot)}{\partial \beta} |_{\beta=1} < 0 \), and that \( \frac{\partial \pi(\cdot)}{\partial \beta} |_{\beta=0} > 0 \) when Assumption 1 holds. Therefore, (5) follows directly from (17). ■

**Proof of Lemma 4.**

From (5):

\[ 1 - \beta^* = \frac{V \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] + I \left[ \phi_L p_L (1-q)^2 t_H + \phi_H p_H q^2 t_L \right]}{2 V \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]}. \]  

(18)

From (16):

\[ \pi(\beta) = \frac{\beta V}{t_L t_H} \left[ 1 - \beta \right] V \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \]

\[ - \frac{\beta V}{t_L t_H} \left[ \phi_L p_L (1-q)^2 t_H + \phi_H p_H q^2 t_L \right] I. \]  

(19)

From (18):

\[ [1 - \beta^*] V \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] = \]

\[ \frac{1}{2} V \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] + \frac{1}{2} I \left[ \phi_L p_L (1-q)^2 t_H + \phi_H p_H q^2 t_L \right]. \]  

(20)

From (19) and (20):

\[ \pi(\beta^*) = \frac{\beta^* V}{2 t_L t_H} \left[ V \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \right] \]
Proof of Lemma 5.

From (3) and Lemma 2:

\[
W_L = \phi_L \left\{ [1-q] p_L V \beta x_L - \int_0^{x_L} t_L x \, dx \right\} = \phi_L \left\{ [1-q] p_L V \beta x_L - \frac{1}{2} t_L x_L^2 \right\} \\
= \phi_L \beta p_L V \left\{ \frac{1-q}{t_L} \right\} \left\{ [1-q] p_L V \beta - \frac{1}{2} t_L \beta p_L V \left\{ \frac{1-q}{t_L} \right\} \right\} \\
= \frac{\phi_L}{2t_L} [1-q]^2 [\beta p_L V]^2. \tag{22}
\]

Substituting from (5) into (22) provides (7). Relation (8) is derived in analogous fashion.

From (6), (7), and (8), equilibrium total welfare is:

\[
W^* = W^*_L + W^*_H + \pi^* = \left\{ \frac{\phi_L p_L [1-q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I]}{8 t_L t_H [\phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L]^2} \cdot \left\{ \phi_L p_L^2 t_H [1-q]^2 + \phi_H p_H^2 q^2 t_L + 2 \left\{ \phi_L p_L^2 t_H [1-q]^2 + \phi_H p_H^2 q^2 t_L \right\} \right\} \right\}^2 \\
= \frac{3 \left\{ \phi_L p_L [1-q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] \right\}^2}{8 t_L t_H [\phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L]}. \tag{21}
\]

Proof of Lemma 6.

It is apparent from (5) that \( \beta^* \) increases as \( V \) increases and as \( I \) decreases. Furthermore:

\[
\frac{\partial \beta^*}{\partial t_H} \equiv \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \phi_L p_L [1-q]^2 [p_L V - I] \\
- \left\{ \phi_L p_L [1-q]^2 [p_L V - I] t_H + \phi_H p_H q^2 [p_H V - I] t_L \right\} \phi_L p_L^2 [1-q]^2 \\
= - \phi_L p_L \phi_H p_H q^2 [1-q]^2 [p_H - p_L] t_L I < 0.
\]

\[
\frac{\partial \beta^*}{\partial t_L} \equiv \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \phi_H p_H q^2 [p_H V - I] \\
- \left\{ \phi_L p_L [1-q]^2 [p_L V - I] t_H + \phi_H p_H q^2 [p_H V - I] t_L \right\} \phi_H p_H^2 q^2 \\
= \phi_L p_L \phi_H p_H q^2 [1-q]^2 [p_H - p_L] t_H I > 0.
\]
$$\frac{\partial \beta^*}{\partial p_H} = \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \phi_H q^2 t_L [2p_H V - I]$$

$$- 2 \phi_H p_H q^2 t_L \left\{ \phi_L p_L [1-q]^2 [p_L V - I] t_H + \phi_H p_H q^2 [p_H V - I] t_L \right\}$$

$$= I \phi_H q^2 t_L \left\{ \phi_L p_L [1-q]^2 t_H [2p_H - p_L] + \phi_H p_H q^2 t_L \right\} > 0.$$

$$\frac{\partial \beta^*}{\partial \phi_H} = \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \left\{ p_H q^2 [p_H V - I] t_L - p_L [1-q]^2 [p_L V - I] t_H \right\}$$

$$- [p_H q^2 t_L - p_L (1-q)^2 t_H] \left\{ \phi_L p_L [1-q]^2 [p_L V - I] t_H + \phi_H p_H q^2 [p_H V - I] t_L \right\}$$

$$= p_H q^2 [1-q]^2 t_L t_H [p_H - p_L] I > 0. \quad \blacksquare$$

Proof of Lemma 7.

From (5):

$$\frac{\partial \beta^*}{\partial p_L} = \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \phi_L [1-q]^2 t_H [2p_L V - I]$$

$$- 2 \phi_L p_L [1-q]^2 t_H \left\{ \phi_L p_L [1-q]^2 [p_L V - I] t_H + \phi_H p_H q^2 [p_H V - I] t_L \right\}$$

$$= I \phi_L [1-q]^2 t_H \left\{ \phi_L p_L^2 [1-q]^2 t_H + \phi_H p_H q^2 t_L [2p_L - p_H] \right\}. \quad (23)$$

From (23):

$$\frac{\partial \beta^*}{\partial p_L} > 0 \iff \phi_L [1-q]^2 t_H p_L^2 + 2 \phi_H p_H q^2 t_L p_L - \phi_H p_H^2 q^2 t_L > 0$$

$$\iff \frac{\phi_L [1-q]^2 t_H}{\phi_H q^2 t_L} \left( \frac{p_L}{p_H} \right)^2 + 2 \left( \frac{p_L}{p_H} \right) - 1 > 0$$

$$\iff \delta_0 y^2 + 2 y - 1 > 0 \text{ where } y = \frac{p_L}{p_H} \text{ and } \delta_0 = \frac{\phi_L [1-q]^2 t_H}{\phi_H q^2 t_L}$$

$$\iff y > \frac{1}{2 \delta_0} \left[ -2 + \sqrt{4 + 4 \delta_0} \right] \iff y > \delta_1 = \frac{\sqrt{1+\delta_0} - 1}{\delta_0}. \quad \blacksquare$$

Proof of Proposition 1.

It is apparent from (6), (7), and (8) that \( \frac{\partial \pi^*}{\partial V} > 0, \frac{\partial W^*}{\partial L} > 0, \frac{\partial W^*}{\partial V} > 0, \frac{\partial \pi^*}{\partial I} < 0, \frac{\partial W^*}{\partial I} < 0, \text{ and } \frac{\partial W^*}{\partial I} < 0. \) Now define:

$$z \equiv \phi_L p_L [1-q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] > 0. \quad (24)$$

The inequality in (24) follows from Assumption 1. Then from (6):

$$\frac{\partial \pi^*}{\partial p_H} = \left[ \phi_L p_L^2 t_H (1-q)^2 + \phi_H p_H^2 t_L q^2 \right] 2 z \phi_H q^2 t_L [p_H V - I] - z^2 2 \phi_H p_H^2 t_L q^2$$
\[ s \equiv \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right] [p_H V - I] \\
- p_H \left\{ \phi_L p_L t_H [1 - q]^2 [p_L V - I] + \phi_H p_H t_L q^2 [p_H V - I] \right\} \\
= \phi_L p_L^2 t_H [1 - q]^2 [p_H V - I] - \phi_L p_L p_H t_H [1 - q]^2 [p_L V - I] > 0. \quad (25) \]

The inequality in (25) holds because \( p_H V - I > 0 > p_L V - I \).

Also from (6):
\[
\frac{\partial \pi'}{\partial t_H} \equiv z t_H \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right] 2 \phi_L p_L [1 - q]^2 [p_L V - I] \\
- z^2 \{2 \phi_L p_L^2 t_H [1 - q]^2 + \phi_H p_H^2 t_L q^2\} \\
= 2 \phi_L p_L [1 - q]^2 t_H [p_L V - I] \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right] \\
- \left\{ 2 \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right\} \cdot \left\{ \phi_L p_L t_H [1 - q]^2 [p_L V - I] + \phi_H p_H t_L q^2 [p_H V - I] \right\} \\
= \phi_L p_L [1 - q]^2 t_H [p_L V - I] \phi_H p_H^2 t_L q^2 \\
- \phi_H p_H t_L q^2 [p_H V - I] \left[ 2 \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right] < 0. \quad (26) \]

From (7):
\[
\frac{\partial W^*_L}{\partial p_H} \equiv \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2 2 z \phi_H q^2 t_L [2 p_H V - I] \\
- \left\{ 2 \phi_H q^2 t_L \right\} 2 \phi_H p_H q^2 t_L \\
\equiv 2 \left\{ 2 p_H V - I \right\} \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \\
- 2 p_H \left\{ \phi_L p_L [1 - q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] \right\} \\
> 2 \left\{ p_H V - I \right\} \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] - 2 \phi_H p_H q^2 t_L [p_H V - I] \quad (27) \]
\[
= 2 \left\{ p_H V - I \right\} \phi_L p_L^2 [1 - q]^2 t_H > 0. \]

The inequality in (27) holds because \( 2 p_H V - I > 2 \left\{ p_H V - I \right\} > 0 \) and \( p_L V - I < 0 \).

Also from (7):
\[
\frac{\partial W^*_L}{\partial t_H} \equiv \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2 2 z \phi_L p_L [1 - q]^2 [p_L V - I] \\
- \left\{ 2 \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right\} \phi_L p_L^2 [1 - q]^2 \\
\equiv \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] [p_L V - I] \\
- p_L \left\{ \phi_L p_L [1 - q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] \right\} \]
\[
\phi_H p_H^2 q^2 t_L [p_L V - I] - \phi_H p_H p_L q^2 t_L [p_H V - I] < 0. \tag{28}
\]

The inequality in (28) holds because \( p_H V - I > 0 > p_L V - I \).

The proof that \( \frac{\partial W^*}{\partial p_H} > 0 \) and \( \frac{\partial W^*}{\partial q_H} > 0 \) follows from (8) in analogous fashion. \( \blacksquare \)

**Proof of Proposition 2.**

From (6):
\[
\frac{\partial \pi^*}{\partial \phi_H} \triangleq \left[ \phi_L p_L^2 t_H (1-q)^2 + \phi_H p_H^2 t_L q^2 \right] 2 z \left\{ p_H t_L q^2 [p_H V - I] - p_L t_H [1-q]^2 [p_L V - I] \right\} 
- z^2 \left[ p_H^2 t_L q^2 - p_L^2 t_H (1-q)^2 \right]
\]

\[
\triangleq 2 \left[ \phi_L p_L^2 t_H (1-q)^2 + \phi_H p_H^2 t_L q^2 \right] \left\{ p_H t_L q^2 [p_H V - I] - p_L t_H [1-q]^2 [p_L V - I] \right\} 
- \left[ p_H^2 t_L q^2 - p_L^2 t_H (1-q)^2 \right] \left\{ \phi_L p_L t_H [1-q]^2 [p_L V - I] + \phi_H p_H t_L q^2 [p_H V - I] \right\}
- p_L t_H [1-q]^2 [p_L V - I] \left\{ \phi_H p^2_L t_H [1-q]^2 + [1 + \phi_H] p^2_H t_L q^2 \right\} > 0. \tag{29}
\]

The inequality in (29) holds because \( p_H V - I > 0 > p_L V - I \).

From (8):
\[
\frac{\partial W^*}{\partial \phi_H} \triangleq \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2 
- \left\{ \phi_H 2 z \left[ p_H q^2 t_L [p_H V - I] - p_L (1-q)^2 t_H [p_L V - I] \right] + z^2 \right\}
- \phi_H z^2 \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \left[ p_H^2 q^2 t_L - p_L^2 (1-q)^2 t_H \right]
\]

\[
\triangleq \left[ \phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \left[ 2 \phi_H p_H q^2 t_L (p_H V - I) - 2 \phi_H p_L (1-q)^2 t_H (p_L V - I) \right] 
+ z \left\{ \phi_L p_L^2 [1-q]^2 t_H + \phi_H p_H^2 q^2 t_L - 2 \phi_H p_H^2 q^2 t_L + 2 \phi_H p_L^2 [1-q]^2 t_H \right\}
+ \left\{ [1 + \phi_H] p_L^2 [1-q]^2 t_H + \phi_H p_H q^2 t_L [p_H V - I] \right\}
- \phi_H p_H q^2 t_L \left[ p_H V - I \right] \left\{ 2 p_L^2 [1-q]^2 t_H + \phi_H p_H^2 q^2 t_L \right\}
- p_L [1-q]^2 t_H \left[ p_L V - I \right] \phi_H \left[ 2 \phi_H + \phi_L \right] p_H^2 q^2 t_L 
+ \phi_L p_L^2 [1-q]^2 t_H \left\{ \phi_H p_L [1-q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] \right\} > 0. \tag{30}
\]

The inequality in (30) holds because \( p_H V - I > 0 > p_L V - I \) and because \( z > 0 \).
Since \( p = \) Proof of Proposition 3.

From (7):

\[
\frac{\partial W^*_L}{\partial \phi_H} = \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2 \\
\cdot \left\{ \phi_L 2 z \left[ p_H q^2 t_H (p_H V - I) - p_L (1 - q)^2 t_H (p_L V - I) \right] - z^2 \right\} \\
- \phi_L z^2 2 \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \left[ p_H^2 q^2 t_L - p_L^2 (1 - q)^2 t_H \right] \\
= \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \left[ 2 \phi_L p_H q^2 t_L (p_H V - I) - 2 \phi_L p_L (1 - q)^2 t_H (p_L V - I) \right] \\
- z \left\{ \phi_L p_L^2 [1 - q]^2 t_H + \phi_H p_H^2 q^2 t_L + 2 \phi_L p_H^2 q^2 t_L - 2 \phi_H p_L^2 [1 - q]^2 t_H \right\} \\
= \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \left[ 2 \phi_L p_H q^2 t_L (p_H V - I) - 2 \phi_L p_L (1 - q)^2 t_H (p_L V - I) \right] \\
- \left\{ \left[ \phi_H + 2 \phi_L \right] p_H^2 q^2 t_L - \phi_L p_L^2 [1 - q]^2 t_H \right\} \\
\cdot \left\{ \phi_L p_L [1 - q]^2 t_H (p_L V - I) + \phi_H p_H q^2 t_L (p_H V - I) \right\} \\
= p_H q^2 t_L \left( p_H V - I \right) \left\{ \phi_L \left[ \phi_L + 2 \phi_L \right] p_L^2 [1 - q]^2 t_H - \phi_H p_H^2 q^2 t_L \right\} \\
- \phi_L p_L [1 - q]^2 t_H \left( p_L V - I \right) \left\{ \left[ 3 \phi_H + 2 \phi_L \right] p_H^2 q^2 t_L + \phi_L p_L^2 [1 - q]^2 t_H \right\}. \quad (31)
\]

Since \( p_H V - I > 0 > p_L V - I \), (31) implies:

\[
\frac{\partial W^*_L}{\partial \phi_H} > 0 \text{ if } \phi_L \left[ \phi_L + 2 \phi_L \right] p_L^2 [1 - q]^2 t_H - \phi_H p_H^2 q^2 t_L > 0 \\
\iff \frac{\phi_L \left[ \phi_H + 2 \phi_L \right]}{\phi_H^2} > \frac{p_H^2 q^2 t_L}{p_L^2 [1 - q]^2 t_H} \\
\iff \frac{[1 - \phi_H] \left[ 2 \phi_H \right]}{\phi_H^2} > \frac{p_H^2 q^2 t_L}{p_L^2 [1 - q]^2 t_H}. \quad \blacksquare \quad (32)
\]

**Proof of Proposition 3.**

From (6):

\[
\frac{\partial \pi^*}{\partial t_L} = t_L \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right] z 2 \phi_H p_H q^2 (p_H V - I) \\
- z^2 \left\{ t_L \phi_H p_H^2 q^2 + \phi_L p_L^2 t_H [1 - q]^2 + \phi_H p_H^2 t_L q^2 \right\} \\
= 2 \phi_H p_H q^2 t_L \left( p_H V - I \right) \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right] \\
- \left\{ \phi_L p_L t_H [1 - q]^2 \left( p_L V - I \right) + \phi_H p_H t_L q^2 (p_H V - I) \right\} \\
\cdot \left[ \phi_L p_L^2 t_H (1 - q)^2 + 2 \phi_H p_H^2 t_L q^2 \right] \\
= \phi_H p_H q^2 t_L \left( p_H V - I \right) \phi_L p_L^2 t_H [1 - q]^2
\]

32
\[ - \phi_L p_L t_H [1 - q]^2 [p_L V - I] [\phi_L p_L^2 t_H (1 - q)^2 + 2 \phi_H p_H^2 t_L q^2] > 0. \]

From (8):
\[
\frac{\partial W^*_H}{\partial t_L} \leq s \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \left[ p_H V - I \right] \\
- z^2 \left[ \phi_L p_L^2 (1 - q)^2 t_H + 2 \phi_H p_H^2 q^2 t_L \right] \phi_H p_H^2 q^2 \\
\leq s \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] [p_H V - I] \\
- p_H \{ \phi_L p_L [1 - q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] \} \\
= \phi_L p_L^2 [1 - q]^2 t_H [p_H V - I] - \phi_L p_L p_H [1 - q]^2 t_H [p_L V - I] > 0.
\]

From (9):
\[
\frac{\partial W^*_l}{\partial t_L} \leq t_L \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \left[ p_H V - I \right] \\
- z^2 \left[ \phi_L p_L^2 (1 - q)^2 t_H + 2 \phi_H p_H^2 q^2 t_L \right] \\
\leq 2 \phi_H p_H q^2 t_L [p_H V - I] \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \\
- \left[ \phi_L p_L^2 (1 - q)^2 t_H + 2 \phi_H p_H^2 q^2 t_L \right] \\
\cdot \{ \phi_L p_L [1 - q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] \} \\
= \phi_H p_H q^2 t_L [p_H V - I] \phi_L p_L [1 - q]^2 t_H \\
- \phi_L p_L [1 - q]^2 t_H [p_L V - I] \left[ \phi_L p_L^2 (1 - q)^2 t_H + 2 \phi_H p_H^2 q^2 t_L \right] > 0.
\]

From (7):
\[
\frac{\partial W^*_l}{\partial t_L} \leq t_L \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \left[ p_H V - I \right] \\
- z^2 \left( 2 t_L \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \phi_H p_H^2 q^2 \\
+ \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2 \right) \\
\leq 2 \phi_H p_H q^2 t_L [p_H V - I] \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \\
- \left[ \phi_L p_L^2 (1 - q)^2 t_H + 3 \phi_H p_H^2 q^2 t_L \right] \\
\cdot \{ \phi_L p_L [1 - q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_H V - I] \} \\
= \phi_H p_H q^2 t_L [p_H V - I] \left[ \phi_L p_L^2 (1 - q)^2 t_H - \phi_H p_H^2 q^2 t_L \right] \\
- \phi_L p_L [1 - q]^2 t_H [p_L V - I] \left[ \phi_L p_L^2 (1 - q)^2 t_H + 3 \phi_H p_H^2 q^2 t_L \right] \equiv g(\phi_L). \tag{33}
\]
From (33): 
\[ g(\phi_L) = p_H q^2 t_L [p_H V - I] \left[ r p_L^2 (1 - q)^2 t_H - p_H^2 q^2 t_L \right] \]
\[ - r p_L [1 - q]^2 t_H [p_L V - I] \left[ r p_L^2 (1 - q)^2 t_H + 3 p_H^2 q^2 t_L \right], \tag{34} \]
where \( r = \frac{\dot{\phi}_L}{\phi_H} \). Since \( p_L V - I < 0 \), it is apparent from (33) and (34) that:
\[ \frac{\partial W^*_L}{\partial t_L} > 0 \text{ if } r = \frac{\phi_L}{\phi_H} \geq \frac{p_H^2 q^2 t_L}{p_L^2 [1 - q]^2 t_L}. \]

**Proof of Proposition 4.**

(i). 
\[ \frac{\partial \pi^*}{\partial p_L} = \frac{N_1 D_1}{2 t_L \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2} \equiv N_1 D_1 \tag{35} \]

where
\[ N_1 \equiv \phi_L [1 - q]^2 \left\{ \phi_H p_H q^2 t_L [p_H V - I] + \phi_L p_L [1 - q]^2 t_H [p_L V - I] \right\} \tag{36} \]
and
\[ D_1 \equiv \phi_L p_L^3 [1 - q]^2 t_H V + \phi_H p_H q^2 t_L [p_L p_H V - I (p_H - p_L)]. \tag{37} \]

**Proof.** Differentiating (6) provides:
\[ \frac{\partial \pi^*}{\partial p_L} = \frac{1}{16 t_L^2 t_H^2 \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2} \left\{ 4 t_L t_H \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right] \right. \]
\[ \cdot 2 \left\{ V \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H q^2 t_L q^2 \right] - I \left[ \phi_L p_L t_H (1 - q)^2 + \phi_H p_H t_L q^2 \right] \right\} \]
\[ \cdot \left[ 2 V \phi_L p_L t_H (1 - q)^2 - I \phi_L t_H (1 - q)^2 \right] \]
\[ - 8 t_L t_H \phi_L p_L [1 - q]^2 \left\{ V \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right] \right. \]
\[ \cdot \left[ \phi_L p_L t_H (1 - q)^2 + \phi_H p_H t_L q^2 \right] \right\} \}
\[ = \frac{V \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 q^2 t_L q^2 \right] - I \left[ \phi_L p_L t_H (1 - q)^2 + \phi_H p_H t_L q^2 \right]}{2 t_L t_H \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2} \]
\[ \cdot \left\{ \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right] \left[ 2 V \phi_L p_L t_H (1 - q)^2 - I \phi_L t_H (1 - q)^2 \right] \right. \]
\[ - \phi_L p_L t_H [1 - q]^2 \left\{ V \left[ \phi_L p_L^2 t_H (1 - q)^2 + \phi_H p_H^2 t_L q^2 \right] \right. \]
\[ \cdot \left[ \phi_L p_L t_H (1 - q)^2 + \phi_H p_H t_L q^2 \right] \right\} \}
\[ = \frac{N_1 / \left[ \phi_L (1 - q)^2 \right]}{2 t_L t_H \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2} \]
\[ \cdot \left\{ V \phi_L^2 p_L^3 t_H [1 - q]^4 + V \phi_L p_L \phi_H p_H^2 t_L t_H q^2 [1 - q]^2 \right\} \]
\[ - I \phi_L \phi_H p_H t_L t_H q^2 [1 - q]^2 [p_H - p_L] \]
\[ = \frac{N_1 D_1}{2 t_L \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2}. \]

(ii). \( D_1 \) is monotonically increasing in \( p_L \). Furthermore, there exists a \( \hat{p}_L \in (0, p_H) \) such that \( D_1 < 0 \) for \( p_L \in (0, \hat{p}_L) \) and \( D_1 > 0 \) for \( p_L \in (\hat{p}_L, p_H) \).

**Proof.** From (37):
\[ \frac{\partial D_1}{\partial p_L} = 3 \phi_L p_L^2 [1 - q]^2 t_H V + \phi_H p_H q^2 t_L [p_H V + I] > 0. \] (38)

Furthermore, from (37):
\[ D_1|_{p_L=0} = - \phi_H p_H^2 q^2 t_L I < 0, \quad \text{and} \]
\[ D_1|_{p_L=p_H} = \phi_L p_H^3 [1 - q]^2 t_H V + \phi_H p_H^3 q^2 t_L V > 0. \] \( \Box \)

(iii). There exists a \( \widehat{p}_L \in (0, p_H) \) such that \( \frac{\partial \pi'}{\partial p_L} < 0 \) for \( p_L \in (0, \widehat{p}_L) \) and \( \frac{\partial \pi'}{\partial p_L} > 0 \) for \( p_L \in (\widehat{p}_L, p_H) \).

**Proof.** The proof follows immediately from (i) and (ii) since \( N_1 > 0 \) for all \( p_L \in (0, p_H) \) under Assumption 1. \( \Box \)

(iv). From (8):
\[ \frac{\partial W^*}{\partial p_L} = \frac{8}{2} \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right]^2 2 z \phi_L [1 - q]^2 t_H \left[ 2 p_L V - I \right] \\
- z^2 \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] 2 \phi_L p_L [1 - q]^2 t_H \\
\frac{8}{2} = \left[ 2 p_L V - I \right] \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] - 2 p_L z \\
\frac{8}{2} = \left[ 2 p_L V - I \right] \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] \\
- 2 p_L \left\{ \phi_H p_H q^2 t_H [p_H V - I] + \phi_L p_L [1 - q]^2 t_H [p_L V - I] \right\} \\
= \phi_L p_L^2 [1 - q]^2 t_H I - \phi_H p_H q^2 t_L [p_H - 2 p_L] I. \] (39)

Relation (39) reveals that \( \frac{\partial W^*_L}{\partial p_L} < 0 \) if \( p_L \leq \frac{I}{2 V} \) (since \( z > 0 \)). Relation (40) reveals that \( \frac{\partial W^*_L}{\partial p_L} > 0 \) if \( p_L > \frac{1}{2} \). p_H 

(v). From (9):
\[ \frac{\partial W^*}{\partial p_L} = \frac{8}{2} \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] 2 z \phi_L [1 - q]^2 t_H \left[ 2 p_L V - I \right] \\
- z^2 \left[ 2 \phi_L p_L [1 - q]^2 t_H \right] \\
\frac{8}{2} = \left[ 2 p_L V - I \right] \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] - p_L z \\
\frac{8}{2} = \left[ 2 p_L V - I \right] \left[ \phi_L p_L^2 (1 - q)^2 t_H + \phi_H p_H^2 q^2 t_L \right] - p_L z \] (41)
\[ \begin{align*}
&= [2p_L V - I] [\phi_L p_L^2 (1-q)^2 t_H + \phi_H p_H^2 q^2 t_L] \\
&\quad - p_L \{ \phi_L p_L [1-q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_L V - I] \} \\
&\quad = \phi_H p_H q^2 t_L [p_L p_H V - (p_H - p_L) I] + \phi_L p_L^2 [1-q]^2 t_H p_L V. \\
\end{align*} \]  

(42)

Since \( z > 0 \), (41) implies that \( \frac{\partial W^*}{\partial p_L} < 0 \) if \( p_L < \frac{I}{2V} \). Also, (43) implies that \( \frac{\partial W^*}{\partial p_L} > 0 \) if \( p_L p_H V > [p_H - p_L] I \Leftrightarrow \frac{p_L p_H V}{p_H - p_L} > \frac{I}{V} \).

(vi). From (7):

\[ \sqrt{W_L^2} = \left[ \frac{(1-q)\sqrt{\phi_L}}{\sqrt{8t_L}} \right] \omega(p_L), \]

where:

\[ \omega(p_L) = p_L \left\{ \phi_L p_L [1-q]^2 t_H [p_L V - I] + \phi_H p_H q^2 t_L [p_L V - I] \right\} \]

\[ \phi_L p_L^2 [1-q]^2 t_H + \phi_H p_H^2 q^2 t_L. \]

From (44) and (45) imply:

\[ \frac{\partial W^*}{\partial p_L} \geq 0 \quad \text{as} \quad \frac{\partial \omega(p_L)}{\partial p_L} \geq 0. \]

From (45):

\[ \omega(p_L) = p_L \left[ V - \left( \frac{\phi_L p_L [1-q]^2 t_H + \phi_H p_H q^2 t_L}{\phi_L p_L^2 [1-q]^2 t_H + \phi_H p_H^2 q^2 t_L} \right) I \right] \]

\[ = p_L \left[ V - \left( \frac{\phi_L p_L^2 [1-q]^2 t_H + \phi_H p_H q^2 t_L}{\phi_L p_L^2 [1-q]^2 t_H + \phi_H p_H q^2 t_L} \right) \frac{I}{I} \right]. \]

(47)

Let \( A \equiv \frac{\phi_L [1-q]^2 t_H}{\phi_H p_H q^2 t_L} \). Then, (47) implies:

\[ \omega(p_L) = p_L \left[ V - \left( \frac{A p_L + 1}{A p_L^2 + p_H} \right) I \right] = V p_L \left[ -\frac{A p_L^2 + p_L}{A p_L^2 + p_H} \right] I \]

\[ = V p_L \left[ \frac{A p_L^2 + p_H}{A p_L^2 + p_H} - p_L - p_H \right] I = V p_L \left[ 1 + \frac{p_L - p_H}{A p_L^2 + p_H} \right] I. \]

(48)

(48) implies:

\[ \frac{\partial \omega(p_L)}{\partial p_L} = V - \left[ \frac{A p_L^2 + p_H - (p_L - p_H) (2 A p_L)}{(A p_L^2 + p_H)^2} \right] I \]

\[ = \frac{[A p_L^2 + p_H]^2 V - [2 A p_L] I}{[A p_L^2 + p_H]^2} > 0 \]

(49)
= A^2 p_L^4 V + A p_L^2 I + p_H [ p_H V - I + 2 A p_L ( p_L V - I ) ] > 0. \tag{50} 

Let \[ h ( p_L ) = p_H V - I + 2 A p_L [ p_L V - I ] \]
\[ = p_H V - I + 2 p_L [ p_L V - I ] \left[ \frac{ \phi_L (1 - q)^2 t_H }{ \phi_H p_H q^2 t_L } \right] \]
\[ = \phi_H p_H q^2 t_L [ p_H V - I ] + 2 \phi_L p_L [ 1 - q ]^2 t_H [ p_L V - I ] . \tag{51} \]

Relations (46), (49), (50), and (51) imply that \( \frac{\partial W}{\partial p_L} > 0 \) when \( q \) is sufficiently close to 1.

(i) – (vi) constitute the proof of all conclusions in the Proposition other than the conclusion regarding the systematic losses of the lender, \( L \) entrepreneurs, and \( H \) entrepreneurs. These losses are illustrated in Table 1 in the text.

\[ \begin{align*} 
\text{Proof of Corollary 1.} & \\
\text{From Assumption 1:} & \\
\phi_H p_H [ p_H V - I ] t_L + \phi_L p_L [ p_L V - I ] t_H > 0 . 
\end{align*} \]

Therefore, \( h ( p_L ) > 0 \) from (51), and so \( \frac{\partial W}{\partial p_L} > 0 \) from (46), (49), and (50) if:
\[ q^2 \geq 2 [ 1 - q ]^2 \Leftrightarrow q \geq \frac{ \sqrt{2} }{ 1 + \sqrt{2} } \approx 0.58579 . \]

\[ \begin{align*} 
\text{Proof of Corollary 2.} & \\
\text{The data in Table 2 provide a proof of the corollary.} 
\end{align*} \]

\[ \begin{align*} 
\text{Proof of Proposition 5.} & \\
\text{As noted in the text, the proposition is an immediate corollary of Proposition 4 since the} \\
\text{relevant increase in} \ p_L \ \text{can be made arbitrarily large relative to the increase in} \ p_H . 
\end{align*} \]

\[ \begin{align*} 
\text{Proof of Proposition 6.} & \\
\text{The proof follows from Table 3 in the text.} 
\end{align*} \]

\[ \begin{align*} 
\text{Proof of Proposition 7.} & \\
\text{From (6), the lender’s profit in this setting when she implements screening accuracy} \ q \ \text{is:} \\
\Pi ( q, C ) = \pi^*(q) - C(q), \tag{52} \\
\text{where:} 
\end{align*} \]
\[ \pi^*(q) = \left[ \frac{\phi_H p_H q^2 t_L (p_H V - I) + \phi_L p_L (1 - q)^2 t_H (p_L V - I)}{4 t_L t_H \left[ \phi_H p_H^2 q^2 t_L + \phi_L p_L^2 (1 - q)^2 t_H \right]} \right]^2. \]  

(53)

Let \( q^* = \arg \max_q \Pi(q, C) \). Also let \( \Pi^* = \Pi(q^*, C) \). Then:

\[
\frac{d\Pi^*}{dp_L} = \left. \frac{\partial \Pi(\cdot)}{\partial p_L} \right|_{q = q^*} + \left\{ \left. \frac{\partial \Pi(\cdot)}{\partial q} \right|_{q = q^*} \right\} \left. \frac{\partial q^*}{\partial p_L} \right|_{q = q^*}.
\]

(54)

The last equality in (54) reflects the envelope theorem. Relations (52) and (54) imply that

\[
\frac{dW^*}{dp_L} = \frac{\partial \pi^*(q^*)}{\partial p_L}. 
\]

From (9), total welfare in this setting when the lender implements screening accuracy \( q \) at cost \( C(q) \) is:

\[
\tilde{W}(q, C) = \frac{3}{2} \pi^*(q) - C(q) = \frac{3}{2} \left[ \pi^*(q) - \frac{2}{3} C(q) \right] 
\]

(55)

From (52), (53), and (55):

\[
\tilde{W}(q, C) = \frac{3}{2} \pi^*(q) - C(q) = \frac{3}{2} \pi^*(q) - C(q), \quad \text{where} \quad \tilde{C}(q) = \frac{2}{3} C(q). 
\]

(56)

Relation (56) implies:

\[
\frac{2}{3} \frac{d\tilde{W}(q, C)}{dp_L} = \left. \frac{\partial \Pi(q^*, \tilde{C})}{\partial p_L} \right|_{q = q^*} = \left. \frac{\partial \pi^*(q)}{\partial p_L} \right|_{q = q^*} 
\]

(57)

The second equality in (57) reflects the envelope theorem. (57) implies that \( \frac{d\tilde{W}(q, C)}{dp_L} = \frac{\partial \pi^*(q)}{\partial p_L} \). Therefore, the conclusion in the proposition follows from Proposition 4.
References


