Technical Appendix to Accompany

"Employing Simple Cost-Sharing Policies to Motivate the Efficient Implementation of Distributed Energy Resources"

by David P. Brown and David E. M. Sappington

This Technical Appendix has three sections. Section 1 describes how the numerical solutions to [P] and [PNC] are derived. Section 2 explains how the numerical solutions to [PM] are derived. Section 3 derives the analytic characterization of the solutions to [Pm] and [PR] and describes how the numerical solutions to these problems are derived.

1 Numerical Solutions to [P] and [PNC].

We begin by defining two problems, labeled [P1] and [P2].

Problem [P1]

$$\underset{\overline{r},\, r_0,\, s}{\text{Maximize}} \quad - \int_{\underline{\delta}}^{\delta_n} \left\{ \overline{r} + \left(\frac{s}{\delta} [\, \overline{c} - \underline{c}\,] \right)^{\frac{1}{\gamma-1}} [\, s-1\,]\, [\, \overline{c} - \underline{c}\,] \right\} dG(\delta) - [\, 1 - G(\, \delta_n\,)\,]\, r_0$$

subject to $r_0 \ \geq \ c_0$, and

$$\overline{r} - \overline{c} + \left(\frac{1}{\delta_n} \right)^{\frac{1}{\gamma-1}} [\, \overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \, (s)^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma - 1}{\gamma} \right] \ = \ r_0 - c_0 \, .$$

$(\, \delta_{n1}, \overline{r}_1,\, r_{01},\, s_1)$ will denote the solution to [P1].

Problem [P2]

$$\underset{\overline{r},\, r_0,\, s}{\text{Maximize}} \quad - \int_{\underline{\delta}}^{\delta_1} \{\, \underline{c} + \overline{r} - \overline{c} + s\, [\, \overline{c} - \underline{c}\,]\} \, dG(\delta)$$

$$- \int_{\delta_1}^{\delta_n} \left\{ \overline{r} + \left(\frac{s}{\delta} [\, \overline{c} - \underline{c}\,] \right)^{\frac{1}{\gamma-1}} [\, s-1\,]\, [\, \overline{c} - \underline{c}\,] \right\} dG(\delta) - \, [\, 1 - G(\, \delta_n\,)\,]\, r_0$$

subject to $r_0 \ \geq \ c_0$, and

$$\overline{r} - \overline{c} + \left(\frac{1}{\delta_n} \right)^{\frac{1}{\gamma-1}} [\, \overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \, (s)^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma - 1}{\gamma} \right] \ = \ r_0 - c_0 \, ,$$

where $\delta_1 \ = \ s\, [\, \overline{c} - \underline{c}\,]$.

$(\, \delta_{12},\, \delta_{n2}, \overline{r}_2,\, r_{02},\, s_2)$ will denote the solution to [P2].

Observation A1. If $\delta_{n1} \leq \overline{\delta}$ and $s_1 \leq \frac{\delta}{\overline{c}-\underline{c}}$ (so $\frac{s_1}{\delta}[\,\overline{c}-\underline{c}\,] \leq 1$ for all $\delta \in [\,\underline{\delta},\,\overline{\delta}\,]$), then $(\,\delta_{n1}, \overline{r}_1,\, r_{01},\, s_1)$ is the solution to the regulator's problem .

Observation A2. If $\delta_{n1} \leq \overline{\delta}$, $s_1 > \frac{\delta}{\overline{c}-\underline{c}}$, $\delta_{12} > \underline{\delta}$, and $\delta_{n2} \leq \overline{\delta}$, then the solution to [P2] is the solution to the regulator's problem.

Otherwise, the solution to [PNC] is the solution to the regulator's problem. We solve the constrained nonlinear programs [P] and [PNC] using the CONOPT algorithm in GAMS (Ferris and Munson, 2000).

<u>Calculations for the Numerical Solutions to [P] and [PNC]</u>

Consider any $\delta_1, \delta_2 \in [\,\underline{\delta}\,, \overline{\delta}\,]$, with $\delta_2 \geq \delta_1$. When δ is uniformly distributed on $[\,\underline{\delta}, \overline{\delta}\,]$:

$$\int_{\delta_1}^{\delta_2} \delta \; dG(\delta) \;=\; \frac{1}{2}\left[\frac{1}{\overline{\delta}-\underline{\delta}}\right]\left[\,(\delta_2)^2 - (\delta_1)^2\,\right]. \tag{1}$$

<u>When $\gamma = 2$</u>:

$$\int_{\delta_1}^{\delta_2} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} dG(\delta) \;=\; \left[\frac{1}{\overline{\delta}-\underline{\delta}}\right]\left[\,\ln(\,\delta_2\,) - \ln(\,\delta_1\,)\,\right]. \tag{2}$$

<u>When $\gamma \neq 2$</u>:

$$\int_{\delta_1}^{\delta_2} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} dG(\delta) \;=\; \left[\frac{1}{\overline{\delta}-\underline{\delta}}\right] \int_{\delta_1}^{\delta_2} (\delta)^{\frac{1}{1-\gamma}}\, d\delta \;=\; \left[\frac{1}{\overline{\delta}-\underline{\delta}}\right]\left[\frac{\gamma-1}{\gamma-2}\right](\delta)^{\frac{2-\gamma}{1-\gamma}}\Big|_{\delta_1}^{\delta_2}$$

$$= \left[\frac{1}{\overline{\delta}-\underline{\delta}}\right]\left[\frac{\gamma-1}{\gamma-2}\right]\left[\,(\,\delta_2\,)^{\frac{\gamma-2}{\gamma-1}} - (\,\delta_1\,)^{\frac{\gamma-2}{\gamma-1}}\,\right]. \tag{3}$$

<u>Case 1</u>. $\delta_n \leq \overline{\delta}$, so the relevant problem is [P].

Expected procurement cost in this case is:

$$E\{\,\text{Proc Cost}\,\} \;=\; \int_{\underline{\delta}}^{\delta_1} \underline{r}\; dG(\delta) + \int_{\delta_1}^{\delta_n} \{\,p(\delta)\,\underline{r} + [\,1-p(\delta)\,]\,\overline{r}\,\}\, dG(\delta) + \int_{\delta_n}^{\overline{\delta}} r_0\; dG(\delta)$$

$$= \int_{\underline{\delta}}^{\delta_1} \{\,\overline{r} + [\,s-1\,]\,[\,\overline{c}-\underline{c}\,]\,\}\; dG(\delta) + [\,1 - G(\,\delta_n\,)\,]\,c_0$$

$$+ \int_{\delta_1}^{\delta_n} \left\{ \overline{r} + \left(\frac{s}{\delta}[\,\overline{c}-\underline{c}\,]\right)^{\frac{1}{\gamma-1}} [\,s-1\,]\,[\,\overline{c}-\underline{c}\,] \right\} dG(\delta)$$

$$= G(\,\delta_1\,)\,\{\,\overline{r} + [\,s-1\,]\,[\,\overline{c}-\underline{c}\,]\,\} + \overline{r}\,[\,G(\,\delta_n\,) - G(\,\delta_1\,)\,] + [\,1 - G(\,\delta_n\,)\,]\,c_0$$

$$+ \; (s)^{\frac{1}{\gamma-1}} \, [\, s-1 \,] \, [\, \overline{c} - \underline{c} \,]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} dG(\delta) \, . \tag{4}$$

The firm's expected profit is:

$$E\{\pi\} \;=\; \int_{\underline{\delta}}^{\delta_1} \left\{ \overline{r} - \overline{c} + s \, [\, \overline{c} - \underline{c} \,] - \frac{\delta}{\gamma} \right\} dG(\delta)$$

$$+ \int_{\delta_1}^{\delta_n} \left\{ \overline{r} - \overline{c} + \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\, \overline{c} - \underline{c} \,]^{\frac{\gamma}{\gamma-1}} \, (s)^{\frac{\gamma}{\gamma-1}} \, \left[\frac{\gamma - 1}{\gamma} \right] \right\} dG(\delta) + \int_{\delta_n}^{\overline{\delta}} [\, r_0 - c_0 \,] \, dG(\delta)$$

$$= \; [\, \overline{r} - \overline{c} + s \, (\overline{c} - \underline{c}) \,] \, G(\, \delta_1 \,) - \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \delta \; dG(\delta) + [\, G(\, \delta_n \,) - G(\, \delta_1 \,) \,] \, [\, \overline{r} - \overline{c} \,]$$

$$+ \; [\, \overline{c} - \underline{c} \,]^{\frac{\gamma}{\gamma-1}} \; (s)^{\frac{\gamma}{\gamma-1}} \, \left[\frac{\gamma - 1}{\gamma} \right] \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} dG(\delta) + [\, 1 - G(\, \delta_n \,) \,] \, [\, r_0 - c_0 \,] \, . \tag{5}$$

The expected success probability is:

$$E\{\, p(\delta) \,\} \;=\; \int_{\underline{\delta}}^{\delta_1} 1 \; dG(\delta) + \int_{\delta_1}^{\delta_n} \left(\frac{s}{\delta} \, [\, \overline{c} - \underline{c} \,] \right)^{\frac{1}{\gamma-1}} dG(\delta)$$

$$= \; G(\, \delta_1 \,) + (s)^{\frac{1}{\gamma-1}} \, [\, \overline{c} - \underline{c} \,]^{\frac{1}{\gamma-1}} \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} dG(\delta) \, . \tag{6}$$

Expected project cost is:

$$E \, \{\, \text{Cost} \,\} \;=\; \int_{\underline{\delta}}^{\delta_1} \left[\underline{c} + \frac{\delta}{\gamma} \right] dG(\delta) + \int_{\delta_1}^{\delta_n} \left\{ \underline{c} \, p(\delta) + \overline{c} \, [\, 1 - p(\delta) \,] + \frac{\delta}{\gamma} \, [\, p(\delta) \,]^{\gamma} \right\} dG(\delta)$$

$$+ \; \int_{\delta_n}^{\overline{\delta}} c_0 \; dG(\delta)$$

$$= \; \underline{c} \; G(\, \delta_1 \,) + \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \delta \; dG(\delta) + \int_{\delta_1}^{\delta_n} \left\{ \overline{c} - p(\delta) \, [\, \overline{c} - \underline{c} \,] + \frac{\delta}{\gamma} \, [\, p(\delta) \,]^{\gamma} \right\} dG(\delta)$$

$$+ \; [\, 1 - G(\, \delta_n \,) \,] \, c_0$$

$$= \; \underline{c} \; G(\, \delta_1 \,) + \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \delta \; dG(\delta) + \int_{\delta_1}^{\delta_n} \left\{ \overline{c} - \left(\frac{s}{\delta} \, [\, \overline{c} - \underline{c} \,] \right)^{\frac{1}{\gamma-1}} [\, \overline{c} - \underline{c} \,] + \frac{\delta}{\gamma} \left(\frac{s}{\delta} \, [\, \overline{c} - \underline{c} \,] \right)^{\frac{\gamma}{\gamma-1}} \right\} dG(\delta)$$

$$+ \; [\, 1 - G(\, \delta_n \,) \,] \, c_0$$

$$= \; \underline{c} \; G(\, \delta_1 \,) + \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \delta \, dG(\delta) + \int_{\delta_1}^{\delta_n} \left\{ \overline{c} - (s)^{\frac{1}{\gamma-1}} \, [\, \overline{c} - \underline{c} \,]^{\frac{\gamma}{\gamma-1}} \left[1 - \frac{s}{\gamma} \right] \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} \right\} dG(\delta)$$

$$+ \,[\,1 - G(\,\delta_n\,)\,]\,c_0$$

$$= \underline{c}\,G(\,\delta_1\,) + \frac{1}{\gamma}\int_{\underline{\delta}}^{\delta_1} \delta\; dG(\delta) + \overline{c}\,[\,G(\,\delta_n\,) - G(\,\delta_1\,)\,]$$

$$- \;(s)^{\frac{1}{\gamma-1}}\,[\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma - s}{\gamma}\right] \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} dG(\delta) + [\,1 - G(\,\delta_n\,)\,]\,c_0\,. \tag{7}$$

Case 2. $\delta_n > \overline{\delta}$, so the relevant problem is [PNC].

Expected procurement cost in this case is:

$$E\{\,\text{Proc Cost}\,\} \;=\; \int_{\underline{\delta}}^{\delta_1} \underline{r}\,dG(\delta) + \int_{\delta_1}^{\overline{\delta}} \{\,p(\delta)\;\underline{r} + [\,1 - p(\delta)\,]\;\overline{r}\,\}\,dG(\delta)$$

$$= \int_{\underline{\delta}}^{\delta_1} \{\,\overline{r} + [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,\}\;dG(\delta) + \int_{\delta_1}^{\overline{\delta}} \left\{ \overline{r} + \left(\frac{s}{\delta}\,[\,\overline{c} - \underline{c}\,]\right)^{\frac{1}{\gamma-1}} [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] \right\} dG(\delta)$$

$$= G(\,\delta_1\,)\,\{\,\overline{r} + [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,\} + \overline{r}\,[\,1 - G(\,\delta_1\,)\,]$$

$$+ \;(s)^{\frac{1}{\gamma-1}}\,[\,s-1\,]\,[\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \int_{\delta_1}^{\overline{\delta}} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} dG(\delta)\,. \tag{8}$$

The firm's expected profit is:

$$E\{\pi\} \;=\; \int_{\underline{\delta}}^{\delta_1} \left\{ \overline{r} - \overline{c} + s\,[\,\overline{c} - \underline{c}\,] - \frac{\delta}{\gamma} \right\} dG(\delta)$$

$$+ \;\int_{\delta_1}^{\overline{\delta}} \left\{ \overline{r} - \overline{c} + \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} [\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}}\,(s)^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma-1}{\gamma}\right] \right\} dG(\delta)$$

$$= \;[\,\overline{r} - \overline{c} + s\,(\,\overline{c} - \underline{c}\,)\,]\,G(\,\delta_1\,) - \frac{1}{\gamma}\int_{\underline{\delta}}^{\delta_1} \delta\; dG(\delta) + [\,1 - G(\,\delta_1\,)\,]\,[\,\overline{r} - \overline{c}\,]$$

$$+ \;[\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}}\;(s)^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma-1}{\gamma}\right] \int_{\delta_1}^{\overline{\delta}} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} dG(\delta)\,. \tag{9}$$

The expected success probability is:

$$E\{\,p(\delta)\,\} \;=\; \int_{\underline{\delta}}^{\delta_1} 1\;dG(\delta) + \int_{\delta_1}^{\overline{\delta}} \left(\frac{s}{\delta}\,[\,\overline{c} - \underline{c}\,]\right)^{\frac{1}{\gamma-1}} dG(\delta)$$

$$= \;G(\,\delta_1\,) + (s)^{\frac{1}{\gamma-1}}\,[\,\overline{c} - \underline{c}\,]^{\frac{1}{\gamma-1}} \int_{\delta_1}^{\overline{\delta}} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} dG(\delta)\,. \tag{10}$$

Expected project cost is:

$$E\{\,\text{Cost}\,\} \;=\; \int_{\underline{\delta}}^{\delta_1}\left[\,\underline{c}+\frac{\delta}{\gamma}\,\right]dG(\delta)+\int_{\delta_1}^{\overline{\delta}}\left\{\,\underline{c}\,p(\delta)+\overline{c}\,[\,1-p(\delta)\,]+\frac{\delta}{\gamma}\,[\,p(\delta)\,]^{\gamma}\,\right\}dG(\delta)$$

$$= \;\underline{c}\,G(\,\delta_1\,)+\frac{1}{\gamma}\int_{\underline{\delta}}^{\delta_1}\delta\;dG(\delta)+\int_{\delta_1}^{\overline{\delta}}\left\{\,\overline{c}-p(\delta)\,[\,\overline{c}-\underline{c}\,]+\frac{\delta}{\gamma}\,[\,p(\delta)\,]^{\gamma}\right\}dG(\delta)$$

$$= \;\underline{c}\,G(\,\delta_1\,)+\frac{1}{\gamma}\int_{\underline{\delta}}^{\delta_1}\delta\;dG(\delta)+\int_{\delta_1}^{\overline{\delta}}\left\{\,\overline{c}-\left(\frac{s}{\delta}\,[\,\overline{c}-\underline{c}\,]\right)^{\frac{1}{\gamma-1}}[\,\overline{c}-\underline{c}\,]+\frac{\delta}{\gamma}\left(\frac{s}{\delta}\,[\,\overline{c}-\underline{c}\,]\right)^{\frac{\gamma}{\gamma-1}}\right\}dG(\delta)$$

$$= \;\underline{c}\,G(\,\delta_1\,)+\frac{1}{\gamma}\int_{\underline{\delta}}^{\delta_1}\delta\,dG(\delta)+\int_{\delta_1}^{\overline{\delta}}\left\{\,\overline{c}-(s)^{\frac{1}{\gamma-1}}\,[\,\overline{c}-\underline{c}\,]^{\frac{\gamma}{\gamma-1}}\left[\,1-\frac{s}{\gamma}\,\right]\left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}}\right\}dG(\delta)$$

$$= \;\underline{c}\,G(\,\delta_1\,)+\frac{1}{\gamma}\int_{\underline{\delta}}^{\delta_1}\delta\;dG(\delta)+\overline{c}\,[\,1-G(\,\delta_1\,)\,]$$

$$-\;(s)^{\frac{1}{\gamma-1}}\,[\,\overline{c}-\underline{c}\,]^{\frac{\gamma}{\gamma-1}}\left[\,\frac{\gamma-s}{\gamma}\,\right]\int_{\delta_1}^{\overline{\delta}}\left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}}dG(\delta)\,. \qquad (11)$$

2 Numerical Solutions to [PM].

To characterize the solution to [PM] numerically, we:

1. Solve for δ_1 and δ_n.
2. Draw 250,000 values randomly from a uniform distribution on the support $[\,\underline{\delta}\,,\overline{\delta}\,]$.
3. For each δ realization, solve for the optimal procurement policy $(\underline{r}(\delta),\,\overline{r}(\delta),\,s(\delta),\,r_0(\delta))$ and the associated level of procurement cost, profit, project cost, probability of success, and effort cost.
4. Summarize the distribution of values by its minimum, mean, maximum, and standard-deviation.

Observe that for $a \neq 1$:

$$\int [\,2\,\delta-\underline{\delta}\,]^{-a}\;d\,\delta \;=\; \frac{1}{2\,[\,1-\,a\,]}\,[\,2\,\delta-\underline{\delta}\,]^{1-a}\,.$$

Therefore, if $\gamma = 2$, then $\frac{\gamma}{\gamma-1} = 2$, and so for $\delta_x,\ \delta_y \in [\,\underline{\delta}\,,\overline{\delta}\,]$ with $\delta_x \leq \delta_y$:

$$\int_{\delta_x}^{\delta_y}[\,2\,\delta-\underline{\delta}\,]^{-\frac{\gamma}{\gamma-1}}\;d\,\delta \;=\; -\frac{1}{2}\,[\,2\,\delta_y-\underline{\delta}\,]^{-1}+\frac{1}{2}\,[\,2\,\delta_x-\underline{\delta}\,]^{-1}$$

$$=\; \frac{1}{2\,\underline{\delta}-4\,\delta_y}-\frac{1}{2\,\underline{\delta}-4\,\delta_x}\,. \qquad (12)$$

Furthermore, if $\gamma = 3$, then $\frac{\gamma}{\gamma-1} = \frac{3}{2}$, and so $\delta_x,\, \delta_y \in [\,\underline{\delta}\,,\overline{\delta}\,]$ with $\delta_x \leq \delta_y$:

$$\int_{\delta_x}^{\delta_y} [\,2\,\delta - \underline{\delta}\,]^{-\frac{\gamma}{\gamma-1}}\, d\,\delta \;=\; -\frac{1}{2}\,[\,2\,\delta_y - \underline{\delta}\,]^{-\frac{1}{2}} + \frac{1}{2}\,[\,2\,\delta_x - \underline{\delta}\,]^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2\,\delta_x - \underline{\delta}}} - \frac{1}{\sqrt{2\,\delta_y - \underline{\delta}}}\,. \tag{13}$$

Because $G(\cdot)$ is uniform:

$$\delta_1 + \frac{G(\,\delta_1\,)}{g(\,\delta_1\,)} \;=\; \overline{c} - \underline{c} \;\;\Leftrightarrow\;\; \delta_1 + \delta_1 \,-\, \underline{\delta} \;=\; \overline{c} - \underline{c} \;\;\Leftrightarrow\;\; \delta_1 \;=\; \frac{1}{2}\,[\,\overline{c} - \underline{c} + \underline{\delta}\,]\,. \tag{14}$$

Because $\delta + \frac{G(\,\delta\,)}{g(\,\delta\,)} \;=\; 2\,\delta - \,\underline{\delta}$:

$$2\,\delta_1 - \,\underline{\delta} \;=\; \left[\frac{\gamma-1}{\gamma}\right]^{\gamma-1} [\,\overline{c} - c_0\,]^{1-\gamma}\,[\,\overline{c} - \underline{c}\,]^{\gamma}$$

$$\Leftrightarrow\;\; \delta_1 \;=\; \frac{1}{2}\left\{\left[\frac{\gamma-1}{\gamma}\right]^{\gamma-1} [\,\overline{c} - c_0\,]^{1-\gamma}\,[\,\overline{c} - \underline{c}\,]^{\gamma} + \underline{\delta}\right\}. \tag{15}$$

From Conclusion 6, when $\delta < \,\delta_1$:

$$\psi(\,\delta\,) = 1, \quad s(\,\delta\,) \;=\; \frac{\delta_1}{2\,\delta_1 - \underline{\delta}}\,, \;\text{ and }\; \overline{r}(\,\delta\,) \;=\; z_1(\,\delta_1\,)\,,$$

where:

$$z_1(\,\delta_1\,) \;=\; \overline{c} + \frac{1}{\gamma}\,[\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \left[\frac{1}{2\,\underline{\delta} \,-\, 4\,\delta_1} - \frac{1}{2\,\underline{\delta} \,-\, 4\,\delta_1}\right]$$

$$-\left(\frac{1}{\delta_1}\right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma}\right] [\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \left[\frac{\delta_1}{2\,\delta_1 \,-\, \underline{\delta}}\right]^{\frac{\gamma}{\gamma-1}} \quad \text{when } \gamma = 2\,; \;\text{ and}$$

$$z_1(\,\delta_1\,) \;=\; \overline{c} + \frac{1}{\gamma}\,[\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \left[\frac{1}{\sqrt{2\,\delta_1 \,-\, \underline{\delta}}} - \frac{1}{\sqrt{2\,\delta_1 \,-\, \underline{\delta}}}\right]$$

$$-\left(\frac{1}{\delta_1}\right)^{\frac{1}{\gamma-1}} \left[\frac{\gamma-1}{\gamma}\right] [\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \left[\frac{\delta_1}{2\,\delta_1 \,-\, \underline{\delta}}\right]^{\frac{\gamma}{\gamma-1}} \quad \text{when } \gamma = 3\,.$$

When $\delta < \,\delta_1$, $p(\delta) = 1$, procurement cost is $\underline{r}(\delta)$, effort cost is $\frac{\delta}{\gamma}$, total project cost is $\underline{c} + \frac{\delta}{\gamma}$, and profit is:

$$\Pi(\delta) \;=\; \overline{r}(\delta) - \overline{c} + s(\delta)\,[\,\overline{c} - \underline{c}\,] - \frac{\delta}{\gamma}\,.$$

From Conclusion 6, when $\delta \in [\,\delta_1,\, \delta_1\,]$:

$$\psi(\,\delta\,) = 1, \quad s(\,\delta\,) \;=\; \frac{\delta}{2\,\delta - \underline{\delta}}\,, \;\text{ and }\; \overline{r}(\,\delta\,) \;=\; z_1(\,\delta\,)\,,$$

where:

$$z_1(\delta_1) = \bar{c} + \frac{1}{\gamma}\left[\bar{c} - \underline{c}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1}{2\underline{\delta} - 4\delta_1} - \frac{1}{2\underline{\delta} - 4\delta}\right]$$

$$- \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}}\left[\frac{\gamma-1}{\gamma}\right]\left[\bar{c} - \underline{c}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\delta}{2\delta - \underline{\delta}}\right]^{\frac{\gamma}{\gamma-1}} \quad \text{when } \gamma = 2\,; \text{ and}$$

$$z_1(\delta_1) = \bar{c} + \frac{1}{\gamma}\left[\bar{c} - \underline{c}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1}{\sqrt{2\delta - \underline{\delta}}} - \frac{1}{\sqrt{2\delta_1 - \underline{\delta}}}\right]$$

$$- \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}}\left[\frac{\gamma-1}{\gamma}\right]\left[\bar{c} - \underline{c}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\delta}{2\delta - \underline{\delta}}\right]^{\frac{\gamma}{\gamma-1}} \quad \text{when } \gamma = 3\,.$$

When $\delta \in [\delta_1, \delta_1]$, $p(\delta) = \left(\frac{s(\delta)}{\delta}\left[\bar{c} - \underline{c}\right]\right)^{\frac{1}{\gamma-1}}$, procurement cost is $\underline{r}(\delta) + p(\delta)\left[\underline{r}(\delta) - \bar{r}(\delta)\right]$, effort cost is $\frac{\delta}{\gamma}[p(\delta)]^{\gamma}$, total project cost is $\underline{c}\,p(\delta) + \bar{c}\left[1 - p(\delta)\right] + \frac{\delta}{\gamma}[p(\delta)]^{\gamma}$, and profit is:

$$\Pi(\delta) = \bar{r}(\delta) - \bar{c} + \left(s(\delta)\left[\bar{c} - \underline{c}\right]\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}}\left[\frac{\gamma-1}{\gamma}\right].$$

Also:

$$\underline{r}(\delta) = \bar{r}(\delta) + \left[s(\delta) - 1\right]\left[\bar{c} - \underline{c}\right].$$

When $\delta > \delta_n$, the utility undertakes the core project and receives payment $r_0(\delta) = c_0$, so procurement cost and total project cost are c_0.

3 Characterizing the Solutions to [Pm] and [PR].

A. Characterizing the Solution to [Pm].

Paralleling the analysis that underlies the solution to [P], it is readily verified that problem [Pm] is:

$$\underset{r_0,\,s}{\text{Maximize}} \quad - \int_{\underline{\delta}}^{\delta_1}\left\{\underline{c} + m + s\left[\bar{c} - \underline{c}\right]\right\} dG(\delta)$$

$$- \int_{\delta_1}^{\delta_n}\left\{\bar{c} + m + \left(\frac{s}{\delta}\left[\bar{c} - \underline{c}\right]\right)^{\frac{1}{\gamma-1}}\left[s - 1\right]\left[\bar{c} - \underline{c}\right]\right\} dG(\delta) - \left[1 - G(\delta_n)\right] r_0$$

$$\text{subject to} \quad r_0 \geq c_0\,, \quad \text{and} \tag{16}$$

$$\pi(\delta_n) = r_0 - c_0\,. \tag{17}$$

To characterize the solution to [Pm], let $\lambda_0 \geq 0$ and λ_r denote the Lagrange multipliers associated with constraints (16) and (17), respectively. Then the Lagrangian function associated with [Pm] is:

$$\begin{aligned}\mathcal{L} \;=\; & -\int_{\underline{\delta}}^{\delta_1} \{\, \underline{c} + m + s\,[\,\overline{c} - \underline{c}\,]\,\}\; dG(\delta) \\ & - \int_{\delta_1}^{\delta_n} \left\{ \overline{c} + m + \left(\frac{s}{\delta}\,[\,\overline{c} - \underline{c}\,] \right)^{\frac{1}{\gamma-1}} [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] \right\} dG(\delta) \\ & - [\,1 - G(\,\delta_n\,)\,]\, r_0 + \lambda_0\,[\,r_0 - c_0\,] + \lambda_r\,[\,r_0 - c_0 - \pi(\,\delta_n\,)\,]\,. \end{aligned} \tag{18}$$

The necessary conditions for a solution to [Pm] include:

$$\begin{aligned} r_0 : \quad & -\frac{\partial\,\delta_n}{\partial r_0}\; g(\,\delta_n\,) \left[\overline{c} + m + \left(\frac{s}{\delta_n}\,[\,\overline{c} - \underline{c}\,] \right)^{\frac{1}{\gamma-1}} [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] - r_0 \right] - [\,1 - G(\,\delta_n\,)\,] \\ & + \lambda_0 + \lambda_r - \lambda_r\, \frac{\partial \pi(\,\delta_n\,)}{\partial r_0} \;=\; 0\,; \end{aligned} \tag{19}$$

$$\begin{aligned} s : \quad & \frac{\partial\,\delta_1}{\partial s}\; g(\,\delta_1\,) \left\{ \overline{c} + m + \left(\frac{s}{\delta_1}\,[\,\overline{c} - \underline{c}\,] \right)^{\frac{1}{\gamma-1}} [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] - (\,\underline{c} + m + s\,[\,\overline{c} - \underline{c}\,]\,) \right\} \\ & - \frac{\partial\,\delta_n}{\partial s}\; g(\,\delta_n\,) \left[\overline{c} + m + \left(\frac{s}{\delta_n}\,[\,\overline{c} - \underline{c}\,] \right)^{\frac{1}{\gamma-1}} [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] - r_0 \right] \\ & - \lambda_r\, \frac{\partial \pi(\,\delta_n\,)}{\partial s} - [\,\overline{c} - \underline{c}\,]\, G(\,\delta_1\,) \\ & - \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \left[(s)^{\frac{1}{\gamma-1}} + [\,s-1\,] \left(\frac{1}{\gamma-1} \right) (s)^{\frac{2-\gamma}{\gamma-1}} \right] dG(\delta) \;=\; 0\,. \end{aligned} \tag{20}$$

From the definition of δ_n:

$$\begin{aligned} & \overline{c} + m + \left(\frac{s}{\delta_n}\,[\,\overline{c} - \underline{c}\,] \right)^{\frac{1}{\gamma-1}} [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] - r_0 \\ & = \overline{c} + m - r_0 + (\,\delta_n\,)^{-\frac{1}{\gamma-1}}\,(\,s\,)^{\frac{1}{\gamma-1}}\,[\,s-1\,]\,[\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \\ & = \overline{c} + m - r_0 + \left(s\,[\,1-s\,]^{\gamma-1}\,[\,\overline{c} - \underline{c}\,]^{\gamma} \left[\frac{1}{\overline{c} + m - r_0} \right]^{\gamma-1} \right)^{-\frac{1}{\gamma-1}} (\,s\,)^{\frac{1}{\gamma-1}}\,[\,s-1\,]\,[\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \\ & = \overline{c} + m - r_0 - s^{-\frac{1}{\gamma-1}}\,[\,1-s\,]^{-1}\,[\,\overline{c} - \underline{c}\,]^{-\frac{\gamma}{\gamma-1}} \left[\frac{1}{\overline{c} + m - r_0} \right]^{-1} (\,s\,)^{\frac{1}{\gamma-1}}\,[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}} \\ & = \overline{c} + m - r_0 - (\,\overline{c} + m - r_0\,) \;=\; 0\,. \end{aligned} \tag{21}$$

Because $\delta_1\,(s) \;=\; s\,[\,\overline{c} - \underline{c}\,]$:

$$\begin{aligned}
&\overline{c} + m + \left(\frac{s}{\delta_1} [\, \overline{c} - \underline{c} \,] \right)^{\frac{1}{\gamma-1}} [\, s - 1 \,] [\, \overline{c} - \underline{c} \,] - (\, \underline{c} + m + s [\, \overline{c} - \underline{c} \,] \,) \\
&\quad = \ \overline{c} + m + [\, s - 1 \,] [\, \overline{c} - \underline{c} \,] - (\, \underline{c} + m + s [\, \overline{c} - \underline{c} \,] \,) \\
&\quad = \ [\, s - 1 \,] [\, \overline{c} - \underline{c} \,] - (\, \underline{c} - \overline{c} + s [\, \overline{c} - \underline{c} \,] \,) \\
&\quad = \ s [\, \overline{c} - \underline{c} \,] - s [\, \overline{c} - \underline{c} \,] \ = \ 0 \, .
\end{aligned} \tag{22}$$

(21) and (22) imply that (19) and (20) can be written as:

$$\lambda_0 + \lambda_r - \lambda_r \, \frac{\partial \pi(\, \delta_n \,)}{\partial r_0} - [\, 1 - G(\, \delta_n \,) \,] \ = \ 0 \, ; \quad \text{and} \tag{23}$$

$$\begin{aligned}
&- \ \lambda_r \, \frac{\partial \pi(\, \delta_n \,)}{\partial s} - [\, \overline{c} - \underline{c} \,] \, G(\, \delta_1 \,) \\
&\qquad - \ [\, \overline{c} - \underline{c} \,]^{\frac{\gamma}{\gamma-1}} \left[s^{\frac{1}{\gamma-1}} + [\, s - 1 \,] \left(\frac{1}{\gamma - 1} \right) (s)^{\frac{2-\gamma}{\gamma-1}} \right] H(\, \delta_1, \, \delta_n \,) \ = \ 0 \, ,
\end{aligned} \tag{24}$$

where $H(\, \delta_1, \, \delta_n \,) \ \equiv \ \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma - 1}}$.

Observe that:

$$\begin{aligned}
s^{\frac{1}{\gamma-1}} + [\, s - 1 \,] \left[\frac{1}{\gamma - 1} \right] (s)^{\frac{2-\gamma}{\gamma-1}} \ &= \ (s)^{\frac{2-\gamma}{\gamma-1}} \left[s + \left(\frac{s - 1}{\gamma - 1} \right) \right] \\
= \ (s)^{\frac{2-\gamma}{\gamma-1}} \left[\frac{s \, (\gamma - 1) + s - 1}{\gamma - 1} \right] \ &= \ (s)^{\frac{2-\gamma}{\gamma-1}} \left[\frac{s \, \gamma - 1}{\gamma - 1} \right] .
\end{aligned} \tag{25}$$

(25) implies that (24) can be written as:

$$\lambda_r \, \frac{\partial \pi(\, \delta_n \,)}{\partial s} + [\, \overline{c} - \underline{c} \,] \, G(\, \delta_1 \,) \ = \ (s)^{\frac{2-\gamma}{\gamma-1}} \left[\frac{1 - s \, \gamma}{\gamma - 1} \right] [\, \overline{c} - \underline{c} \,]^{\frac{\gamma}{\gamma-1}} \ H(\, \delta_1, \, \delta_n \,) \, . \tag{26}$$

Observe that:

$$\begin{aligned}
\pi(\, \delta_n \,) \ &= \ m + \left(\frac{1}{\delta_n} \right)^{\frac{1}{\gamma-1}} [\, \overline{c} - \underline{c} \,]^{\frac{\gamma}{\gamma-1}} \ (s)^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma - 1}{\gamma} \right] \\
&= \ m + \left(\frac{[\, \overline{c} + m - r_0 \,]^{\gamma-1}}{s \, [\, 1 - s \,]^{\gamma-1} \, [\, \overline{c} - \underline{c} \,]^{\gamma}} \right)^{\frac{1}{\gamma-1}} [\, \overline{c} - \underline{c} \,]^{\frac{\gamma}{\gamma-1}} \ (s)^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma - 1}{\gamma} \right] \\
&= \ m + \left[\frac{\gamma - 1}{\gamma} \right] [\, \overline{c} + m - r_0 \,] \left[\frac{s}{1 - s} \right] .
\end{aligned} \tag{27}$$

From (27):

$$\frac{\partial \pi(\delta_n)}{\partial r_0} = - \left[\frac{\gamma - 1}{\gamma} \right] \left[\frac{s}{1-s} \right] = - \frac{s\,[\,\gamma - 1\,]}{\gamma\,[\,1-s\,]}\,; \text{ and} \tag{28}$$

$$\frac{\partial \pi(\delta_n)}{\partial s} = \left[\frac{\gamma - 1}{\gamma} \right] [\,\overline{c} + m - r_0\,] \left[\frac{1}{1-s} \right]^2 . \tag{29}$$

(28) and (29) imply that (23) and (26) can be written as:

$$\lambda_0 + \lambda_r \left[1 + \frac{s\,(\,\gamma - 1\,)}{\gamma\,(\,1-s\,)} \right] = 1 - G(\,\delta_n\,)\,; \quad \text{and} \tag{30}$$

$$\lambda_r \left[\frac{\gamma - 1}{\gamma} \right] [\,\overline{c} + m - r_0\,] \left[\frac{1}{1-s} \right]^2 + [\,\overline{c} - \underline{c}\,]\,G(\,\delta_1\,)$$
$$= (s)^{\frac{2-\gamma}{\gamma-1}} \left[\frac{1 - s\,\gamma}{\gamma - 1} \right] [\,\overline{c} - \underline{c}\,]^{\frac{\gamma}{\gamma-1}}\; H(\,\delta_1,\, \delta_n\,)\,. \tag{31}$$

To characterize the solution to the nonlinear mixed complementarity program [Pm], we solve for the values of $\{s, r_0,\, \lambda_0,\, \lambda_r,\, \delta_1,\, \delta_n\}$ using the PATH algorithm in GAMS (Ferris and Munson, 2000).

B. Characterizing the Solution to [PR].

To characterize the solution to [PR], define $\underline{w} \equiv \underline{r} - \underline{c} = \overline{r} - \overline{c} + s\,[\,\overline{c} - \underline{c}\,]$ and $\overline{w} \equiv \overline{r} - \overline{c}$.

Lemma 1. *When δ is realized and the firm implements the non-core project, it will implement success probability*

$$p(\delta) = \min \left\{ 1\,,\, \left(\frac{1}{\delta}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,] \right)^{\frac{1}{\gamma - 1}} \right\} . \tag{32}$$

<u>Proof</u>. When the firm operates under the non-core project, it will choose p to

$$\underset{p}{\text{Maximize}} \quad p\,u(\,\underline{w}\,) + [\,1-p\,]\,u(\,\overline{w}\,) - D(p,\delta)$$

When $p \in (0,1)$, the utility's choice of p is determined by:

$$u(\,\underline{w}\,) - u(\,\overline{w}\,) = \delta\,p^{\gamma-1} \;\Rightarrow\; p^{\gamma-1} = \frac{1}{\delta}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]$$
$$\Rightarrow\; p(\delta) = \left(\frac{1}{\delta}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,] \right)^{\frac{1}{\gamma - 1}} . \tag{33}$$

The conclusion follows from (33) because $p(\cdot)$ cannot exceed 1. ■

Lemma 2. *$p(\delta) = 1$ for all $\delta \leq \delta_1(s) \equiv u(\underline{w}) - u(\overline{w})$ when δ is realized and the utility implements the non-core project.*

Proof. (33) implies that $p(\,\delta_1(s)\,) = 1$. Therefore, the conclusion follows from (33) because, holding $\underline{w}$ and s constant:

$$p'(\delta) \stackrel{s}{=} \left[\frac{1}{\gamma - 1}\right] \left(\frac{1}{\delta}\right)^{\frac{2-\gamma}{\gamma-1}} \left(-\frac{1}{\delta^2}\right) < 0\,. \quad \blacksquare$$

Lemma 3. *The firm's expected utility under the non-core project when it implements the success probability specified in (32) is:*

$$U(\delta) = \begin{cases} u(\overline{w}) + \left[\frac{\gamma-1}{\gamma}\right] \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} [\,u(\underline{w}) - u(\overline{w})\,]^{\frac{\gamma}{\gamma-1}} & \text{if } \delta > \delta_1(s), \\ u(\underline{w}) - \frac{\delta}{\gamma} & \text{if } \delta \leq \delta_1(s). \end{cases} \tag{34}$$

Proof. Lemmas 1 and 2 imply that the utility's expected profit under the specified conditions when $\delta > \delta_1(s)$ is:

$$p(\delta)\, u(\underline{w}) + [\,1 - p(\delta)\,]\, u(\overline{w}) - D(p, \delta) = u(\overline{w}) + p(\delta)\,[\,u(\underline{w}) - u(\overline{w})\,] - \frac{\delta}{\gamma}\,[\,p(\delta)\,]^{\gamma}$$

$$= u(\overline{w}) + \left(\frac{1}{\delta}\,[\,u(\underline{w}) - u(\overline{w})\,]\right)^{\frac{1}{\gamma-1}} [\,u(\underline{w}) - u(\overline{w})\,] - \frac{\delta}{\gamma} \left(\frac{1}{\delta}\,[\,u(\underline{w}) - u(\overline{w})\,]\right)^{\frac{\gamma}{\gamma-1}}$$

$$= u(\overline{w}) + \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} [\,u(\underline{w}) - u(\overline{w})\,]^{\frac{\gamma}{\gamma-1}} - \frac{1}{\gamma} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} [\,u(\underline{w}) - u(\overline{w})\,]^{\frac{\gamma}{\gamma-1}}$$

$$= u(\overline{w}) + \left[\frac{\gamma - 1}{\gamma}\right] \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} [\,u(\underline{w}) - u(\overline{w})\,]^{\frac{\gamma}{\gamma-1}}\,.$$

Lemmas 1 and 2 imply that the utility's expected profit under the specified conditions when $\delta \leq \delta_1(s)$ is:

$$p(\delta)\, u(\underline{w}) + [\,1 - p(\delta)\,]\, u(\underline{w}) - D(p, \delta) = u(\underline{w}) - \frac{\delta}{\gamma}\,. \quad \blacksquare$$

Lemma 4. *Suppose $\overline{r} > r_0$. Then the regulator prefers to implement the non-core project rather than the core project for all $\delta \leq \delta_n(s)$, where:*

$$\delta_n(s) \equiv [\,u(\underline{w}) - u(\overline{w})\,] \left(\frac{[\,1 - s\,]\,[\,\overline{c} - \underline{c}\,]}{\overline{r} - r_0}\right)^{\gamma-1}. \tag{35}$$

Furthermore, $\delta_n(s) \geq \delta_1(s)$ if $\underline{r} \leq r_0 < \overline{r}$.

Proof. Lemma 1 implies that the regulator's expected procurement cost when the utility undertakes the non-core project and $p(\delta) \in (0,1)$ is:

$$\overline{r} + p(\delta)\,[\,\underline{r} - \overline{r}\,] \;=\; \overline{r} + \left(\frac{1}{\delta}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]\right)^{\frac{1}{\gamma-1}} [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] \;\leq\; r_0$$

$$\Leftrightarrow \quad \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\,[\,s-1\,]\,[\,\overline{c} - \underline{c}\,] \;\leq\; r_0 - \overline{r}$$

$$\Leftrightarrow \quad \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\,[\,1-s\,]\,[\,\overline{c} - \underline{c}\,] \;\geq\; \overline{r} - r_0$$

$$\Leftrightarrow \quad [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\,\frac{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]}{\overline{r} - r_0} \;\geq\; (\delta)^{\frac{1}{\gamma-1}}$$

$$\Leftrightarrow \quad \delta \;\leq\; [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]\left(\frac{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]}{\overline{r} - r_0}\right)^{\gamma-1} \;\equiv\; \delta_n(s).$$

Therefore, Lemma 2 implies that for $s > 0$ and $\overline{r} > r_0$:

$$\delta_n\,(s) \;\geq\; \delta_1\,(s) \quad \Leftrightarrow \quad [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]\left(\frac{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]}{\overline{r} - r_0}\right)^{\gamma-1} \;\geq\; u(\,\underline{w}\,) - u(\,\overline{w}\,)$$

$$\Leftrightarrow \quad \left(\frac{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]}{\overline{r} - r_0}\right)^{\gamma-1} \;\geq\; 1 \quad \Leftrightarrow \quad [\,1-s\,]\,[\,\overline{c} - \underline{c}\,] \;\geq\; \overline{r} - r_0$$

$$\Leftrightarrow \quad \overline{r} - \underline{r} \;\geq\; \overline{r} - r_0 \quad \Leftrightarrow \quad \underline{r} \;\leq\; r_0\,. \quad \blacksquare$$

Lemma 5. *When $\delta_1(s) > \underline{\delta}$, the regulator's problem, [PR], is:*

$$\underset{\overline{r},\,r_0,\,s}{\mathit{Maximize}} \quad -\int_{\delta_1}^{\delta_n} \left\{ \overline{r} + \left(\frac{1}{\delta}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]\right)^{\frac{1}{\gamma-1}} [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] \right\} dG(\delta)$$

$$-\; G(\,\delta_1\,)\,\{\,\overline{r} + [\,s-1\,]\;[\,\overline{c} - \underline{c}\,]\,\} \;-\; [\,1 - G(\,\delta_n\,)\,]\,r_0$$

$$\textit{subject to:} \quad u(r_0 - c_0) \;\geq\; \overline{U}\,, \quad \textit{and} \tag{36}$$

$$U(\,\delta_n\,) \;=\; u(r_0 - c_0)\;\;. \tag{37}$$

Proof. Lemmas 1 – 4 imply that the regulator's problem is:

$$\underset{\overline{r},\,r_0,\,s}{\text{Minimize}} \quad \int_{\underline{\delta}}^{\delta_1} \underline{r}\; dG(\delta) + \int_{\delta_1}^{\delta_n} \{\,\overline{r} + p(\delta)\,[\,\underline{r} - \overline{r}\,]\,\}\, dG(\delta) + \int_{\delta_n}^{\overline{\delta}} r_0\; dG(\delta)$$

$$\text{subject to:} \quad u(r_0 - c_0) \;\geq\; \overline{U}\,,$$

$$U(\delta) \;\geq\; u(r_0 - c_0) \qquad \text{for all } \delta \;\leq\; \delta_n\,, \text{ and}$$

$$u(r_0 - c_0) \;\geq\; U(\delta) \qquad \text{for all } \delta \;>\; \delta_n\,. \tag{38}$$

It is apparent from (34) that $U'(\delta) < 0$ for all $\delta \in \left[\,\underline{\delta}\,,\overline{\delta}\,\right]$. Therefore, the constraints in (38) imply that $u(r_0 - c_0) \;=\; U(\,\delta_n\,)$ at the solution to the regulator's problem. Consequently, Lemma 1 implies that the regulator's problem is as specified in [PR]. ■

(34) and (35) imply:

$$\begin{aligned}
U(\,\delta_n\,) \;&=\; u(\,\overline{w}\,) + \left[\frac{\gamma-1}{\gamma}\right]\left(\frac{1}{\delta_n}\right)^{\frac{1}{\gamma-1}}\left[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,\right]^{\frac{\gamma}{\gamma-1}} \\
&=\; u(\,\overline{w}\,) + \left[\frac{\gamma-1}{\gamma}\right]\left(\frac{1}{u(\,\underline{w}\,) - u(\,\overline{w}\,)}\left(\frac{\overline{r} - r_0}{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]}\right)^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\left[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,\right]^{\frac{\gamma}{\gamma-1}} \\
&=\; u(\,\overline{w}\,) + \left[\frac{\gamma-1}{\gamma}\right]\left(\frac{1}{u(\,\underline{w}\,) - u(\,\overline{w}\,)}\right)^{\frac{1}{\gamma-1}}\left[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,\right]^{\frac{\gamma}{\gamma-1}}\;\frac{\overline{r} - r_0}{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]} \\
&=\; u(\,\overline{w}\,) + \left[\frac{\gamma-1}{\gamma}\right]\left[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,\right]\frac{\overline{r} - r_0}{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]}\,.
\end{aligned} \tag{39}$$

Because $u(\,\underline{w}\,) - u(\,\overline{w}\,) \;=\; u(\,\overline{r} - \overline{c} + s\,[\,\overline{c} - \underline{c}\,]\,) - u(\,\overline{r} - \overline{c}\,)$, (39) implies:

$$\begin{aligned}
\frac{\partial U(\,\delta_n\,)}{\partial \overline{r}} \;&=\; u'(\,\overline{w}\,) \\
&\quad + \left[\frac{\gamma-1}{\gamma}\right]\frac{1}{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]}\left[\,u(\,\underline{w}\,) - u(\,\overline{w}\,) + (\,\overline{r} - r_0\,)\,(\,u'(\,\underline{w}\,) - u'(\,\overline{w}\,)\,)\,\right];
\end{aligned} \tag{40}$$

$$\frac{\partial U(\,\delta_n\,)}{\partial r_0} \;=\; -\left[\frac{\gamma-1}{\gamma}\right]\left[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,\right]\frac{1}{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]} \;<\; 0\,; \;\; \text{and} \tag{41}$$

$$\frac{\partial U(\,\delta_n\,)}{\partial s} \;=\; \left[\frac{\gamma-1}{\gamma}\right]\left[\frac{\overline{r} - r_0}{\overline{c} - \underline{c}}\right]\left[\frac{(1-s\,)\;u'(\,\underline{w}\,)\,[\,\overline{c} - \underline{c}\,] + u(\,\underline{w}\,) - u(\,\overline{w}\,)}{(1-s)^2}\right] \;\overset{s}{=}\; \overline{r} - r_0. \tag{42}$$

The inequality in (41) holds because $s \in (0,1\,]$ and $\underline{w} \;>\; \overline{w}$.

The following definitions are helpful in characterizing the solution to [PR].

$$\varphi \;\equiv\; [\,\overline{c} - \underline{c}\,]\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{2-\gamma}{\gamma-1}}\left[\,u(\,\underline{w}\,) - u(\,\overline{w}\,) \;-\; \frac{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]}{\gamma-1}\;u'(\,\underline{w}\,)\right]. \tag{43}$$

$$H(\,\delta_1, \delta_n\,) \;\equiv\; \int_{\delta_1}^{\delta_n}\left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} dG(\delta)\,. \tag{44}$$

Let $\lambda_1 \geq 0$ and λ_2 denote the Lagrange multipliers associated with constraints (36) and (37), respectively. Then the Lagrangian function associated with [PR] is:

$$\mathcal{L} = -\overline{r}\,[\,G(\delta_n) - G(\delta_1)\,] - H(\delta_1, \delta_n)\,[\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\underline{w}) - u(\overline{w})\,]^{\frac{1}{\gamma-1}} - [\,1 - G(\delta_n)\,]\; r_0$$

$$- \; G(\delta_1)\,\{\,\overline{r} + [\,s-1\,]\;[\,\overline{c} - \underline{c}\,]\,\} + \lambda_1\left[\,u(r_0 - c_0) - \overline{U}\,\right] + \lambda_2\,[\,u(r_0 - c_0) - U(\delta_n)\,]\,. \quad (45)$$

$\delta_1 = \delta_1(s, \overline{r})$ and $\delta_n = \delta_n(\overline{r}, s, r_0)$ from Lemmas 2 and 4. Therefore, the necessary conditions for a solution to [PR] include:

$$r_0: \; \left[r_0 - \overline{r} - [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\underline{w}) - u(\overline{w})\,]^{\frac{1}{\gamma-1}} \left(\frac{1}{\delta_n} \right)^{\frac{1}{\gamma-1}} \right] \frac{\partial \delta_n}{\partial r_0}\, g(\delta_n)$$

$$- \; [\,1 - G(\delta_n)\,] + \lambda_1\, u'(r_0 - c_0) + \lambda_2\, u'(r_0 - c_0) - \lambda_2\, \frac{\partial U(\delta_n)}{\partial r_0} \;=\; 0\,; \quad (46)$$

$$\overline{r}: \; \left[r_0 - \overline{r} - [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\underline{w}) - u(\overline{w})\,]^{\frac{1}{\gamma-1}} \left(\frac{1}{\delta_n} \right)^{\frac{1}{\gamma-1}} \right] \frac{\partial \delta_n}{\partial \overline{r}}\, g(\delta_n)$$

$$+ \left[\overline{r} - \overline{r} - [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] + [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\underline{w}) - u(\overline{w})\,]^{\frac{1}{\gamma-1}} \left(\frac{1}{\delta_1} \right)^{\frac{1}{\gamma-1}} \right] \frac{\partial \delta_1}{\partial \overline{r}}\, g(\delta_1)$$

$$- \; H(\delta_1, \delta_n)\,[\,s-1\,]\,[\,\overline{c} - \underline{c}\,] \left[\frac{1}{\gamma - 1} \right] [\,u(\underline{w}) - u(\overline{w})\,]^{\frac{2-\gamma}{\gamma-1}}\,[\,u'(\underline{w}) - u'(\overline{w})\,]$$

$$- \; [\,G(\delta_n) - G(\delta_1)\,] - \; G(\delta_1) - \; \lambda_2\, \frac{\partial U(\delta_n)}{\partial \overline{r}} \;=\; 0\,; \quad (47)$$

$$s: \; \left[r_0 - \overline{r} - [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\underline{w}) - u(\overline{w})\,]^{\frac{1}{\gamma-1}} \left(\frac{1}{\delta_n} \right)^{\frac{1}{\gamma-1}} \right] \frac{\partial \delta_n}{\partial s}\, g(\delta_n) - \lambda_2\, \frac{\partial U(\delta_n)}{\partial s}$$

$$+ \left[\overline{r} - \overline{r} - [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] + [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\underline{w}) - u(\overline{w})\,]^{\frac{1}{\gamma-1}} \left(\frac{1}{\delta_1} \right)^{\frac{1}{\gamma-1}} \right] \frac{\partial \delta_1}{\partial s}\, g(\delta_1)$$

$$- \; G(\delta_1)\,[\,\overline{c} - \underline{c}\,] - \; H(\delta_1, \delta_n)\;[\,\overline{c} - \underline{c}\,]\;\{\,[\,u(\underline{w}) - u(\overline{w})\,]^{\frac{1}{\gamma-1}}$$

$$+ \; [\,s-1\,] \left[\frac{1}{\gamma - 1} \right] [\,u(\underline{w}) - u(\overline{w})\,]^{\frac{2-\gamma}{\gamma-1}}\; u'(\underline{w})\,[\,\overline{c} - \underline{c}\,]\,\} \;=\; 0\,. \quad (48)$$

The expression in (47) reflects the fact that:

$$\frac{d}{d\overline{r}}\,[\,u(\underline{w}) - u(\overline{w})\,] \;=\; u'(\overline{r} - \overline{c} + s\,[\,\overline{c} - \underline{c}\,])\, \frac{d\underline{w}}{d\overline{r}} - u'(\overline{r} - \overline{c})\, \frac{d\overline{w}}{d\overline{r}} \;=\; u'(\underline{w}) - u'(\overline{w})\,.$$

The expression in (48) reflects the fact that:

$$\frac{d}{ds}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,] \;=\; u'(\,\overline{r} - \overline{c} + s\,[\,\overline{c} - \underline{c}\,]\,)\,\frac{d\,\underline{w}}{ds} - u'(\,\overline{r} - \overline{c}\,)\,\frac{d\,\overline{w}}{ds} \;=\; u'(\,\underline{w}\,)\,[\,\overline{c} - \underline{c}\,]\,.$$

(35) implies:

$$(\,\delta_n\,)^{\frac{1}{\gamma-1}} \;\equiv\; [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\;\frac{[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]}{\overline{r} - r_0}$$

$$\Rightarrow\;\; \overline{r} - r_0 \;=\; \left(\frac{1}{\delta_n}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]\right)^{\frac{1}{\gamma-1}}\,[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]$$

$$\Rightarrow\;\; r_0 - \overline{r} - [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\left(\frac{1}{\delta_n}\right)^{\frac{1}{\gamma-1}} \;=\; 0\,. \tag{49}$$

Because $\delta_1 \;=\; u(\,\underline{w}\,) - u(\,\overline{w}\,)$:

$$\overline{r} - \overline{r} - [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] + [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\left(\frac{1}{\delta_1}\right)^{\frac{1}{\gamma-1}}$$

$$=\; [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\left(-1 + [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\left(\frac{1}{u(\,\underline{w}\,) - u(\,\overline{w}\,)}\right)^{\frac{1}{\gamma-1}}\right) \;=\; 0\,. \tag{50}$$

(49) and (50) imply that (46) – (48) can be written as:

$$-\,[\,1 - G(\,\delta_n\,)\,] + [\,\lambda_1 + \lambda_2\,]\,u'(r_0 - c_0) - \lambda_2\,\frac{\partial U(\,\delta_n\,)}{\partial r_0} \;=\; 0\,; \tag{51}$$

$$-\;H(\,\delta_1, \delta_n\,)\,[\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\left[\frac{1}{\gamma-1}\right][\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{2-\gamma}{\gamma-1}}\,[\,u'(\,\underline{w}\,) - u'(\,\overline{w}\,)\,]$$

$$-\;G(\,\delta_n\,) -\; \lambda_2\,\frac{\partial U(\,\delta_n\,)}{\partial\,\overline{r}} \;=\; 0\,;\;\; \text{and} \tag{52}$$

$$-\,\lambda_2\,\frac{\partial U(\,\delta_n\,)}{\partial s} -\;\; G(\delta_1)\,[\,\overline{c} - \underline{c}\,] - \,\varphi\,H(\,\delta_1, \delta_n\,) \;=\; 0\,. \tag{53}$$

Suppose $\lambda_2 = 0$. Then (52) implies:

$$-\,H(\,\delta_1, \delta_n\,)\,[\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\left[\frac{1}{\gamma-1}\right][\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{2-\gamma}{\gamma-1}}\,[\,u'(\,\underline{w}\,) - u'(\,\overline{w}\,)\,] = G(\,\delta_n\,)$$

$$\Rightarrow\;\; -\;H(\,\delta_1, \delta_n\,)\,[\,1-s\,]\,[\,\overline{c} - \underline{c}\,]\left[\frac{1}{\gamma-1}\right][\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{2-\gamma}{\gamma-1}}\,[\,u'(\,\overline{w}\,) - u'(\,\underline{w}\,)\,] \;=\; G(\,\delta_n\,)\,. \tag{54}$$

The left-hand side of (54) is non-positive because $s \in (0, 1\,]$ and $\underline{w} > \overline{w}$. The right-hand side of (54) is strictly positive because $\delta_n > \underline{\delta}$, by assumption. This contradiction ensures $\lambda_2 \neq 0$. Therefore, $U(\,\delta_n\,) \;=\; u(r_0 - c_0)$, from (37).

(53) implies:

$$\lambda_2 \;=\; -\,\frac{\varphi\, H(\,\delta_1, \delta_n\,) + G(\,\delta_1\,)\,[\,\overline{c} - \underline{c}\,]}{\frac{\partial\, U(\,\delta_n\,)}{\partial\, s}}\,. \tag{55}$$

Problem [PNC-R] where the Risk Averse Firm Always Implements the Non-Core Project

Lemma 6. *The regulator's problem, [PNC-R], is:*

$$\underset{\overline{r},\, s}{Maximize} \quad -\int_{\delta_1}^{\overline{\delta}} \left\{ \overline{r} + \left(\frac{1}{\delta}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,] \right)^{\frac{1}{\gamma-1}} [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] \right\} dG(\delta)$$

$$-\; G(\,\delta_1\,)\{\,\overline{r} + [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,\}$$

$$\textit{subject to} \quad U(\,\overline{\delta}\,) \;\geq\; \overline{U}\,. \tag{56}$$

Proof. Lemmas 1 – 3 imply that under the specified conditions, the regulator's problem is:

$$\underset{\overline{r},\, s}{\text{Minimize}} \quad \int_{\underline{\delta}}^{\delta_1} \underline{r}\; dG(\delta) + \int_{\delta_1}^{\overline{\delta}} \{\,\overline{r} + p(\delta)\,[\,\underline{r} - \overline{r}\,]\,\}\; dG(\delta)$$

$$\text{subject to} \quad U(\,\delta\,) \;\geq\; \overline{U} \;\text{ for all }\; \delta \in \left[\,\underline{\delta}, \overline{\delta}\,\right]. \tag{57}$$

The conclusion follows from Lemma 1 because $U'(\delta) < 0$, from (34). ■

Let $\lambda \geq 0$ denote the Lagrange multiplier associated with constraint (56). Define:

$$H(\,\delta_1\,) \;=\; \int_{\delta_1}^{\overline{\delta}} \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} dG(\delta) \tag{58}$$

where $\delta_1 = \delta_1(s, \overline{r})$. Then the Lagrangian function associated with [PNC-R] is:

$$\mathcal{L} \;=\; -\,G(\,\delta_1\,)\{\,\overline{r} + [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,\} - [\,1 - G(\,\delta_1\,)\,]\;\overline{r}$$

$$-\; H(\,\delta_1\,)\;[\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}} + \lambda\left[\,U(\,\overline{\delta}\,) - \overline{U}\,\right]. \tag{59}$$

The necessary conditions for a solution to [PNC-R] include:

$$\overline{r}: \quad \left\{ \overline{r} - \overline{r} - [\,s-1\,]\,[\,\overline{c} - \underline{c}\,] + [\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}} \left(\frac{1}{\delta_1} \right)^{\frac{1}{\gamma-1}} \right\} g(\,\delta_1\,)\,\frac{\partial\,\delta_1}{\partial\,\overline{r}}$$

$$+\; \lambda\,\frac{\partial U(\,\overline{\delta}\,)}{\partial\,\overline{r}} - G(\,\delta_1\,) - [\,1 - G(\,\delta_1\,)\,]$$

$$-\; H(\,\delta_1\,)\;[\,s-1\,]\,[\,\overline{c} - \underline{c}\,]\left[\frac{1}{\gamma-1} \right][\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{2-\gamma}{\gamma-1}}\,[\,u'(\,\underline{w}\,) - u'(\,\overline{w}\,)\,] \;=\; 0\,; \tag{60}$$

$$s: \quad \left\{ \overline{r} - \overline{r} - [\, s-1\,]\,[\,\overline{c} - \underline{c}\,] + [\, s-1\,]\,[\,\overline{c} - \underline{c}\,]\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}} \left(\frac{1}{\delta_1} \right)^{\frac{1}{\gamma-1}} \right\} g(\,\delta_1\,)\, \frac{\partial\, \delta_1}{\partial s}$$

$$+ \; \lambda\, \frac{\partial U(\,\overline{\delta}\,)}{\partial s} - G(\,\delta_1\,)\,[\,\overline{c} - \underline{c}\,] - \; H(\delta_1)\,[\,\overline{c} - \underline{c}\,] \; \{\, [\, u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}$$

$$+ \;\; [\, s-1\,] \left[\frac{1}{\gamma - 1} \right] [\, u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{2-\gamma}{\gamma-1}} \; u'(\,\underline{w}\,)\,[\,\overline{c} - \underline{c}\,]\,\} \;=\; 0\,. \quad (61)$$

(50) implies that (60) and (61) can be written as:

$$\lambda\, \frac{\partial U(\,\overline{\delta}\,)}{\partial \overline{r}} - 1$$

$$- \, H(\,\delta_1\,)\; [\, s-1\,]\,[\,\overline{c} - \underline{c}\,] \left[\frac{1}{\gamma - 1} \right] [\, u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{2-\gamma}{\gamma-1}} \,[\, u'(\,\underline{w}\,) - u'(\,\overline{w}\,)\,] \;=\; 0\,; \qquad (62)$$

$$\lambda\, \frac{\partial U(\,\overline{\delta}\,)}{\partial s} - G(\,\delta_1\,)\,[\,\overline{c} - \underline{c}\,] - \; H(\,\delta_1\,)\,[\,\overline{c} - \underline{c}\,] \; \{\, [\, u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}$$

$$+ \;\; [\, s-1\,] \left[\frac{1}{\gamma - 1} \right] [\, u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{2-\gamma}{\gamma-1}} \; u'(\,\underline{w}\,)\,[\,\overline{c} - \underline{c}\,]\,\} \;=\; 0\,. \qquad (63)$$

(62) implies:

$$\lambda\, \frac{\partial U(\,\overline{\delta}\,)}{\partial \overline{r}} \;=\; 1 + H(\,\delta_1\,)\,[\, s-1\,]\,[\,\overline{c} - \underline{c}\,] \left[\frac{1}{\gamma - 1} \right] [\, u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{2-\gamma}{\gamma-1}} \,[\, u'(\,\underline{w}\,) - u'(\,\overline{w}\,)\,]$$

$$=\; 1 + H(\,\delta_1\,)\,[\, 1-s\,]\,[\,\overline{c} - \underline{c}\,] \left[\frac{1}{\gamma - 1} \right] [\, u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{2-\gamma}{\gamma-1}} \,[\, u'(\,\overline{w}\,) - u'(\,\underline{w}\,)\,] \;>\; 0\,. \quad (64)$$

The inequality in (64) holds because $s \in (0, 1\,]$ and $\underline{w} > \overline{w}$. (64) implies that $\lambda > 0$. Therefore, $U(\,\overline{\delta}\,) = \overline{U}$, from (56).

(63) implies:

$$\lambda\, \frac{\partial U(\,\overline{\delta}\,)}{\partial s} - \; H(\,\delta_1\,)\,\varphi - G(\,\delta_1\,)\,[\,\overline{c} - \underline{c}\,] \;=\; 0 \;\;\Rightarrow\;\; \lambda \;=\; \frac{H(\,\delta_1\,)\,\varphi + G(\,\delta_1\,)\,[\,\overline{c} - \underline{c}\,]}{\frac{\partial U(\,\overline{\delta}\,)}{\partial s}}\,. \quad (65)$$

Deriving the Numerical Solution to [PR]

(1), (2), and (3), respectively, provide the values of $\int_{\delta_1}^{\delta_2} \delta \; dG(\delta)$, $\int_{\delta_1}^{\delta_2} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} dG(\delta)$, and $\int_{\delta_1}^{\delta_2} \left(\frac{1}{\delta}\right)^{\frac{1}{\gamma-1}} dG(\delta)$ when δ is uniformly distributed on $\left[\, \underline{\delta}, \overline{\delta} \,\right]$.

Suppose the firm's utility function for wealth is:

$$u(w) \;=\; w^{\alpha} \;\;\Rightarrow\;\; u'(w) \;=\; \alpha\, w^{\alpha - 1} \;>\; 0 \;\;\Rightarrow\;\; u''(w) \;=\; \alpha\,[\,\alpha - 1\,]\, w^{\alpha - 2} \;<\; 0\,, \quad (66)$$

where $\alpha \in (0,1)$ reflects the degree of risk-aversion.

For this utility function:

$$u(\underline{w}) - u(\overline{w}) \;=\; \underline{w}^{\alpha} - \overline{w}^{\,\alpha}\,, \text{ and} \tag{67}$$

$$u'(\underline{w}) - u'(\overline{w}) \;=\; \alpha\,\underline{w}^{\alpha-1} - \alpha\,\overline{w}^{\,\alpha-1}\,. \tag{68}$$

<u>The Solution to [PR]</u>

(67) and Lemma 2 imply:

$$\delta_1(s) \;=\; \underline{w}^{\alpha} - \overline{w}^{\,\alpha}\,. \tag{69}$$

(35) and (67) imply:

$$\delta_n(s) \;=\; [\,\underline{w}^{\alpha} - \overline{w}^{\,\alpha}\,]\left(\frac{[\,1-s\,]\,[\,\overline{c}-\underline{c}\,]}{\overline{r}-r_0}\right)^{\gamma-1}. \tag{70}$$

(36) and (66) require:

$$[\,r_0 - c_0\,]^{\alpha} \;\geq\; \overline{U} \quad \text{and} \quad \lambda_1\left[\,(r_0-c_0)^{\alpha} - \overline{U}\,\right] \;=\; 0\,. \tag{71}$$

(39), (66), and (67) imply that (37) can be written as:

$$\overline{w}^{\,\alpha} + \left[\frac{\gamma-1}{\gamma}\right][\,\underline{w}^{\alpha} - \overline{w}^{\,\alpha}\,]\,\frac{\overline{r}-r_0}{[\,1-s\,]\,[\,\overline{c}-\underline{c}\,]} \;=\; [\,r_0 - c_0\,]^{\alpha}\,. \tag{72}$$

(41), (66), and (67) imply that (51) can be written as:

$$-\left[\frac{\overline{\delta}-\delta_n}{\overline{\delta}-\underline{\delta}}\right] + [\,\lambda_1 + \lambda_2\,]\;\alpha\,[\,r_0 - c_0\,]^{\alpha-1} + \lambda_2\left[\frac{\gamma-1}{\gamma}\right][\,\underline{w}^{\alpha} - \overline{w}^{\,\alpha}\,]\,\frac{1}{[\,1-s\,]\,[\,\overline{c}-\underline{c}\,]} \;=\; 0\,. \tag{73}$$

(40), (66), (67), and (68) imply that (52) can be written as:

$$\begin{aligned} &- H(\delta_1,\delta_n)\,[\,s-1\,]\,[\,\overline{c}-\underline{c}\,]\left[\frac{1}{\gamma-1}\right][\,\underline{w}^{\alpha} - \overline{w}^{\,\alpha}\,]^{\frac{2-\gamma}{\gamma-1}}\left[\,\alpha\,\underline{w}^{\alpha-1} - \alpha\,\overline{w}^{\,\alpha-1}\,\right] \\ &- \lambda_2\left[\alpha\,\overline{w}^{\,\alpha-1} + \left[\frac{\gamma-1}{\gamma}\right]\frac{1}{[\,1-s\,]\,[\,\overline{c}-\underline{c}\,]}\left[\,\underline{w}^{\alpha} - \overline{w}^{\,\alpha} + (\,\overline{r}-r_0\,)\left(\,\alpha\,\underline{w}^{\alpha-1} - \alpha\,\overline{w}^{\,\alpha-1}\,\right)\right]\right] \\ &- \left(\frac{\delta_n - \underline{\delta}}{\overline{\delta}-\underline{\delta}}\right) \;=\; 0\,. \end{aligned} \tag{74}$$

(42), (66), and (67) imply that (53) can be written as:

$$-\left[\frac{\delta_1 - \underline{\delta}}{\overline{\delta}-\underline{\delta}}\right][\,\overline{c}-\underline{c}\,] - \varphi\,H(\delta_1,\delta_n)$$

$$= \lambda_2 \left[\frac{\gamma - 1}{\gamma} \right] \left[\frac{\overline{r} - r_0}{\overline{c} - \underline{c}} \right] \left[\frac{[\, 1 - s \,]\, \alpha\, \underline{w}^{\alpha - 1}\, [\, \overline{c} - \underline{c}\,] + \underline{w}^{\alpha} - \overline{w}^{\alpha}}{[\, 1 - s\,]^2} \right]. \tag{75}$$

(66) and (67) imply that (43) can be written as:

$$\varphi \equiv [\, \overline{c} - \underline{c}\,]\, [\, \underline{w}^{\alpha} - \overline{w}^{\alpha}\,]^{\frac{2-\gamma}{\gamma-1}} \left[\underline{w}^{\alpha} - \overline{w}^{\alpha} - \frac{[\, 1 - s\,]\, [\, \overline{c} - \underline{c}\,]}{\gamma - 1}\, \alpha\, \underline{w}^{\alpha - 1} \right]. \tag{76}$$

To characterize the solution to [P-R], we need to characterize the values of the variables $\{\, \delta_1, \, \delta_n, \, \lambda_1, \, \lambda_2, \, r_0, \, \overline{r}, \, s\, \}$. Expressions for these variables are defined in (69) – (75). $H(\cdot)$ and φ are defined in (44) and (76), respectively. We can solve this nonlinear mixed complementarity program using the PATH algorithm in GAMS.

Lemmas 1 – 5 imply that expected procurement cost is:

$$\begin{aligned}
E\{\, \text{Proc Cost}\, \} &= \int_{\underline{\delta}}^{\delta_1} \underline{r}\; dG(\delta) + \int_{\delta_1}^{\delta_n} \{\, p(\delta)\, \underline{r} + [\, 1 - p(\delta)\,]\, \overline{r}\, \}\, dG(\delta) + \int_{\delta_n}^{\overline{\delta}} r_0\; dG(\delta) \\
&= \{\, \overline{r} + [\, s - 1\,]\, [\, \overline{c} - \underline{c}\,]\, \}\, G(\, \delta_1\,) + \int_{\delta_1}^{\delta_n} \{\, \overline{r} + p(\delta)\, [\, \underline{r} - \overline{r}\,]\, \}\, dG(\delta) + [\, 1 - G(\, \delta_n\,)\,]\, r_0 \\
&= \{\, \overline{r} + [\, s - 1\,]\, [\, \overline{c} - \underline{c}\,]\, \}\, G(\, \delta_1\,) + [\, 1 - G(\, \delta_n\,)\,]\; r_0 \\
&\quad + \int_{\delta_1}^{\delta_n} \left\{ \overline{r} + \left(\frac{1}{\delta}\, [\, u(\, \underline{w}\,) - u(\, \overline{w}\,)\,] \right)^{\frac{1}{\gamma-1}} [\, s - 1\,]\, [\, \overline{c} - \underline{c}\,] \right\} dG(\delta) \\
&= \{\, \overline{r} + [\, s - 1\,]\, [\, \overline{c} - \underline{c}\,]\, \}\, G(\, \delta_1\,) + [1 - G(\, \delta_n\,)]\, r_0 \\
&\quad + \overline{r}\, [\, G(\, \delta_n\,) - G(\, \delta_1\,)\,] + [\, s - 1\,]\, [\, \overline{c} - \underline{c}\,]\, [\, u(\, \underline{w}\,) - u(\, \overline{w}\,)\,]^{\frac{1}{\gamma-1}}\; H(\, \delta_1, \delta_n\,)\, .
\end{aligned}$$

(34) and Lemmas 1 – 5 imply:

$$\begin{aligned}
E\{\pi\} &= \int_{\underline{\delta}}^{\delta_1} \left[u(\, \underline{w}\,) - \frac{\delta}{\gamma} \right] dG(\delta) + \int_{\delta_n}^{\overline{\delta}} u(r_0 - c_0)\, dG(\delta) \\
&\quad + \int_{\delta_1}^{\delta_n} \left\{ u(\, \overline{w}\,) + \left[\frac{\gamma - 1}{\gamma} \right] \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma-1}} [\, u(\, \underline{w}\,) - u(\, \overline{w}\,)\,]^{\frac{\gamma}{\gamma-1}} \right\} dG(\delta) \\
&= u(\, \underline{w}\,)\, G(\, \delta_1\,) - \frac{1}{\gamma} \int_{\delta_1}^{\delta_n} \delta\; dG(\delta) + u(r_0 - c_0)\, [\, 1 - G(\, \delta_n\,)\,] \\
&\quad + u(\, \overline{w}\,)\, [\, G(\, \delta_n\,) - G(\, \delta_1\,)\,] + \left[\frac{\gamma - 1}{\gamma} \right] [\, u(\, \underline{w}\,) - u(\, \overline{w}\,)\,]^{\frac{\gamma}{\gamma-1}}\; H(\, \delta_1, \delta_n\,)\, .
\end{aligned}$$

Lemma 1 implies:

$$E\{\,p(\delta)\,\} \;=\; \int_{\underline{\delta}}^{\delta_1} 1\; dG(\delta) + \int_{\delta_1}^{\delta_n} \left(\frac{1}{\delta}\,[\,u(\underline{w}) - u(\overline{w})\,] \right)^{\frac{1}{\gamma-1}} dG(\delta)$$

$$=\; G(\,\delta_1\,) + [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\; H(\,\delta_1, \delta_n\,).$$

Lemmas 1 – 5 imply:

$$E\,\{\,\text{Cost}\,\} \;=\; \int_{\underline{\delta}}^{\delta_1} \left[\underline{c} + \frac{\delta}{\gamma} \right] dG(\delta) + \int_{\delta_1}^{\delta_n} \left\{ \underline{c}\; p(\delta) + \overline{c}\;[\,1 - p(\delta)\,] + \frac{\delta}{\gamma}\,[\,p(\delta)\,]^{\gamma} \right\} dG(\delta)$$

$$+\; \int_{\delta_n}^{\overline{\delta}} c_0\; dG(\delta)$$

$$=\; \underline{c}\; G(\,\delta_1\,) + \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \delta\; dG(\delta) +\; [\,1 - G(\,\delta_n\,)\,]\, c_0$$

$$+ \int_{\delta_1}^{\delta_n} \left\{ \overline{c} - \left(\frac{1}{\delta}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,] \right)^{\frac{1}{\gamma-1}} [\,\overline{c} - \underline{c}\,] + \frac{\delta}{\gamma} \left(\frac{1}{\delta}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,] \right)^{\frac{\gamma}{\gamma-1}} \right\} dG(\delta)$$

$$=\; \underline{c}\; G(\,\delta_1\,) + \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \delta\; dG(\delta) +\; [\,1 - G(\,\delta_n\,)\,]\, c_0 + \overline{c}\,[\,G(\,\delta_n\,) - G(\,\delta_1\,)\,]$$

$$-\,[\,\overline{c} - \underline{c}\,]\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\; H(\,\delta_1, \delta_n\,) + \frac{1}{\gamma}\,[\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{\gamma}{\gamma-1}}\; H(\,\delta_1, \delta_n\,)\,.$$

The Solution to [PNC-R]

Because $\lambda > 0$, (34), (66), and (67) imply that (56) can be written as:

$$U(\,\overline{\delta}\,) \;=\; u(\,\overline{w}\,) + \left[\frac{\gamma - 1}{\gamma} \right] \left(\frac{1}{\overline{\overline{\delta}}} \right)^{\frac{1}{\gamma-1}} [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{\gamma}{\gamma-1}} \;=\; \overline{U}$$

$$\Leftrightarrow\;\; \overline{w}^{\alpha} + \left[\frac{\gamma - 1}{\gamma} \right] \left(\frac{1}{\overline{\overline{\delta}}} \right)^{\frac{1}{\gamma-1}} [\,\underline{w}^{\alpha} - \overline{w}^{\,\alpha}\,]^{\frac{\gamma}{\gamma-1}} \;=\; \overline{U}. \tag{77}$$

$$\Rightarrow\;\; \frac{\partial U(\,\overline{\delta}\,)}{\partial\,\overline{r}} \;=\; u'(\overline{w}) + \left(\frac{1}{\overline{\overline{\delta}}} \right)^{\frac{1}{\gamma-1}} [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\,[\,u'(\,\underline{w}\,) - u'(\,\overline{w}\,)\,]\,; \tag{78}$$

$$\frac{\partial U(\,\overline{\delta}\,)}{\partial s} \;=\; \left(\frac{1}{\overline{\overline{\delta}}} \right)^{\frac{1}{\gamma-1}} [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\; u'(\,\underline{w}\,)[\,\overline{c} - \underline{c}\,]. \tag{79}$$

(67), (68), and (78) imply that (62) can be written as:

$$\lambda \left[u'(\,\overline{w}\,) + \left(\frac{1}{\overline{\overline{\delta}}} \right)^{\frac{1}{\gamma-1}} [\,u(\,\underline{w}\,) - u(\,\overline{w}\,)\,]^{\frac{1}{\gamma-1}}\,[\,u'(\,\underline{w}\,) - u'(\,\overline{w}\,)\,] \right] - 1$$

$$- H(\delta_1)\ [s-1][\overline{c}-\underline{c}]\left[\frac{1}{\gamma-1}\right][u(\underline{w})-u(\overline{w})]^{\frac{2-\gamma}{\gamma-1}}[u'(\underline{w})-u'(\overline{w})] \;=\; 0$$

$$\Leftrightarrow\ \lambda\left[\alpha\,\overline{w}^{\alpha-1}+\left(\frac{1}{\overline{\overline{\delta}}}\right)^{\frac{1}{\gamma-1}}[\underline{w}^{\alpha}-\overline{w}^{\alpha}]^{\frac{1}{\gamma-1}}\left[\alpha\,\underline{w}^{\alpha-1}-\alpha\,\overline{w}^{\alpha-1}\right]\right]-1$$

$$- \ H(\delta_1)[s-1][\overline{c}-\underline{c}]\left[\frac{1}{\gamma-1}\right][\underline{w}^{\alpha}-\overline{w}^{\alpha}]^{\frac{2-\gamma}{\gamma-1}}\left[\alpha\,\underline{w}^{\alpha-1}-\alpha\,\overline{w}^{\alpha-1}\right] \;=\; 0\,. \qquad (80)$$

(66) – (68) and (79) imply that (63) can be written as:

$$\lambda\left[\left(\frac{1}{\overline{\overline{\delta}}}\right)^{\frac{1}{\gamma-1}}[u(\underline{w})-u(\overline{w})]^{\frac{1}{\gamma-1}}\ u'(\underline{w})[\overline{c}-\underline{c}]\right]-\left[\frac{\delta_1-\underline{\delta}}{\overline{\delta}-\underline{\delta}}\right][\overline{c}-\underline{c}]$$

$$- H(\delta_1)[\overline{c}-\underline{c}]\ \{[u(\underline{w})-u(\overline{w})]^{\frac{1}{\gamma-1}}$$

$$+\ [s-1]\left[\frac{1}{\gamma-1}\right][u(\underline{w})-u(\overline{w})]^{\frac{2-\gamma}{\gamma-1}}\ u'(\underline{w})[\overline{c}-\underline{c}]\} \;=\; 0$$

$$\Leftrightarrow\ \lambda\left[\left(\frac{1}{\overline{\overline{\delta}}}\right)^{\frac{1}{\gamma-1}}[\underline{w}^{\alpha}-\overline{w}^{\alpha}]^{\frac{1}{\gamma-1}}\ \alpha\,\underline{w}^{\alpha-1}[\overline{c}-\underline{c}]\right]-\left[\frac{\delta_1-\underline{\delta}}{\overline{\delta}-\underline{\delta}}\right][\overline{c}-\underline{c}]$$

$$- H(\delta_1)[\overline{c}-\underline{c}]\ \{[\underline{w}^{\alpha}-\overline{w}^{\alpha}]^{\frac{1}{\gamma-1}}$$

$$+\ [s-1]\left[\frac{1}{\gamma-1}\right][\underline{w}^{\alpha}-\overline{w}^{\alpha}]^{\frac{2-\gamma}{\gamma-1}}\ \alpha\,\underline{w}^{\alpha-1}[\overline{c}-\underline{c}]\} \;=\; 0\,. \qquad (81)$$

To characterize the solution to [PNC-R], we need to specify the values of $\{\delta_1, \lambda, \overline{r}, s\}$. These variables are specified in the conditions (69), (77), (80), and (81). The variable $H(\delta_1)$ is defined in (58). We can solve for this nonlinear mixed complementarity program using the PATH algorithm in GAMS.

Lemmas 1 – 3 and 6 imply:

$$E\{\text{Proc Cost}\} \;=\; \int_{\underline{\delta}}^{\delta_1}\underline{r}\,dG(\delta)+\int_{\delta_1}^{\overline{\delta}}\{p(\delta)\,\underline{r}+[1-p(\delta)]\,\overline{r}\}\,dG(\delta)$$

$$=\ \{\overline{r}+[s-1][\overline{c}-\underline{c}]\}\,G(\delta_1)+\int_{\delta_1}^{\overline{\delta}}\{\overline{r}+p(\delta)[\underline{r}-\overline{r}]\}\,dG(\delta)$$

$$=\ \{\overline{r}+[s-1][\overline{c}-\underline{c}]\}\,G(\delta_1)$$

$$+\int_{\delta_1}^{\overline{\delta}}\left\{\overline{r}+\left(\frac{1}{\delta}[u(\underline{w})-u(\overline{w})]\right)^{\frac{1}{\gamma-1}}[s-1][\overline{c}-\underline{c}]\right\}dG(\delta)$$

$$=\ \{\overline{r}+[s-1][\overline{c}-\underline{c}]\}\,G(\delta_1)$$

$$+ \, \overline{r} \, [\, 1 - G(\, \delta_1 \,) \,] + [\, s - 1 \,] \, [\, \overline{c} - \underline{c} \,] \, [\, u(\, \underline{w} \,) - u(\, \overline{w} \,) \,]^{\frac{1}{\gamma - 1}} \; H(\, \delta_1 \,) \, .$$

(34) and Lemmas 1 – 3 imply:

$$\begin{aligned} E\{\pi\} \;&=\; \int_{\underline{\delta}}^{\delta_1} \left[u(\, \underline{w} \,) - \frac{\delta}{\gamma} \right] dG(\delta) \\ &\quad + \int_{\delta_1}^{\overline{\delta}} \left\{ u(\, \overline{w} \,) + \left[\frac{\gamma - 1}{\gamma} \right] \left(\frac{1}{\delta} \right)^{\frac{1}{\gamma - 1}} [\, u(\, \underline{w} \,) - u(\, \overline{w} \,) \,]^{\frac{\gamma}{\gamma - 1}} \right\} dG(\delta) \\ &= \; u(\, \underline{w} \,) \, G(\, \delta_1 \,) - \frac{1}{\gamma} \int_{\delta_1}^{\overline{\delta}} \delta \, dG(\delta) \\ &\quad + \; u(\, \overline{w} \,) \, [\, 1 - G(\, \delta_1 \,) \,] + \left[\frac{\gamma - 1}{\gamma} \right] [\, u(\, \underline{w} \,) - u(\, \overline{w} \,) \,]^{\frac{\gamma}{\gamma - 1}} \; H(\, \delta_1 \,). \end{aligned}$$

Lemma 1 implies:

$$\begin{aligned} E\{\, p(\delta) \,\} \;&=\; \int_{\underline{\delta}}^{\delta_1} 1 \; dG(\delta) + \int_{\delta_1}^{\overline{\delta}} \left(\frac{1}{\delta} \, [\, u(\underline{w}) - u(\overline{w}) \,] \right)^{\frac{1}{\gamma - 1}} dG(\delta) \\ &= \; G(\, \delta_1 \,) + [\, u(\, \underline{w} \,) - u(\, \overline{w} \,) \,]^{\frac{1}{\gamma - 1}} \; H(\, \delta_1 \,) \, . \end{aligned}$$

Lemmas 1 – 3 imply:

$$E\,\{\, \text{Cost} \,\} \;=\; \int_{\underline{\delta}}^{\delta_1} \left[\underline{c} + \frac{\delta}{\gamma} \right] dG(\delta) + \int_{\delta_1}^{\overline{\delta}} \left\{ \underline{c} \; p(\delta) + \overline{c} \; [\, 1 - p(\delta) \,] + \frac{\delta}{\gamma} \, [\, p(\delta) \,]^{\gamma} \right\} dG(\delta)$$

$$\begin{aligned} &= \; \underline{c} \; G(\, \delta_1 \,) + \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \delta \; dG(\delta) \\ &\quad + \int_{\delta_1}^{\overline{\delta}} \left\{ \overline{c} - \left(\frac{1}{\delta} \, [\, u(\, \underline{w} \,) - u(\, \overline{w} \,) \,] \right)^{\frac{1}{\gamma - 1}} [\, \overline{c} - \underline{c} \,] + \frac{\delta}{\gamma} \left(\frac{1}{\delta} \, [\, u(\, \underline{w} \,) - u(\, \overline{w} \,) \,] \right)^{\frac{\gamma}{\gamma - 1}} \right\} dG(\delta) \\ &= \; \underline{c} \; G(\, \delta_1 \,) + \frac{1}{\gamma} \int_{\underline{\delta}}^{\delta_1} \delta \; dG(\delta) + \overline{c} \; [\, 1 - G(\, \delta_1 \,) \,] \\ &\quad - [\, \overline{c} - \underline{c} \,] \, [\, u(\, \underline{w} \,) - u(\, \overline{w} \,) \,]^{\frac{1}{\gamma - 1}} \; H(\delta_1) + \frac{1}{\gamma} \, [\, u(\, \underline{w} \,) - u(\, \overline{w} \,) \,]^{\frac{\gamma}{\gamma - 1}} \; H(\delta_1) \, . \end{aligned}$$

References

Ferris, Michael and Todd Munson, "Complementarity Problems in GAMS and the PATH Solver," *Journal of Economic Dynamics and Control*, 24(2), February 2000, 165-188.