Technical Appendix to Accompany

"Asymmetric Treatment of Identical Agents in Teams"

by A. Bose, D. Pal, and D. Sappington

Findings 1 – 4 below provide sufficient conditions for the values of p^A, p^B, and $p^A + p^B + \gamma\, p^A p^B$ that constitute the solutions to [P-S] and [P-SQ] to all lie in the $(0, 1)$ interval and for these values of p^A and p^B to be uniquely defined by the agent's relevant first-order conditions. The proofs of these Findings proceed for the case where agent i's cost (k^i) can differ from agent j's cost (k^j). The ensuing analysis presumes that $V_F = 0$, without essential loss of generality.

To begin, define [P-S]^I to be problem [P-S] where constraints (4) and (7) in the text are replaced by:

$$[1 + \gamma\, p^j]\, T_S^i \;=\; k^i \left(p^i\right)^{\theta-1} \quad \Rightarrow \quad T_S^i \;=\; \frac{k^i \left(p^i\right)^{\theta-1}}{1 + \gamma\, p^j} \quad \text{for } j \neq i,\;\; i, j \in \{A, B\}. \tag{A1}$$

The solution to [P-S]^I will be the solution to [P-S] if: (i) $p^A \in (0, 1)$, $p^B \in (0, 1)$, and $p^A + p^B + \gamma\, p^A p^B \in (0, 1)$ at the solution to [P-S]^I; and (ii) agent i's choice of p^i as defined implicitly by (A1) uniquely maximizes his objective function.

Finding 1. *$p^A \in (0, 1)$, $p^B \in (0, 1)$, and $p^A + p^B + \gamma\, p^A p^B \in (0, 1)$ at the solution to [P-S]^I if: (i) $k^A = k^B = k \,>\, V_S\,(2)^{\theta-1}$ when $\gamma = 0$; (ii) $[1 + \gamma]^{\frac{1}{\theta-1}} \left[2 \left(\frac{V_S}{k}\right)^{\frac{1}{\theta-1}} + \gamma\, [1 + \gamma]^{\frac{1}{\theta-1}} \left(\frac{V_S}{k}\right)^{\frac{2}{\theta-1}} \right]$ < 1 when $\gamma > 0$; and (iii) $k > V_S\,(2)^{\theta-1}$ when $\gamma \in (-1, 0)$.*

<u>Proof</u>. Given p^j, agent i chooses p^i to maximize:

$$\Pi^i(p^i | p^j) \;=\; p(p^i, p^j)\, T_S^i \;-\; \frac{k^i}{\theta} (p^i)^\theta. \tag{A2}$$

Differentiating (A2) provides:

$$\frac{\partial \Pi^i(\cdot)}{\partial p^i} \;=\; \frac{\partial p(\cdot)}{\partial p^i}\, T_S^i \;-\; k^i \left(p^i\right)^{\theta-1}. \tag{A3}$$

Equation (1) in the text implies $\frac{\partial p(\cdot)}{\partial p^i} \in \{0,\, 1 + \gamma\, p^j\}$ for $\widetilde{p} \neq 1$, where $\widetilde{p} \equiv p^A + p^B + \gamma\, p^A p^B$. Consequently, (A3) implies that for $\widetilde{p} < 1$:

$$\frac{\partial^2 \Pi^i(\cdot)}{\partial \left(p^i\right)^2} \;=\; -\, k^i\, [\theta - 1] \left(p^i\right)^{\theta-2} \;<\; 0. \tag{A4}$$

Furthermore, from (A3):

$$\frac{\partial \Pi^i(\cdot)}{\partial p^i} \;=\; \begin{cases} [1 + \gamma\, p^j]\, T_S^i - k^i \left(p^i\right)^{\theta-1} & \text{when } \;\; \widetilde{p} \in (0, 1) \\ -\, k^i \left(p^i\right)^{\theta-1} & \text{when } \;\; \widetilde{p} > 1. \end{cases} \tag{A5}$$

Equation (1) in the text and (A5) imply:

$$p^i\left(p^j\right) = \begin{cases} \left(\frac{[1+\gamma\, p^j]T_S^i}{k^i}\right)^{\frac{1}{\theta-1}} & \text{when} \quad \widetilde{p} \in (0,1) \\ 1 - p^j & \text{when} \quad \widetilde{p} > 1. \end{cases} \tag{A6}$$

Case 1. $\gamma = 0$.

(A6) implies that $p^A < 1$, $p^B < 1$, and $p^A + p^B < 1$ in this case if $\left[\frac{T_S^i}{ki}\right]^{\frac{1}{\theta-1}} < \frac{1}{2}$ for $i = A, B$. Therefore, because the principal will never pay an agent more than the value of success (V_S), a sufficient condition for unique, interior values of p^A and p^B is:

$$\left[\frac{V_S}{k^i}\right]^{\frac{1}{\theta-1}} < \frac{1}{2} \quad \Leftrightarrow \quad 2\left[V_S\right]^{\frac{1}{\theta-1}} < \left[k^i\right]^{\frac{1}{\theta-1}} \quad \Leftrightarrow \quad k^i > V_S\,(2)^{\theta-1} \quad \text{for } i = A, B.$$

Case 2. $\gamma > 0$.

(A6) implies that if $p^A + p^B + \gamma\, p^A p^B \in (0,1)$, then :

$$\frac{\partial p^i(\cdot)}{\partial p^j} = \left(\frac{T_S^i}{k^i}\right)^{\frac{1}{\theta-1}} \left[\frac{\gamma}{\theta-1}\right] \left[1+\gamma\, p^j\right]^{\frac{2-\theta}{\theta-1}} > 0, \qquad \text{and} \tag{A7}$$

$$\frac{\partial^2 p^i(\cdot)}{\partial\left(p^j\right)^2} = -\left(\frac{T_S^i}{k^i}\right)^{\frac{1}{\theta-1}} \left[\frac{\theta-2}{[\theta-1]^2}\right] \gamma^2 \left[1+\gamma\, p^j\right]^{\frac{3-2\theta}{\theta-1}} < 0. \tag{A8}$$

(A7) and (A8) imply that as long as $p(\cdot) \in (0,1)$, agent i's best response function is an increasing, concave function of p^j. Also, (A6) implies that $p^i = \left[\frac{T_S^i}{k^i}\right]^{\frac{1}{\theta-1}}$ when $p^j = 0$.

We seek conditions sufficient to ensure the best response functions of agents A and B intersect at a point that lies (strictly) within the region in the (p^A, p^B)-plane bounded by the $p^A = 0$ axis, the $p^B = 0$ axis, and the curve $p^A + p^B + \gamma\, p^A p^B = 1$. Formally, we seek conditions that ensure the solutions to (A9) and (A10) also satisfy (A11).

$$\left[1+\gamma\, p^B\right] T_S^A - k^A \left(p^A\right)^{\theta-1} = 0. \tag{A9}$$

$$\left[1+\gamma\, p^A\right] T_S^B - k^B \left(p^B\right)^{\theta-1} = 0. \tag{A10}$$

$$p^A + p^B + \gamma\, p^A p^B \in (0,1). \tag{A11}$$

When $k^A = k^B = k$, it suffices to identify conditions that ensure the solutions to (A12) and (A13) satisfy (A11).

$$\left[1+\gamma\, p^B\right] V_S - k\left(p^A\right)^{\theta-1} = 0. \tag{A12}$$

$$\left[1+\gamma\, p^A\right] V_S - k\left(p^B\right)^{\theta-1} = 0. \tag{A13}$$

This is the case because $T^A \le V_S$ and $T^B \le V_S$ at the solution to [P-S] and because, from (A9) and (A10):

$$\frac{dp^i}{dT_S^i} = \frac{1+\gamma\, p^j}{k^i[\theta-1](p^i)^{\theta-2}} > 0 \;\text{ when } \gamma > 0. \tag{A14}$$

Therefore, the values of p^A and p^B that solve (A9) and (A10) will be less than the corresponding values of p^A and p^B that solve (A12) and (A13) when $\gamma > 0$.

(A9) implies:

$$k^A \left(p^A\right)^{\theta-1} = [1+\gamma\, p^B]\, T_S^A \le [1+\gamma]\, T_S^A$$

$$\Rightarrow \left(p^A\right)^{\theta-1} \le \frac{[1+\gamma]\, T_S^A}{k^A} \Rightarrow p^A \le \left(\frac{[1+\gamma]\, T_S^A}{k^A}\right)^{\frac{1}{\theta-1}}. \tag{A15}$$

Similarly, (A10) implies:

$$p^B \le \left(\frac{[1+\gamma]\, T_S^B}{k^B}\right)^{\frac{1}{\theta-1}}. \tag{A16}$$

(A15) and (A16) imply:

$$\begin{aligned} & p^A + p^B + \gamma\, p^A p^B \\ \le\ & \left(\frac{[1+\gamma]\, T_S^A}{k^A}\right)^{\frac{1}{\theta-1}} + \left(\frac{[1+\gamma]\, T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} + \gamma \left(\frac{[1+\gamma]\, T_S^A}{k^A}\right)^{\frac{1}{\theta-1}} \left(\frac{[1+\gamma]\, T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} \\ =\ & [1+\gamma]^{\frac{1}{\theta-1}} \left[\left(\frac{T_S^A}{k^A}\right)^{\frac{1}{\theta-1}} + \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} + \gamma\, [1+\gamma]^{\frac{1}{\theta-1}} \left(\frac{T_S^A}{k^A}\right)^{\frac{1}{\theta-1}} \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}}\right] \\ \le\ & [1+\gamma]^{\frac{1}{\theta-1}} \left[\left(\frac{V_S}{k^A}\right)^{\frac{1}{\theta-1}} + \left(\frac{V_S}{k^B}\right)^{\frac{1}{\theta-1}} + \gamma\, [1+\gamma]^{\frac{1}{\theta-1}} \left(\frac{V_S}{k^A}\right)^{\frac{1}{\theta-1}} \left(\frac{V_S}{k^B}\right)^{\frac{1}{\theta-1}}\right]. \end{aligned} \tag{A17}$$

(A15) – (A17) imply that if $k^A = k^B = k$ and if $[1+\gamma]^{\frac{1}{\theta-1}} \left[2\left(\frac{V_S}{k}\right)^{\frac{1}{\theta-1}} + \gamma\,[1+\gamma]^{\frac{1}{\theta-1}} \left(\frac{V_S}{k}\right)^{\frac{2}{\theta-1}}\right]$ < 1, then $p^A < 1$, $p^B < 1$ and $p^A + p^B + \gamma\, p^A p^B < 1$ at the solution to [P-S]^I.

Case 3. $\gamma < 0$.

(A7) and (A8) imply that as long as $p \in (0,1)$, agent i's optimal choice of p^i is a decreasing, convex function of p^j. Also, (A6) implies that $p^i = \left[\frac{T_S^i}{k^i}\right]^{\frac{1}{\theta-1}}$ when $p^j = 0$. From (A14), if $|\gamma| < 1$, then $\frac{dp^i}{dT_S^i} > 0$ when $p^j < 1$. Consequently, it suffices to ensure that the solutions to (A12) and (A13) satisfy (A11). (A9), (A10), (A12), and (A13) imply:

$$p^i = \left(\frac{[1+\gamma\, p^j]\, V_S}{k^i}\right)^{\frac{1}{\theta-1}} < \left(\frac{V_S}{k^i}\right)^{\frac{1}{\theta-1}} < \frac{1}{2} \text{ for } i \in \{A,B\} \text{ when } k^i > V_S\,(2)^{\theta-1}.$$

Therefore, $p^A < 1$, $p^B < 1$, and $p^A + p^B + \gamma p^A p^B < 1$ when $k^i > V_S\,(2)^{\theta-1}$ for $i = A, B$. Also, if $|\gamma| < 1$, then (A9) and (A10) imply $p^A > 0$ and $p^B > 0$. Furthermore, $p^A + p^B + \gamma\, p^A p^B = p^A[1+\gamma\, p^B] + p^B > 0$ when $|\gamma| < 1$. ■

Finding 2. *Agent $i \in \{A,B\}$'s choice of p^i as defined by (A1) uniquely maximizes his objective function when the conditions identified in Finding 1 hold.*

Proof. From (A4), $\Pi^i(p^i|p^j)$ is a strictly concave function of p^i for $p^j \in (0,1)$ for $j \neq i$,

$i, j \in \{A, B\}$. Therefore, when the conditions identified in Finding 1 hold, p^i as defined by (A1) uniquely maximizes (A2). ■

Now define [P-SQ]^I to be problem [P-SQ] where constraint (4) in the text is replaced by (A1) for $i = B$ and where constraint (5) in the text is replaced by:

$$T_S^A = \frac{k^A \left(p^A\right)^{\theta-1}}{1 + \gamma\, p^B + \frac{\gamma\, p^B}{\theta-1}} = \frac{k^A \left(p^A\right)^{\theta-1}}{1 + \left[\frac{\theta}{\theta-1}\right] \gamma\, p^B}. \tag{A18}$$

(A18) is derived by first differentiating (A1) when $i = B$ to obtain:

$$\gamma\, T_S^B dp^A = k^B[\theta-1] \left(p^B\right)^{\theta-2} dp^B \quad \Rightarrow \quad \frac{dp^B}{dp^A} = \frac{\gamma\, T_S^B}{k^B[\theta-1] \left(p^B\right)^{\theta-2}}. \tag{A19}$$

Then, differentiating constraint (5) in the text (with $k = k^A$) reveals that for $p(\cdot) \in (0,1)$:

$$\left[1 + \gamma\, p^B + \left(1 + \gamma\, p^A\right) \frac{dp^B}{dp^A}\right] T_S^A = k^A \left(p^A\right)^{\theta-1} \quad \Rightarrow T_S^A = \frac{k^A \left(p^A\right)^{\theta-1}}{1 + \gamma\, p^B + \left[1 + \gamma\, p^A\right] \frac{dp^B}{dp^A}}. \tag{A20}$$

(A1) and (A19) imply:

$$\left[1 + \gamma\, p^A\right] \frac{dp^B}{dp^A} = \frac{\left[1 + \gamma\, p^A\right] \gamma\, T_S^B}{k^B[\theta-1] \left(p^B\right)^{\theta-2}} = \frac{\gamma\, k^B \left(p^B\right)^{\theta-1}}{k^B[\theta-1] \left(p^B\right)^{\theta-2}} = \frac{\gamma\, p^B}{\theta-1}. \tag{A21}$$

Substituting (A21) into (A20) provides (A18).

Finding 3. *$p^A \in (0,1)$, $p^B \in (0,1)$, and $p^A + p^B + \gamma\, p^A p^B \in (0,1)$ at the solution to [P-SQ]^I if: (i) $k^A = k^B = k > V_S\,(2)^{\theta-1}$ when $\gamma = 0$; (ii) $k > \widetilde{k}$ when $\gamma > 0$, where $\widetilde{k}$ is defined by $\left(\widetilde{p}^A\right)^{\theta-1} = \frac{V_S}{\widetilde{k}} + \left[\frac{\gamma\,(1+\gamma)\,\theta}{\theta-1}\right] (V_S)^{\frac{\theta}{\theta-1}} \left(\frac{1}{\widetilde{k}}\right)^{\frac{\theta}{\theta-1}}$ and $\widetilde{p}^A$ is defined by $\widetilde{p}^A + \left[1 + +\gamma\, \widetilde{p}^A\right] \left(\frac{[1+\gamma\, \widetilde{p}^A]\, V_S}{k}\right)^{\frac{1}{\theta-1}} = 1$; and (iii) $k > V_S\,(2)^{\theta-1}$ when $\gamma \in (-1, 0)$.*

Proof.

Case 1. $\gamma = 0$.

The analysis in this case is identical to the analysis in the proof of Finding 1.

Case 2. $\gamma > 0$.

The analysis for agent B coincides with the corresponding analysis in the proof of Finding 1.

Agent A will pick his preferred point on agent B's reaction function as defined by (A6) for $i = B$. Let $\pi^A(p^A) \equiv \Pi^A(p^A | p^B(p^A))$ denote agent A's expected profit in the present setting. Also let $\widehat{p}^A \leq 1$ denote the smallest value of p^A on the portion of agent B's reaction function defined by the equation $p^A + p^B + \gamma\, p^A p^B = 1$.

Agent A will never choose to deliver a contribution in excess of $\widehat{p}^A$ because he can secure the same aggregate probability of success ($p(\cdot) = 1$) at lower personal cost by setting $p^A = \widehat{p}^A$. Therefore, conditions that ensure $\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=0} > 0$ and $\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=\widehat{p}^A} < 0$ are sufficient to ensure agent A will choose a $p^A \in (0,1)$.

From (A20), for $p^A + p^B + \gamma\, p^A p^B \in (0,1)$:

$$\frac{d\pi^A(\cdot)}{dp^A} = [1+\gamma\, p^B]\, T_S^A + [1+\gamma\, p^A]\, T_S^A \,\frac{dp^B}{dp^A} - k^A \left(p^A\right)^{\theta-1}. \tag{A22}$$

Solving (A10) for p^B provides:

$$p^B = \left(\frac{[1+\gamma\, p^A]\, T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} \Rightarrow \frac{dp^B}{dp^A} = \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} \left[\frac{\gamma}{\theta-1}\right] [1+\gamma\, p^A]^{\frac{2-\theta}{\theta-1}}. \tag{A23}$$

Substituting from (A23) into (A22) provides:

$$\begin{aligned}\frac{d\pi^A(\cdot)}{dp^A} &= T_S^A + \gamma\, T_S^A \left(\frac{[1+\gamma\, p^A]\, T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} \\ &\quad + [1+\gamma\, p^A]\, T_S^A \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} \left[\frac{\gamma}{\theta-1}\right] [1+\gamma\, p^A]^{\frac{2-\theta}{\theta-1}} - k^A \left(p^A\right)^{\theta-1} \\ &= T_S^A + \gamma \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} [1+\gamma\, p^A]^{\frac{1}{\theta-1}}\, T_S^A \left[\frac{\theta}{\theta-1}\right] - k^A \left(p^A\right)^{\theta-1}.\end{aligned} \tag{A24}$$

(A24) implies:

$$\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=0} = T_S^A + \gamma \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} T_S^A \left[\frac{\theta}{\theta-1}\right] > 0; \quad \text{and} \tag{A25}$$

$$\begin{aligned}\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=\widehat{p}^A} &= T_S^A + \gamma \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} [1+\gamma\, \widehat{p}^A]^{\frac{1}{\theta-1}}\, T_S^A \left[\frac{\theta}{\theta-1}\right] - k^A \left(\widehat{p}^A\right)^{\theta-1} \\ &< V_S + \gamma \left(\frac{V_S}{k^B}\right)^{\frac{1}{\theta-1}} [1+\gamma\, \widehat{p}^A]^{\frac{1}{\theta-1}}\, V_S \left[\frac{\theta}{\theta-1}\right] - k^A \left(\widehat{p}^A\right)^{\theta-1}.\end{aligned} \tag{A26}$$

The inequality in (A26) holds because $T_S^A \le V_S$ and $T_S^B \le V_S$ at the solution to $[\text{P-SQ}]^I$. (A26) implies:

$$\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=\widehat{p}^A} < 0 \;\text{ if }\; k^A \left(\widehat{p}^A\right)^{\theta-1} \ge V_S + \left[\frac{\gamma\, \theta}{\theta-1}\right] \left(\frac{V_S}{k^B}\right)^{\frac{1}{\theta-1}} [1+\gamma\, \widehat{p}^A]^{\frac{1}{\theta-1}}\, V_S. \tag{A27}$$

Since $\widehat{p}^A \le 1$, the weak inequality in (A27) will hold when $k^A = k^B = k$ if:

$$\left(\widehat{p}^A\right)^{\theta-1} \;\geq\; \frac{V_S}{k}+\left[\frac{\gamma\,(1+\gamma)\,\theta}{\theta-1}\right](V_S)^{\frac{\theta}{\theta-1}}\left(\frac{1}{k}\right)^{\frac{\theta}{\theta-1}}. \tag{A28}$$

From equation (1) in the text and (A23), $\widehat{p}^A$ is the solution to:

$$\widehat{p}^A+p^B+\gamma\,\widehat{p}^A p^B=1 \quad \text{and} \quad p^B \;=\; \left(\frac{\left[1+\gamma\,\widehat{p}^A\right]T^B}{k^B}\right)^{\frac{1}{\theta-1}}. \tag{A29}$$

(A29) implies that $\widehat{p}^A$ is the solution to:

$$\widehat{p}^A+\left(\frac{\left[1+\gamma\,\widehat{p}^A\right]T^B}{k^B}\right)^{\frac{1}{\theta-1}}+\gamma\,\widehat{p}^A\left(\frac{\left[1+\gamma\,\widehat{p}^A\right]T^B}{k^B}\right)^{\frac{1}{\theta-1}} \;=\; 1. \tag{A30}$$

Let $\widetilde{p}^A$ be the solution to:

$$\widetilde{p}^A+\left(\frac{\left[1+\gamma\,\widetilde{p}^A\right]V_S}{k^B}\right)^{\frac{1}{\theta-1}}+\gamma\,\widetilde{p}^A\left(\frac{\left[1+\gamma\,\widetilde{p}^A\right]V_S}{k^B}\right)^{\frac{1}{\theta-1}} \;=\; 1. \tag{A31}$$

Since $V_S \geq T^B$, (A30) and (A31) imply that $\widetilde{p}^A \leq \widehat{p}^A$ for given $\gamma > 0$ and $k^B > 0$. Therefore, when $k^A = k^B = k$, the inequality in (A28) holds if:

$$\left(\widetilde{p}^A\right)^{\theta-1} \;\geq\; \frac{V_S}{k}+\left[\frac{\gamma\,(1+\gamma)\,\theta}{\theta-1}\right](V_S)^{\frac{\theta}{\theta-1}}\left(\frac{1}{k}\right)^{\frac{\theta}{\theta-1}}. \tag{A32}$$

Define $\widetilde{k}$ by:

$$\left(\widetilde{p}^A\right)^{\theta-1} \;=\; \frac{V_S}{\widetilde{k}}+\left[\frac{\gamma\,(1+\gamma)\,\theta}{\theta-1}\right](V_S)^{\frac{\theta}{\theta-1}}\left(\frac{1}{\widetilde{k}}\right)^{\frac{\theta}{\theta-1}}. \tag{A33}$$

(A31) implies that $\widetilde{p}^A$ is an increasing function of $k^A = k^B = k$. Furthermore, the expression to the right of the inequality in (A32) is a decreasing function of k. Therefore, as k is increased above $\widetilde{k}$, the inequality in (A32) will continue to hold, and so $\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=\widehat{p}^A} < 0$ for $k > \widetilde{k}$.

Case 3. $\gamma < 0$.

From (A25):

$$\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=0} \;=\; T^A\left[1+\gamma\left(\frac{T^B}{k^B}\right)^{\frac{1}{\theta-1}}\left(\frac{\theta}{\theta-1}\right)\right]$$

$$\geq\; T^A\left[1+\gamma\left(\frac{V_S}{k^B}\right)^{\frac{1}{\theta-1}}\left(\frac{\theta}{\theta-1}\right)\right] \;>\; T^A\left[1+\frac{\gamma}{2}\left(\frac{\theta}{\theta-1}\right)\right]. \tag{A34}$$

The strict inequality in (A34) holds because $\gamma < 0$ and because $k^i > V_S\,(2)^{\theta-1}$ for $i = A, B$ implies $\left(\frac{V_S}{k^B}\right)^{\frac{1}{\theta-1}} < \frac{1}{2}$. (A34) implies:

$$\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=0} \;>\; 0 \quad \text{if} \;\; 1+\frac{\gamma}{2}\left[\frac{\theta}{\theta-1}\right] \;>\; 0 \quad \Leftrightarrow \quad \gamma \;>\; -\,\frac{2\,[\theta-1]}{\theta}. \tag{A35}$$

(A35) implies that $\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=0} > 0$ if $\gamma \in (-\frac{2[\theta-1]}{\theta}, 0)$ when $k^A = k^B = k > V_S\,(2)^{\theta-1}$.

From (A23), $\gamma > -1$ ensures $1 + \gamma p^A > 0$, and so ensures $p^B > 0$. Also, since $\theta > 2$, $\frac{\theta-1}{\theta} > \frac{1}{2}$, which implies $\frac{2[\theta-1]}{\theta} > 1$. Therefore, if $|\gamma| < 1$, then $|\gamma| < \frac{2[\theta-1]}{\theta}$. Consequently, $\gamma > -1$ ensures $\gamma > -\frac{2[\theta-1]}{\theta}$ when $\gamma < 0$. Therefore, $\gamma \in (-1, 0)$ ensures $\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=0} > 0$ and also ensures $p^B > 0$.

To derive conditions that ensure $\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=\widehat{p}^A} < 0$, let $\widehat{p}^A_{\gamma<0}$ denote the value of $\widehat{p}^A$ for given $\gamma < 0$. (A24) implies:

$$\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=\widehat{p}^A_{\gamma<0}} = T_S^A + \gamma\left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}}\left[1+\gamma\,\widehat{p}^A_{\gamma<0}\right]^{\frac{1}{\theta-1}} T_S^A\left[\frac{\theta}{\theta-1}\right] - k^A\left(\widehat{p}^A_{\gamma<0}\right)^{\theta-1}$$

$$< T_S^A - k^A\left(\widehat{p}^A_{\gamma<0}\right)^{\theta-1} \quad \text{since } \gamma \in (-1,0)$$

$$= k^A\left[\frac{T_S^A}{k^A} - \left(\widehat{p}^A_{\gamma<0}\right)^{\theta-1}\right] \leq k^A\left[\frac{V_S}{k^A} - \left(\widehat{p}^A_{\gamma<0}\right)^{\theta-1}\right]$$

$$\Rightarrow \left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=\widehat{p}^A_{\gamma<0}} < 0 \text{ if } \frac{V_S}{k^A} - \left(\widehat{p}^A_{\gamma<0}\right)^{\theta-1} < 0 \quad \Leftrightarrow \quad \widehat{p}^A_{\gamma<0} > \left(\frac{V_S}{k^A}\right)^{\frac{1}{\theta-1}}. \tag{A36}$$

The first equality in (A29) implies that $\widehat{p}^A_{\gamma<0} > \widehat{p}^A_{\gamma=0}$ for a fixed $\widehat{p}^B$. The second equality in (A29) implies that $\widehat{p}^B$ decreases as $\widehat{p}^A$ increases when $\gamma < 0$. Therefore, $\widehat{p}^A_{\gamma<0} > \widehat{p}^A_{\gamma=0}$. Consequently:

$$\widehat{p}^A_{\gamma=0} > \left(\frac{V_S}{k^A}\right)^{\frac{1}{\theta-1}} \Rightarrow \widehat{p}^A_{\gamma<0} > \left(\frac{V_S}{k^A}\right)^{\frac{1}{\theta-1}} \Rightarrow \frac{V_S}{k^A} - \left(\widehat{p}^A_{\gamma<0}\right)^{\theta-1} < 0. \tag{A37}$$

(A29) also implies:

$$\widehat{p}^B_{\gamma=0} = \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}}, \quad \text{and so } \widehat{p}^A_{\gamma=0} = 1 - \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}}. \tag{A38}$$

(A37) and (A38) imply:

$$1 - \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} > \left(\frac{V_S}{k^A}\right)^{\frac{1}{\theta-1}} \Rightarrow \frac{V_S}{k^A} - \left(\widehat{p}^A_{\gamma<0}\right)^{\theta-1} < 0. \tag{A39}$$

$1 - \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} \geq 1 - \left(\frac{V_S}{k^B}\right)^{\frac{1}{\theta-1}}$ since $T_S^B \leq V_S$. Therefore, from (A39):

$$1 - \left(\frac{V_S}{k^B}\right)^{\frac{1}{\theta-1}} > \left(\frac{V_S}{k^A}\right)^{\frac{1}{\theta-1}} \Rightarrow \frac{V_S}{k^A} - \left(\widehat{p}^A_{\gamma<0}\right)^{\theta-1} < 0. \tag{A40}$$

(A40) implies:

$$\left(\frac{V_S}{k^B}\right)^{\frac{1}{\theta-1}} + \left(\frac{V_S}{k^A}\right)^{\frac{1}{\theta-1}} \;<\; 1 \quad\Rightarrow\quad \frac{V_S}{k^A} - \left(\widehat{p}^A_{\gamma<0}\right)^{\theta-1} \;<\; 0. \tag{A41}$$

(A36) and (A41) imply:

$$\left(\frac{V_S}{k^A}\right)^{\frac{1}{\theta-1}} + \left(\frac{V_S}{k^B}\right)^{\frac{1}{\theta-1}} \;<\; 1 \quad\Rightarrow\quad \left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=\widehat{p}^A_{\gamma<0}} \;<\; 0. \tag{A42}$$

Therefore, $\left.\frac{d\pi^A(\cdot)}{dp^A}\right|_{p^A=\widehat{p}^A_{\gamma<0}} < 0$ if $k^i > V_S\,(2)^{\theta-1} \Leftrightarrow \left(\frac{V_S}{k^i}\right)^{\frac{1}{\theta-1}} < \frac{1}{2}$ for $i = A, B$. ■

Finding 4. *Suppose the conditions identified in Finding 3 hold. Further suppose $|\gamma|$ is sufficiently small when $\gamma < 0$. Then agent B's choice of p^B as defined by (A1) with $i = B$ uniquely maximizes his objective function, and agent A's choice of p^A as defined by (A18) uniquely maximizes his objective function.*

Proof. From (A4), $\Pi^B(p^B|p^A)$ is a strictly concave function of p^B. Therefore, the value of p^B identified in (A1) with $i = B$ uniquely maximizes $\Pi^B(\cdot)$ when the conditions identified in Finding 3 hold.

Differentiating (A24) provides:

$$\frac{d^2\pi^A(\cdot)}{d\left(p^A\right)^2} \;=\; \left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} T_S^A \left[\frac{\gamma^2\,\theta}{(\theta-1)^2}\right] \left[1+\gamma\,p^A\right]^{\frac{2-\theta}{\theta-1}} - k^A[\theta-1]\left(p^A\right)^{\theta-2}. \tag{A43}$$

Differentiating (A43) provides:

$$\frac{d^3\pi^A(\cdot)}{d\left(p^A\right)^3} \;=\; -\left(\frac{T_S^B}{k^B}\right)^{\frac{1}{\theta-1}} T_S^A \left[\frac{\gamma^3\,\theta\,(\theta-2)}{(\theta-1)^3}\right] \left[1+\gamma\,p^A\right]^{\frac{3-2\theta}{\theta-1}} - k^A[\theta-1]\,[\theta-2]\left(p^A\right)^{\theta-3}. \tag{A44}$$

(A44) implies that $\frac{d^3\pi^A(\cdot)}{d(p^A)^3} < 0$ when $\gamma > 0$, since $\theta > 2$.

(A24) and (A43) imply that $\pi^A(\cdot)$ is increasing and convex at $p^A = 0$ when $\gamma > 0$. Furthermore, $\pi^A(\cdot)$ is decreasing at $\widehat{p}^A$ when $\gamma > 0$ and the conditions in Finding 3 hold. Therefore, since $\frac{d^3\pi^A(\cdot)}{d(p^A)^3} < 0$ when $\gamma > 0$, the value of $p^A \in (0, \widehat{p}^A)$ defined by (A18) uniquely maximizes agent A's expected profit.

It is also apparent from (A44) that $\frac{d^2\pi^A(\cdot)}{d(p^A)^2}$ is a continuous, decreasing function of p^A when $\gamma = 0$. Therefore, if $\pi^A(\cdot)$ is increasing at $p^A = 0$ and decreasing at $\widehat{p}^A$ when $\gamma = 0$, then there will exist a $\widetilde{\gamma} < 0$ such that for all $\gamma \in [\widetilde{\gamma}, 0)$, the value of $p^A \in (0, \widehat{p}^A)$ at which the expression in (A24) is zero uniquely maximizes agent A's expected profit. Consequently, agent A's choice of p^A as defined by (A18) uniquely maximizes his objective function when the conditions identified in Finding 3 hold and when $\gamma \in [\widetilde{\gamma}, 0)$. ■