Technical Appendix to Accompany

"Optimal Policies to Promote Efficient Distributed Generation of Electricity"

by David P. Brown and David E. M. Sappington

Part A of this Appendix states and proves Proposition 6. Part B outlines the analysis that underlies the numerical solutions in Section 7 of the paper and in Part C of this Appendix. Part C presents additional numerical solutions.

A. The Optimal Regulatory Policy with State-Specific Pricing with Externalities

Proposition 6 characterizes the optimal regulatory policy when the regulator can set: (i) customer-specific TOU unit retail prices (r_{jt}); (ii) customer-specific fixed retail charges (R_j); (iii) technology-specific and time-varying DG payments (w_{jt}); and (iv) technology-specific capacity payments (k_y). The regulator seeks to maximize the difference between expected consumer welfare and expected losses from environmental externalities while ensuring S non-negative expected profit. Formally, the regulator's problem, [RP-te], is:

$$\underset{r_{jt},\, R_j,\, k_y,\, w_{jt},\, K_G}{Maximize} \quad E\left\{ U^D(\cdot) + U^N(\cdot) \right\} - E\left\{ \psi(\cdot) \right\} \tag{1}$$

$$\text{subject to:} \quad E\{\pi\} \;\geq\; 0 \;. \tag{2}$$

Proposition 6. At the solution to [RP-te]:

(i) $$\sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left| \frac{\partial C_t^G(Q_t^v(\cdot,\theta_t),K_G)}{\partial K_G} \right| dF_t(\theta_t) \;=\; C^{K\prime}(K_G) + \frac{\partial T(\cdot)}{\partial K_G}\;;$$

(ii) $$\int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[r_{jt} - \frac{\partial C_t^G(Q_t^v(\cdot,\theta_t),K_G)}{\partial Q_t^v(\cdot,\theta_t)} - \psi_{tv}(\cdot) \right] \frac{\partial X_t^j(\cdot,\theta_t)}{\partial r_{jt}}\, dF_t(\theta_t)$$

$$+ \int_{\underline{\theta}_{t'}}^{\overline{\theta}_{t'}} \left[r_{jt'} - \frac{\partial C_t^G(Q_t^v(\cdot,\theta_{t'}),K_G)}{\partial Q_t^v(\cdot,\theta_{t'})} - \psi_{t'v}(\cdot) \right] \frac{\partial X_{t'}^j(\cdot,\theta_{t'})}{\partial r_{jt}}\, dF_{t'}(\theta_{t'}) \;=\; 0$$

for $t' \neq t,\ t,t' \in \{\psi, H\}$;

(iii) $$w_{nt} \;=\; \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\frac{\partial C_t^G(\cdot)}{\partial Q_t^v} + \psi_{tv}(\cdot) - \psi_{tn}(\cdot) \right] dF_t(\theta_t) \quad \text{for } t \in \{\psi,\, H\}\,;$$

(iv) $$k_n \;=\; \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\frac{\partial C_t^G(\cdot)}{\partial Q_t^v} - w_{nt} + \psi_{tv}(\cdot) - \psi_{tn}(\cdot) \right] \frac{\partial Q_t^n}{\partial K_{Dn}}\, dF_t(\theta_t) - \frac{\partial T(\cdot)}{\partial K_{Dn}}\;; \text{ and}$$

(v) $$k_i \;=\; \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\frac{\partial C_t^G(\cdot)}{\partial Q_t^v} - w_{it} + \psi_{tv}(\cdot) - \psi_{ti}(\cdot) \right] \theta_t\, dF_t(\theta_t) - \frac{\partial T(\cdot)}{\partial K_{Di}}\,.$$

Proof. Let $\lambda \geq 0$ denote the Lagrange multiplier associated with constraint (2). Then the necessary conditions for an interior solution to [RP-te] include:[1]

$$k_i: \quad K_{Di} - \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(\psi_{ti}(\cdot) \frac{\partial Q_t^i}{\partial K_{Di}} \frac{\partial K_{Di}}{\partial k_i} + \psi_{tv}(\cdot) \frac{dQ_t^v(\cdot)}{dk_i} \right) dF_t(\theta_t)$$

$$- \lambda \Bigg[\sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(w_{it}\,\theta_t \frac{\partial K_{Di}}{\partial k_i} + \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} \frac{dQ_t^v}{dk_i} \right) dF_t(\theta_t)$$

$$+ \; K_{Di} + k_i \frac{\partial K_{Di}}{\partial k_i} + \frac{\partial T(\cdot)}{\partial K_{Di}} \frac{\partial K_{Di}}{\partial k_i} \Bigg] \; = \; 0\,; \qquad (3)$$

$$k_n: \quad K_{Dn} - \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(\psi_{tn}(\cdot) \frac{\partial Q_t^n}{\partial K_{Dn}} \frac{\partial K_{Dn}}{\partial k_n} + \psi_{tv}(\cdot) \frac{dQ_t^v(\cdot)}{dk_n} \right) dF_t(\theta_t)$$

$$- \lambda \Bigg[\sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(w_{nt} \frac{\partial Q_t^n}{\partial K_{Dn}} \frac{\partial K_{Dn}}{\partial k_n} + \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} \frac{dQ_t^v}{dk_n} \right) dF_t(\theta_t)$$

$$+ \; K_{Dn} + k_n \frac{\partial K_{Dn}}{\partial k_n} + \frac{\partial T(\cdot)}{\partial K_{Dn}} \frac{\partial K_{Dn}}{\partial k_n} \Bigg] \; = \; 0\,; \qquad (4)$$

$$K_G: \quad \lambda \Bigg[\sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(- \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} \frac{dQ_t^v}{dK_G} - \frac{\partial C_t^G(\cdot)}{\partial K_G} \right) dF_t(\theta_t)$$

$$- \; C^{K\prime}(K_G) - \frac{\partial T(\cdot)}{\partial K_G} \Bigg] - \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \psi_{tv}(\cdot) \frac{dQ_t^v}{dK_G} \, dF_t(\theta_t) \; = \; 0\,; \qquad (5)$$

$$w_{it}: \quad \int_{\underline{\theta}_t}^{\overline{\theta}_t} \theta_t \, K_{Di} \; dF_t(\theta_t)$$

$$- \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(\psi_{ti}(\cdot) \frac{\partial Q_t^i}{\partial K_{Di}} \frac{\partial K_{Di}}{\partial w_{it}} + \psi_{tv}(\cdot) \frac{\partial Q_t^v(\cdot)}{\partial K_{Di}} \frac{\partial K_{Di}}{\partial w_{it}} \right) dF_t(\theta_t)$$

$$- \; \lambda \Bigg[\int_{\underline{\theta}_t}^{\overline{\theta}_t} \theta_t \, K_{Di} \; dF_t(\theta_t)$$

[1] We assume $K_{Di} > 0$ and $K_{Dn} > 0$ at the solution to [RP-e].

$$+ \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(w_{it}\, \theta_t \, \frac{\partial K_{Di}}{\partial w_{it}} + \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} \, \frac{\partial Q_t^v}{\partial K_{Di}} \, \frac{\partial K_{Di}}{\partial w_{it}} \right) dF_t(\theta_t)$$

$$+ \; k_i \, \frac{\partial K_{Di}}{\partial w_{it}} + \frac{\partial T(\cdot)}{\partial K_{Di}} \, \frac{\partial K_{Di}}{\partial w_{it}} \Bigg] \; = \; 0\,; \qquad (6)$$

$$w_{nt}: \quad \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^n \, dF_t(\theta_t) - \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(\psi_{tn}(\cdot) + \psi_{tv}(\cdot) \, \frac{\partial Q_t^v(\cdot)}{\partial Q_t^n} \right) \frac{\partial Q_t^n}{\partial w_{nt}} \, dF_t(\theta_t)$$

$$- \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(\psi_{tn}(\cdot) + \psi_{tv}(\cdot) \, \frac{\partial Q_t^v(\cdot)}{\partial Q_t^n} \right) \frac{\partial Q_t^n}{\partial K_{Dn}} \, \frac{\partial K_{Dn}}{\partial w_{nt}} \, dF_t(\theta_t)$$

$$- \; \lambda \Bigg[\sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(w_{nt} \, \frac{\partial Q_t^n}{\partial K_{Dn}} + \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} \, \frac{\partial Q_t^v}{\partial Q_t^n} \, \frac{\partial Q_t^n}{\partial K_{Dn}} \right) \frac{\partial K_{Dn}}{\partial w_{nt}} \, dF_t(\theta_t)$$

$$+ \; \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(Q_t^n + w_{nt} \, \frac{\partial Q_t^n}{\partial w_{nt}} + \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} \, \frac{\partial Q_t^v}{\partial Q_t^n} \, \frac{\partial Q_t^n}{\partial w_{nt}} \right) dF_t(\theta_t)$$

$$+ \; k_n \, \frac{\partial K_{Dn}}{\partial w_{nt}} + \frac{\partial T(\cdot)}{\partial K_{Dn}} \, \frac{\partial K_{Dn}}{\partial w_{nt}} \Bigg] \; = \; 0\,; \qquad (7)$$

$$R_j: \quad -\, 1 + \lambda \; = \; 0\,; \qquad (8)$$

$$r_{jt}: \quad \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(\left[\frac{\partial V_t^j(X_t^j(\cdot), \theta_t)}{\partial X_t^j} - r_{jt} \right] \frac{\partial X_t^j}{\partial r_{jt}} - X_t^j(\cdot) \right) dF_t(\theta_t)$$

$$+ \; \int_{\underline{\theta}_{t'}}^{\overline{\theta}_{t'}} \left(\frac{\partial V_{t'}^j(X_{t'}^j(\cdot), \theta_{t'})}{\partial X_{t'}^j} - r_{jt'} \right) \frac{\partial X_{t'}^j}{\partial r_{jt}} \; dF_{t'}(\theta_{t'})$$

$$- \; \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(\psi_{tv}(\cdot) \, \frac{\partial Q_t^v(\cdot)}{\partial X_t} \, \frac{\partial X_t}{\partial X_t^j} \, \frac{\partial X_t^j}{\partial r_{jt}} \right) dF_t(\theta_t)$$

$$- \; \int_{\underline{\theta}_{t'}}^{\overline{\theta}_{t'}} \left(\psi_{t'v}(\cdot) \, \frac{\partial Q_{t'}^v(\cdot)}{\partial X_{t'}} \, \frac{\partial X_{t'}}{\partial X_{t'}^j} \, \frac{\partial X_{t'}^j}{\partial r_{jt}} \right) dF_{t'}(\theta_{t'})$$

$$+ \; \lambda \Bigg[\int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(r_{jt} \, \frac{\partial X_t^j}{\partial r_{jt}} + X_t^j(\cdot) \right) dF_t(\theta_t) + \int_{\underline{\theta}_{t'}}^{\overline{\theta}_{t'}} r_{jt'} \; \frac{\partial X_{t'}^j}{\partial r_{jt}} \; dF_{t'}(\theta_{t'})$$

$$- \int_{\underline{\theta}_t}^{\overline{\theta}_t} \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} \frac{\partial Q_t^v}{\partial X_t^j} \frac{\partial X_t^j}{\partial r_{jt}} \, dF_t(\theta_t) - \int_{\underline{\theta}_{t'}}^{\overline{\theta}_{t'}} \frac{\partial C_{t'}^G(\cdot)}{\partial Q_{t'}^v} \frac{\partial Q_{t'}^v}{\partial X_{t'}^j} \frac{\partial X_{t'}^j}{\partial r_{jt}} \, dF_{t'}(\theta_{t'}) \Bigg] \;=\; 0 \, . \tag{9}$$

(8) implies that $\lambda = 1$ at the solution to [RP-te].

$\frac{\partial V_t^j(X_t^j(r))}{\partial X_t^j} = r_t$ for $j \in \{D, N\}$ and $t \in \{L, H\}$ since $V_t^j(X)$ is the gross surplus consumer j derives from output X in period t.[2] Also, because $\lambda = 1$ and $\frac{\partial X_t}{\partial X_t^j} = \frac{\partial Q_t^v}{\partial X_t^j} = \frac{\partial Q_t^v}{\partial X_t} = 1$ (since $Q_t^v = X_t^N + X_t^D - \theta_t K_{Di} - Q_t^n$ and $X_t = X_t^N + X_t^D$), (9) can be written as:

$$\int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[r_{jt} - \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} - \psi_{tv}(\cdot) \right] \frac{\partial X_t^j}{\partial r_{jt}} \, dF_t(\theta_t)$$

$$+ \int_{\underline{\theta}_{t'}}^{\overline{\theta}_{t'}} \left[r_{jt'} - \frac{\partial C_{t'}^G(\cdot)}{\partial Q_{t'}^v} - \psi_{t'v}(\cdot) \right] \frac{\partial X_{t'}^j}{\partial r_{jt}} \, dF_{t'}(\theta_{t'}) \;=\; 0 \text{ for } t, t' \in \{\psi, H\} \; (t' \neq t). \tag{10}$$

Because $\lambda = 1$ and $\frac{dQ_t^v}{dK_G} = 0$, (5) can be written as:

$$\sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} - \frac{\partial C_t^G(\cdot)}{\partial K_G} \, dF_t(\theta_t) \;=\; \frac{\partial C^K(\cdot)}{\partial K_G} + \frac{\partial T(\cdot)}{\partial K_G} \, . \tag{11}$$

Because $\lambda = 1$, $\frac{\partial K_{Di}}{\partial k_i}$ is not a function of θ_t, $Q_t^i = \theta_t K_{Di}$, and k_i only affects Q_t^v through K_{Di}, (3) can be written as:

$$- \Bigg[\sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(w_{it} \, \theta_t + \psi_{ti}(\cdot) \, \theta_t + \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} \frac{dQ_t^v}{dK_{Di}} + \psi_{tv}(\cdot) \frac{dQ_t^v}{dK_{Di}} \right) dF_t(\theta_t)$$

$$+ k_i + \frac{\partial T(\cdot)}{\partial K_{Di}} \Bigg] \frac{\partial K_{Di}}{\partial k_i} \;=\; 0 \, . \tag{12}$$

Because $\frac{dQ_t^v}{dK_{Di}} = -\theta_t$, (12) can be written as:

$$k_i \;=\; \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\frac{\partial C_t^G(\cdot)}{\partial Q_t^v} - w_{it} + \psi_{tv}(\cdot) - \psi_{ti}(\cdot) \right] \theta_t \, dF_t(\theta_t) - \frac{\partial T(\cdot)}{\partial K_{Di}} \, . \tag{13}$$

Observe that $\lambda = 1$, $\frac{\partial K_{Dn}}{\partial k_n}$ is not a function of θ_t, and k_n only affects Q_t^v through K_{Dn}

[2] This will be the case if $\frac{\partial X_t(r_t, r_s)}{\partial r_t} < 0$ for all r_t, r_s ($s \neq t$, $s, t \in \{L, H\}$), which we assume to be true.

and the corresponding impact on Q_t^n. Therefore, (4) can be rewritten as:

$$- \Bigg[\sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(w_{nt} + \psi_{tn}(\cdot) + \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} \frac{\partial Q_t^v}{\partial Q_t^n} + \psi_{tv}(\cdot) \frac{\partial Q_t^v}{\partial Q_t^n} \right) \frac{\partial Q_t^n}{\partial K_{Dn}} \, dF_t(\theta_t)$$

$$+ k_n + \frac{\partial T(\cdot)}{\partial K_{Dn}} \Bigg] \frac{\partial K_{Dn}}{\partial k_n} = 0 . \tag{14}$$

Because $\frac{\partial Q_t^v}{\partial Q_t^n} = -1$, (14) can be written as:

$$k_n = \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\frac{\partial C_t^G(\cdot)}{\partial Q_t^v} - w_{nt} + \psi_{tv}(\cdot) - \psi_{tn}(\cdot) \right] \frac{\partial Q_t^n}{\partial K_{Dn}} \, dF_t(\theta_t) - \frac{\partial T(\cdot)}{\partial K_{Dn}} . \tag{15}$$

Because $\lambda = 1$, $Q_t^i = \theta_t K_{Di}$, and $\frac{\partial K_{Di}}{\partial w_{it}}$ is not a function of θ_s for $s, t \in \{L, H\}$, (6) can be written as:

$$- \Bigg[\sum_{s \in \{\psi, H\}} \int_{\underline{\theta}_s}^{\overline{\theta}_s} \left(w_{is} \, \theta_s + \psi_{si}(\cdot) \, \theta_s + \frac{\partial C_s^G(\cdot)}{\partial Q_s^v} \frac{\partial Q_s^v}{\partial K_{Di}} + \psi_{sv}(\cdot) \frac{\partial Q_s^v}{\partial K_{Di}} \right) dF_s(\theta_s)$$

$$+ k_i + \frac{\partial T(\cdot)}{\partial K_{Di}} \Bigg] \frac{\partial K_{Di}}{\partial w_{it}} = 0 . \tag{16}$$

Because $\frac{\partial Q_s^v}{\partial K_{Di}} = -\theta_s$, (16) can be written as (13).

Because $\lambda = 1$, $\frac{\partial K_{Dn}}{\partial w_{nt}}$ is not a function of θ_s for any $s \in \{\psi, H\}$, and $\frac{\partial Q_t^v}{\partial Q_t^n} = -1$, (7) can be written as:

$$- \Bigg[\sum_{s \in \{\psi, H\}} \int_{\underline{\theta}_s}^{\overline{\theta}_s} \left(w_{ns} - \frac{\partial C_s^G(\cdot)}{\partial Q_s^v} + \psi_{sn}(\cdot) - \psi_{sv}(\cdot) \right) \frac{\partial Q_s^n}{\partial K_{Dn}} \, dF_s(\theta_s) + k_n + \frac{\partial T(\cdot)}{\partial K_{Dn}} \Bigg] \frac{\partial K_{Dn}}{\partial w_{nt}}$$

$$- \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(w_{nt} - \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} + \psi_{tn}(\cdot) - \psi_{tv}(\cdot) \right) \frac{\partial Q_t^n}{\partial w_{nt}} \, dF_t(\theta_t) = 0$$

$$\Rightarrow \quad \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[w_{nt} - \frac{\partial C_t^G(\cdot)}{\partial Q_t^v} + \psi_{tn}(\cdot) - \psi_{tv}(\cdot) \right] \frac{\partial Q_t^n}{\partial w_{nt}} \, dF_t(\theta_t) = 0 . \tag{17}$$

(17) reflects (15). Because $\frac{\partial Q_t^n}{\partial w_{nt}}$ is not a function of θ_t, (17) can be written as:

$$w_{nt} = \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\frac{\partial C_t^G(\cdot)}{\partial Q_t^v} + \psi_{tv}(\cdot) - \psi_{tn}(\cdot) \right] dF_t(\theta_t) . \quad \blacksquare \tag{18}$$

B. Elements of the Numerical Solutions.

The numerical solutions in Section 7 of the paper and in Part C of this Appendix employ the following functional forms:

$$C_t^G(Q_t^v(\cdot), K_G) \;=\; a_v\, Q_t^v(\cdot) + b_v\, [\, Q_t^v(\cdot)\,]^2 + c_v\, \frac{Q_t^v(\cdot)}{K_G}\,; \tag{19}$$

$$C_t^D(Q_t^n(\cdot), K_{Dn}) \;=\; a_n\, Q_t^n(\cdot) + b_n\, \frac{[\, Q_t^n(\cdot)\,]^2}{K_{Dn}}\,; \tag{20}$$

$$T(K_G, K_D) \;=\; a_T^G\, K_G + a_T^{Di}\, K_{Di} + a_T^{Dn}\, K_{Dn}\,; \tag{21}$$

$$C^K(K_G) \;=\; a_K\, K_G + b_K\, [\, K_G\,]^2\,; \tag{22}$$

$$C_D^K(K_{Di}, K_{Dn}) \;=\; a_{Di}\, K_{Di} + b_{Di}\, [\, K_{Di}\,]^2 + a_{Dn}\, K_{Dn} + b_{Dn}\, [\, K_{Dn}\,]^2\,; \tag{23}$$

$$\psi_t(Q_t^v(\cdot), Q_t^i(\cdot), Q_t^n(\cdot)) \;=\; e_v\, Q_t^v(\cdot) + e_i\, Q_t^i(\cdot) + e_n\, Q_t^n(\cdot)\,; \text{ and} \tag{24}$$

$$X_t^j(r, \theta_t) \;=\; m_{jt}\left[\, \beta_{0j} + \theta_t^{\beta_{jt}}\, \right] - \alpha_{jt}\, r\,, \tag{25}$$

where $Q_t^i(\cdot) = \theta_t\, K_{Di}$, $Q_t^v(\cdot) = X_t^D(r, \theta_t) + X_t^N(r, \theta_t) - \theta_t\, K_{Di} - Q_t^n(\cdot)$, and a_v, b_v, c_v, a_n, b_n, c_n, a_T^G, a_T^{Di}, a_T^{Dn}, a_K, b_K, e_v, e_i, e_n, a_{Di}, b_{Di}, a_{Dn}, b_{Dn}, β_{0j}, β_{jt}, α_{jt}, and m_{jt} are constants, for $j \in \{D, N\}$ and $t \in \{L, H\}$.[3]

Using (19) – (25) and the fact that K_{Di}, Q_t^n, and r are not functions of θ_t at the solution to the problems under consideration:

$$\theta_t^E \;=\; \int_{\underline{\theta}_t}^{\overline{\theta}_t} \theta_t\, dF(\theta_t)\,; \tag{26}$$

$$E\{\, Q_t^v(\cdot)\, \} \;=\; \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(X_t^D(r, \theta_t) + X_t^N(r, \theta_t) - \theta_t\, K_{Di} - Q_t^n(\cdot) \right) dF(\theta_t)$$

$$= \int_{\underline{\theta}_t}^{\overline{\theta}_t} \Big(m_{Dt}\left[\, \beta_{0D} + \theta_t^{\beta_{Dt}}\, \right] - \alpha_{Dt}\, r + m_{Nt}\left[\, \beta_{0N} + \theta_t^{\beta_{Nt}}\, \right] - \alpha_{Nt}\, r - \theta_t\, K_{Di} - Q_t^n(\cdot) \Big)\, dF(\theta_t)$$

$$= m_{Dt} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\, \beta_{0D} + \theta_t^{\beta_{Dt}}\, \right] dF(\theta_t) - \alpha_{Dt}\, r$$

$$+ \; m_{Nt} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\, \beta_{0N} + \theta_t^{\beta_{Nt}}\, \right] dF(\theta_t) - \alpha_{Nt}\, r \,-\, \theta_t^E\, K_{Di} - Q_t^n(\cdot)\,; \tag{27}$$

$$E\{\, Q_t^v(\cdot)\, \theta_t\, \} \;=\; \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\, X_t^D(r, \theta_t) + X_t^N(r, \theta_t) - \theta_t\, K_{Di} - Q_t^n(\cdot)\, \right] \theta_t\, dF(\theta_t)$$

[3] As noted in the paper, $\beta_{0D} = \beta_{0N} = 1$ in the baseline setting.

$$= \int_{\underline{\theta}_t}^{\overline{\theta}_t} \Big[m_{Dt} \left(\beta_{0D} + \theta_t^{\beta_{Dt}} \right) \theta_t - \alpha_{Dt}\, r\, \theta_t + m_{Nt} \left(\beta_{0N} + \theta_t^{\beta_{Nt}} \right) \theta_t$$

$$- \alpha_{Nt}\, r\, \theta_t - \theta_t^2\, K_{Di} - \theta_t\, Q_t^n(\cdot) \Big] dF(\theta_t)$$

$$= m_{Dt} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0D} + \theta_t^{\beta_{Dt}} \right] \theta_t\, dF(\theta_t) - \alpha_{Dt}\, r\, \theta_t^E + m_{Nt} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0N} + \theta_t^{\beta_{Nt}} \right] \theta_t\, dF(\theta_t)$$

$$- \alpha_{Nt}\, r\, \theta_t^E - K_{Di} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \theta_t^2\, dF(\theta_t) - \theta_t^E\, Q_t^n(\cdot)\,; \tag{28}$$

$$E\{X_t^j(\cdot)\} = \int_{\underline{\theta}_t}^{\overline{\theta}_t} \Big[m_{jt}(\beta_{0j} + \theta_t^{\beta_{jt}}) - \alpha_{jt}\, r \Big] dF(\theta_t)$$

$$= m_{jt} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0j} + \theta_t^{\beta_{jt}} \right] dF(\theta_t) - \alpha_{jt}\, r\,; \tag{29}$$

$$E \left\{ \frac{\partial X_t^j(\cdot)}{\partial r} \right\} = -\, \alpha_{jt}\,; \tag{30}$$

$$E \left\{ \frac{\partial X_t^j(\cdot)}{\partial r}\, Q_t^v(\cdot) \right\} = -\, \alpha_{jt}\, E\{\, Q_t^v(\cdot)\, \}\,; \tag{31}$$

$$E\{\, [\, Q_t^v(\cdot)\,]^2\, \} = \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\, X_t^D(r,\theta_t) + X_t^N(r,\theta_t) - \theta_t\, K_{Di} - Q_t^n(\cdot)\, \right]^2 dF(\theta_t)$$

$$= \int_{\underline{\theta}_t}^{\overline{\theta}_t} \{\, \left[\, X_t^D(\cdot)\, \right]^2 + \left[\, X_t^N(\cdot)\, \right]^2 + 2\, X_t^D(\cdot)\, X_t^N(\cdot)$$

$$-\, 2\, X_t^D(\cdot)\, \theta_t\, K_{Di} - 2\, X_t^N(\cdot)\, \theta_t\, K_{Di} + [\, \theta_t\, K_{Di}\,]^2 - 2\, X_t^D(\cdot)\, Q_t^n(\cdot)$$

$$-\, 2\, X_t^N(\cdot)\, Q_t^n(\cdot) + 2\, Q_t^n(\cdot)\, \theta_t\, K_{Di} + [\, Q_t^n(\cdot)\,]^2\, \}\, dF(\theta_t)$$

$$= \int_{\underline{\theta}_t}^{\overline{\theta}_t} \{\, \left[m_{Dt} \left(\beta_{0D} + \theta_t^{\beta_{Dt}} \right) - \alpha_{Dt}\, r \right]^2 + \left[m_{Nt} \left(\beta_{0N} + \theta_t^{\beta_{Nt}} \right) - \alpha_{Nt}\, r \right]^2$$

$$+\, 2 \left[m_{Dt} \left(\beta_{0D} + \theta_t^{\beta_{Dt}} \right) - \alpha_{Dt}\, r \right] \left[m_{Nt} \left(\beta_{0N} + \theta_t^{\beta_{Nt}} \right) - \alpha_{Nt}\, r \right]$$

$$-\, 2 \left[m_{Dt} \left(\beta_{0D} + \theta_t^{\beta_{Dt}} \right) - \alpha_{Dt}\, r \right] \theta_t\, K_{Di}$$

$$-\, 2 \left[m_{Nt} \left(\beta_{0N} + \theta_t^{\beta_{Nt}} \right) - \alpha_{Nt}\, r \right] \theta_t\, K_{Di} + [\, \theta_t\, K_{Di}\,]^2$$

$$
\begin{aligned}
& - 2\left[m_{Dt}\left(\beta_{0D} + \theta_t^{\beta_{Dt}}\right) - \alpha_{Dt}\, r \right] Q_t^n(\cdot) - 2\left[m_{Nt}\left(\beta_{0N} + \theta_t^{\beta_{Nt}}\right) - \alpha_{Nt}\, r \right] Q_t^n(\cdot) \,\}\, dF(\theta_t) \\
& + 2\, Q_t^n(\cdot)\, \theta_t^E\, K_{Di} + [\, Q_t^n(\cdot)\,]^2 \\
= \; & [\, m_{Dt}\,]^2 \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0D} + \theta_t^{\beta_{Dt}} \right]^2 dF(\theta_t) + [\, m_{Nt}\,]^2 \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0N} + \theta_t^{\beta_{Nt}} \right]^2 dF(\theta_t) \\
- \; & 2\, \alpha_{Dt}\, m_{Dt}\, r \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0D} + \theta_t^{\beta_{Dt}} \right] dF(\theta_t) + [\, \alpha_{Dt}\, r\,]^2 \\
- \; & 2\, \alpha_{Nt}\, m_{Nt}\, r \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0N} + \theta_t^{\beta_{Nt}} \right] dF(\theta_t) + [\, \alpha_{Nt}\, r\,]^2 \\
+ \; & 2\, m_{Dt}\, m_{Nt} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0D} + \theta_t^{\beta_{Dt}} \right] \left[\beta_{0N} + \theta_t^{\beta_{Nt}} \right] dF(\theta_t) \\
- \; & 2\, m_{Nt}\, \alpha_{Dt}\, r \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0N} + \theta_t^{\beta_{Nt}} \right] dF(\theta_t) \\
- \; & 2\, m_{Dt}\, \alpha_{Nt}\, r \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0D} + \theta_t^{\beta_{Dt}} \right] dF(\theta_t) \; + 2\, \alpha_{Dt}\, \alpha_{Nt}\, (r)^2 \\
- \; & 2\, m_{Dt}\, K_{Di} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0D} + \theta_t^{\beta_{Dt}} \right] \theta_t\, dF(\theta_t) \; + 2\, \alpha_{Dt}\, r\, K_{Di}\, \theta_t^E \\
- \; & 2\, m_{Nt}\, K_{Di} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0N} + \theta_t^{\beta_{Nt}} \right] \theta_t\, dF(\theta_t) \; + 2\, \alpha_{Nt}\, r\, K_{Di}\, \theta_t^E \\
- \; & 2\, m_{Dt}\, Q_t^n(\cdot) \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0D} + \theta_t^{\beta_{Dt}} \right] dF(\theta_t) + 2\, Q_t^n(\cdot)\, \alpha_{Dt}\, r + K_{Di}^2 \int_{\underline{\theta}_t}^{\overline{\theta}_t} \theta_t^2\, dF(\theta_t) \\
- \; & 2\, m_{Nt}\, Q_t^n(\cdot) \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\beta_{0N} + \theta_t^{\beta_{Nt}} \right] dF(\theta_t) + 2\, Q_t^n(\cdot)\, \alpha_{Nt}\, r + 2\, Q_t^n(\cdot)\, \theta_t^E\, K_{Di} + [\, Q_t^n(\cdot)\,]^2 .
\end{aligned} \tag{32}
$$

The conclusions in (26) – (32) allow us to numerically integrate out θ_t separately from the computation of the endogenous variables using the Newton-Raphson Method.

Given θ_t, there is a critical unit retail price, $\bar{r}_{jt}(\theta_t)$, at which $X_t^j(\bar{r}_{jt}(\theta_t), \theta_t) = 0$:

$$
m_{jt}\left[\beta_{0j} + \theta_t^{\beta_{jt}} \right] - \alpha_{jt}\, \bar{r}_{jt}(\theta_t) \; = \; 0 \quad \Rightarrow \quad \bar{r}_{jt}(\theta_t) \; = \; \frac{m_{jt}\left[\beta_{0j} + \theta_t^{\beta_{jt}} \right]}{\alpha_{jt}} . \tag{33}
$$

Let $\widehat{r}$ denote the established unit retail price. (33) implies that consumer j's expected gross surplus in period t is:

$$E\{\,V_t^j(X_t^j(\widehat{r},\theta_t),\theta_t)\,\} \;=\; \int_{\underline{\theta}_t}^{\bar{\theta}_t}\left[\int_{\widehat{r}}^{\bar{r}_{jt}(\theta_t)} X_t^j(r,\theta_t)\,dr + \widehat{r}\,X_t^j(\widehat{r},\theta_t)\right]dF(\theta_t) \tag{34}$$

where, from (25) and (33):

$$\begin{aligned}
\int_{\widehat{r}}^{\bar{r}_{jt}(\theta_t)} X_t^j(r,\theta_t)\,dr \;&=\; \int_{\widehat{r}}^{\bar{r}_{jt}(\theta_t)} \Big\{\, m_{jt}[\,\beta_{0j}+\theta_t^{\beta_{jt}}\,] - \alpha_{jt}\,r\,\Big\}\,dr \\
&= m_{jt}\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]\,\bar{r}_{jt}(\theta_t) - \frac{\alpha_{jt}\,[\,\bar{r}_{jt}(\theta_t)\,]^2}{2} - \left(m_{jt}\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]\widehat{r} - \frac{\alpha_{jt}\,[\,\widehat{r}\,]^2}{2}\right) \\
&= \frac{(m_{jt})^2\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]^2}{\alpha_{jt}} - \frac{(m_{jt})^2\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]^2}{2\,\alpha_{jt}} - \left(m_{jt}\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]\widehat{r} - \frac{\alpha_{jt}\,[\,\widehat{r}\,]^2}{2}\right) \\
&= \frac{(m_{jt})^2\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]^2}{2\,\alpha_{jt}} - m_{jt}\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]\widehat{r} + \frac{\alpha_{jt}\,[\,\widehat{r}\,]^2}{2}\,. \qquad (35)
\end{aligned}$$

Using (35), (34) can be written as:

$$\begin{aligned}
&\int_{\underline{\theta}_t}^{\bar{\theta}_t}\left\{\frac{(m_{jt})^2\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]^2}{2\,\alpha_{jt}} - m_{jt}\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]\widehat{r} + \frac{\alpha_{jt}\,[\,\widehat{r}\,]^2}{2} + \widehat{r}\left(m_{jt}\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right] - \alpha_{jt}\,\widehat{r}\right)\right\}dF(\theta_t) \\
&\qquad = \int_{\underline{\theta}_t}^{\bar{\theta}_t}\left[\frac{(m_{jt})^2\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]^2}{2\,\alpha_{jt}} - \frac{\alpha_{jt}\,\widehat{r}^2}{2}\right]dF(\theta_t) \\
&\qquad = \frac{(m_{jt})^2}{2\,\alpha_{jt}}\int_{\underline{\theta}_t}^{\bar{\theta}_t}\left[\beta_{0j}+\theta_t^{\beta_{jt}}\right]^2 dF(\theta_t) - \frac{\alpha_{jt}\,\widehat{r}^2}{2}. \qquad (36)
\end{aligned}$$

Because there is a probability mass in the empirical distribution function at $\theta_t = 0$ for both $t \in \{L, H\}$, we estimate (26) – (36) using a two-part conditional expectation approach. Specifically:

$$\begin{aligned}
E\{\,Q_t^v(\cdot)\,\} \;&=\; E\{\,Q_t^v(\cdot)\,|\,\theta_t > 0\}\,P(\theta_t > 0) + E\{\,Q_t^v(\cdot)\,|\,\theta_t = 0\}\,P(\theta_t = 0) \\
&= \left[\int_{\underline{\theta}_t}^{\bar{\theta}_t} Q_t^v(\cdot)dF_t^+(\theta_t)\right] P(\theta_t > 0) + Q_t^v(\cdot,\theta_t = 0)\,P(\theta_t = 0)
\end{aligned}$$

where $F_t^+(\theta)$ is the density function for the strictly positive values of θ_t, and where $P(\theta_t > 0)$ and $P(\theta_t = 0)$, respectively, are the probabilities that $\theta_t > 0$ and that $\theta_t = 0$.

State-Specific Retail Prices and DG Payments

Here we consider the setting where the regulator can set unit retail prices and DG output payments that vary across states but not across consumers. To characterize the optimal regulatory policy in this setting, we first solve analytically for the state-specific variables $r_t(\theta_t)$, $Q_t^i(\theta_t)$, $Q_t^n(\theta_t)$, and $Q_t^v(\theta_t)$. Then we characterize the optimal non-stochastic variables K_{Di}, K_{Dn}, K_G as a system of non-linear equations that can be solved via nonlinear programming and numerical integration. Finally, we employ a quasi-Monte Carlo approach to compute expected utility, social losses from externalities, and welfare.

It is readily verified that at the solution to the regulator's problem in this setting:

$$r_t(\theta_t) \;=\; r_{Nt}(\theta_t) \;=\; r_{Dt}(\theta_t) \;=\; a_v + 2\,b_v\,Q_t^v(\theta_t) + \frac{c_v}{K_G} + e_v\,;$$

$$w_{nt}(\theta_t) \;=\; a_v + 2\,b_v\,Q_t^v(\theta_t) + \frac{c_v}{K_G} + e_v - e_n\,;$$

$$w_{it}(\theta_t) \;=\; a_v + 2\,b_v\,Q_t^v(\theta_t) + \frac{c_v}{K_G} + e_v - e_i\,;$$

$$k_i \;=\; -\,a_T^{Di}; \quad \text{and} \;\; k_n \;=\; -\,a_T^{Dn} \quad \text{for} \;\; t \in \{L, H\}\,. \tag{37}$$

Because $Q_t^v(\theta_t) = X_t^D(\cdot) + X_t^N(\cdot) - \theta_t\,K_{Di} - Q_t^n(\theta_t)$:

$$w_{nt}(\theta_t) \;=\; a_n + 2\,b_n\,\frac{Q_t^n(\theta_t)}{K_{Dn}}$$

$$\Rightarrow \quad a_v + 2\,b_v\,Q_t^v(\theta_t) + \frac{c_v}{K_G} + e_v - e_n \;=\; a_n + 2\,b_n\,\frac{Q_t^n(\theta_t)}{K_{Dn}}$$

$$\Rightarrow \quad Q_t^n(\theta_t) \;=\; \frac{K_{Dn}}{2\,b_n}\left[a_v + 2\,b_v\left(X_t^D(\cdot) + X_t^N(\cdot) - \theta_t\,K_{Di} - Q_t^n(\theta_t)\right) + \frac{c_v}{K_G} + e_v - e_n - a_n\right]$$

$$\Rightarrow \quad Q_t^n(\theta_t) \;=\; \frac{K_{Dn}}{2\,[\,b_n + b_v\,K_{Dn}\,]}\left[a_v + 2\,b_v\left(X_t^D(\cdot) + X_t^N(\cdot) - \theta_t\,K_{Di}\right) + \frac{c_v}{K_G} + e_v - e_n - a_n\right]$$

$$=\; Z_1 + \frac{b_v\,K_{Dn}}{b_n + b_v\,K_{Dn}}\;\left[\,X_t^D(\cdot) + X_t^N(\cdot) - \theta_t\,K_{Di}\,\right] \tag{38}$$

where $\quad Z_1 \;\equiv\; \dfrac{K_{Dn}}{2\,[\,b_n + b_v\,K_{Dn}\,]}\left[a_v + \dfrac{c_v}{K_G} + e_v - e_n - a_n\right].$

Because $Q_t^v(\theta_t) = X_t^D(\cdot) + X_t^N(\cdot) - \theta_t\,K_{Di} - Q_t^n(\theta_t)$:

$$r_t(\theta_t) \;=\; a_v + 2\,b_v\left[\,X_t^D(\cdot) + X_t^N(\cdot) - \theta_t\,K_{Di} - Q_t^n(\theta_t)\,\right] + \frac{c_v}{K_G} + e_v$$

$$=\; a_v + 2\,b_v\left[m_{Dt}(\beta_{0D} + \theta_t^{\beta_D}) - \alpha_{Dt}\,r_t(\theta_t)\right] + 2\,b_v\left[m_{Nt}(\beta_{0N} + \theta_t^{\beta_N}) - \alpha_{Nt}\,r_t(\theta_t)\right]$$

$$
\begin{aligned}
& - 2\, b_v\, \theta_t\, K_{Di} - 2\, b_v\, Z_1 + \frac{c_v}{K_G} + e_v \\
& - \frac{2\,(b_v)^2\, K_{Dn}}{b_n + b_v\, K_{Dn}} \left[m_{Dt}(\beta_{0D} + \theta_t^{\beta_D}) - \alpha_{Dt}\, r_t(\theta_t) + m_{Nt}(\beta_{0N} + \theta_t^{\beta_N}) - \alpha_{Nt}\, r_t(\theta_t) - \theta_t\, K_{Di} \right] \\
= \;& a_v + \frac{c_v}{K_G} + e_v - 2\, b_v\, Z_1 \\
& + \left[2\, b_v - \frac{2\,(b_v)^2\, K_{Dn}}{b_n + b_v\, K_{Dn}} \right] \Bigg[m_{Dt} \left(\beta_{0D} + \theta_t^{\beta_D} \right) + m_{Nt} \left(\beta_{0N} + \theta_t^{\beta_N} \right) \\
& \qquad - \left(\alpha_{Dt} + \alpha_{Nt} \right) r_t(\theta_t) - \theta_t\, K_{Di} \Bigg] \\
= \;& a_v + \frac{c_v}{K_G} + e_v - 2\, b_v\, Z_1 \\
& + \left[\frac{2\, b_v\, b_n}{b_n + b_v\, K_{Dn}} \right] \left[m_{Dt} \left(\beta_{0D} + \theta_t^{\beta_D} \right) + m_{Nt} \left(\beta_{0N} + \theta_t^{\beta_N} \right) - \left(\alpha_{Dt} + \alpha_{Nt} \right) r_t(\theta_t) - \theta_t\, K_{Di} \right]
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \quad r_t(\theta_t) \left[1 + \frac{2\, b_v\, b_n\, (\alpha_{Dt} + \alpha_{Nt})}{b_n + b_v\, K_{Dn}} \right] \;&=\; a_v + \frac{c_v}{K_G} + e_v - 2\, b_v\, Z_1 \\
& + \left[\frac{2\, b_v\, b_n}{b_n + b_v\, K_{Dn}} \right] \left[m_{Dt} \left(\beta_{0D} + \theta_t^{\beta_D} \right) + m_{Nt} \left(\beta_{0N} + \theta_t^{\beta_N} \right) - \theta_t\, K_{Di} \right] \\
\Rightarrow \quad r_t(\theta_t) \;&=\; Z_2 + Z_3(\theta_t) \qquad (39)
\end{aligned}
$$

where $\quad Z_2 \;\equiv\; \left[1 + \frac{2\, b_v\, b_n (\alpha_{Dt} + \alpha_{Nt})}{b_n + b_v\, K_{Dn}} \right]^{-1} \left[a_v + \frac{c_v}{K_G} + e_v - 2\, b_v\, Z_1 \right]; \quad$ and

$$
\begin{aligned}
Z_3(\theta_t) \;\equiv\; & \left[1 + \frac{2\, b_v\, b_n (\alpha_{Dt} + \alpha_{Nt})}{b_n + b_v\, K_{Dn}} \right]^{-1} \left[\frac{2\, b_v\, b_n}{b_n + b_v\, K_{Dn}} \right] \\
& \cdot \left[m_{Dt} \left(\beta_{0D} + \theta_t^{\beta_D} \right) + m_{Nt} \left(\beta_{0N} + \theta_t^{\beta_N} \right) - \theta_t\, K_{Di} \right]. \qquad (40)
\end{aligned}
$$

Using (25) and (39), (38) can be written as:

$$
\begin{aligned}
Q_t^n(\theta_t) \;=\; Z_4(\theta_t) \;\equiv\; Z_1 + \frac{b_v\, K_{Dn}}{b_n + b_v\, K_{Dn}} \Bigg[& m_{Dt} \left(\beta_{0N} + \theta_t^{\beta_D} \right) + m_{Nt} \left(\beta_{0N} + \theta_t^{\beta_N} \right) \\
& - \theta_t\, K_{Di} - \left(\alpha_{Dt} + \alpha_{Nt} \right) \left(Z_2 + Z_3(\theta_t) \right) \Bigg]. \qquad (41)
\end{aligned}
$$

(37) and the necessary condition for an optimal choice of K_{Di} imply:

$$\sum_{t \in \{\psi, H\}} \left\{ \int_{\underline{\theta}_t}^{\overline{\theta}_t} w_{it}(\theta_t)\, \theta_t \, dF_t(\theta_t) \right\} + k_i \;=\; a_{Di} + 2\, b_{Di}\, K_{Di}$$

$$\Rightarrow \quad \sum_{t \in \{\psi, H\}} \left\{ \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[a_v + 2\, b_v\, Q_t^v(\theta_t) + \frac{c_v}{K_G} + e_v - e_i \right] \theta_t \, dF_t(\theta_t) \right\} - a_T^{Di} \;=\; a_{Di} + 2\, b_{Di}\, K_{Di}$$

$$\Rightarrow \quad \sum_{t \in \{\psi, H\}} \left\{ \left[a_v + \frac{c_v}{K_G} + e_v - e_i \right] \theta_t^E + 2\, b_v \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\theta_t)\, \theta_t \, dF_t(\theta_t) \right\} - a_T^{Di}$$

$$= \; a_{Di} + 2\, b_{Di}\, K_{Di} \tag{42}$$

where, from (39) and (41), because $Q_t^v(\theta_t) \;=\; X_t^D(\cdot) + X_t^N(\cdot) - \theta_t\, K_{Di} - Q_t^n(\theta_t)$:

$$\int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\theta_t)\, \theta_t \, dF_t(\theta_t) \;=\; \int_{\underline{\theta}_t}^{\overline{\theta}_t} \Big\{ m_{Dt} \, [\, \beta_{0D} + \theta_t^{\beta_D} \,]\, \theta_t + m_{Nt} \, [\, \beta_{0N} + \theta_t^{\beta_N} \,]\, \theta_t$$

$$- \; (\theta_t)^2 \, K_{Di} - [\, \alpha_{Dt} + \alpha_{Nt} \,]\, r_t(\theta_t)\, \theta_t - Q_t^n(\theta_t)\, \theta_t \Big\} \, dF_t(\theta_t)$$

$$= \; \int_{\underline{\theta}_t}^{\overline{\theta}_t} \Big\{ m_{Dt} \, [\, \beta_{0D} + \theta_t^{\beta_D} \,]\, \theta_t + m_{Nt} \, [\, \beta_{0N} + \theta_t^{\beta_N} \,]\, \theta_t$$

$$- \; (\theta_t)^2 \, K_{Di} - [\, \alpha_{Dt} + \alpha_{Nt} \,]\, [\, Z_2 + Z_3(\theta_t) \,]\, \theta_t - Z_4(\theta_t)\, \theta_t \Big\} \, dF_t(\theta_t)\,.$$

(20), (23), (37), (41) and the necessary condition for the optimal choice of K_{Dn} imply:

$$\sum_{t \in \{\psi, H\}} \left\{ \int_{\underline{\theta}_t}^{\overline{\theta}_t} \frac{b_n \, [\, Q_t^n(\theta_t) \,]^2}{[\, K_{Dn} \,]^2} \, dF_t(\theta_t) \right\} + k_n \;=\; a_{Dn} + 2\, b_{Dn}\, K_{Dn}$$

$$\Rightarrow \quad \frac{b_n}{[\, K_{Dn} \,]^2} \sum_{t \in \{\psi, H\}} \left\{ \int_{\underline{\theta}_t}^{\overline{\theta}_t} [\, Z_4(\theta_t) \,]^2 \, dF_t(\theta_t) \right\} - a_T^{Dn} \;=\; a_{Dn} + 2\, b_{Dn}\, K_{Dn}\,. \tag{43}$$

(19), (21), (22), and (37) and the necessary condition for the optimal choice of K_G imply:

$$- \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \frac{\partial C_t^G(\cdot)}{\partial K_G} \, dF_t(\theta_t) \;=\; C^{K\prime}(K_G) + \frac{\partial T(\cdot)}{\partial K_G}$$

$$\Rightarrow \quad \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \frac{c_v \, Q_t^v(\cdot)}{[\, K_G \,]^2} \, dF_t(\theta_t) \;=\; a_K + 2\, b_K \, K_G + a_T^G$$

$$\Rightarrow \quad \frac{c_v}{[K_G]^2} \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t) \;=\; a_K + 2\, b_K\, K_G + a_T^G \tag{44}$$

where, because $Q_t^v(\theta_t) \;=\; X_t^D(\cdot) + X_t^N(\cdot) - \theta_t K_{Di} - Q_t^n(\theta_t)$, (25), (39), and (41) imply:

$$\begin{aligned}
\int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t) \;&=\; \int_{\underline{\theta}_t}^{\overline{\theta}_t} \{\, m_{Dt}\, [\, \beta_{0D} + \theta_t^{\beta_D}\,] + m_{Nt}\, [\, \beta_{0N} + \theta_t^{\beta_N}\,] \\
&\qquad - \,[\, \alpha_{Dt} + \alpha_{Nt}\,]\, r_t(\theta_t) - \theta_t\, K_{Di} - Q_t^n \,\}\; dF_t(\theta_t) \\
&=\; \int_{\underline{\theta}_t}^{\overline{\theta}_t} \{\, m_{Dt}\, [\, \beta_{0D} + \theta_t^{\beta_D}\,] + m_{Nt}\, [\, \beta_{0N} + \theta_t^{\beta_N}\,] \\
&\qquad - \,[\, \alpha_{Dt} + \alpha_{Nt}\,]\, [\, Z_2 + Z_3(\theta_t)\,] - \theta_t\, K_{Di} - Z_4(\theta_t)\,\}\, dF_t(\theta_t)\,.
\end{aligned} \tag{45}$$

Solution Procedure

Step 1. Solve for the non-stochastic variables $\{K_G, K_{Di}, K_{Dn}\}$ defined in the nonlinear system (42), (43), and (44) using the PATH algorithm and numerical integration.

Step 2. Using the identified values of $\{K_G, K_{Di}, K_{Dn}\}$, calculate point estimates for the stochastic variables $r_t(\theta_t)$, $w_{it}(\theta_t)$, $w_{nt}(\theta_t)$, and $Q_t^n(\theta_t)$ defined in (37), (39), and (41). We employ a Monte Carlo algorithm in each period $t \in \{L, H\}$. We select a set of $M = 1,000$ θ_t values $(\theta_{1t}, \theta_{2t}, ..., \theta_{Mt})$ and solve for the state-specific variables for each $t \in \{L, H\}$. The M θ_t values are chosen via the Halton Sequence. Then we average these stochastic solutions, using weights that reflect the likelihood of the relevant θ_{jt}'s for each $j = 1, 2, ..., M$. For instance, the point estimate for $w_{it}(\theta_t)$ is determined by:

$$\widehat{w}_{it} \;\approx\; \sum_{j=1}^{M} \rho_{jt}(\theta_{jt})\, w_{it}(\theta_{jt}) \quad \text{where} \quad \rho_{jt}(\theta_{jt}) \;=\; \frac{f(\theta_{jt})}{\sum_{i=1}^{M}\, f(\theta_{it})}\,.$$

Step 3. Compute the values of expected utility, expected welfare, expected losses from environmental externalities, and the fixed retail charges using a Quasi-Monte Carlo Integration method. We select $M = 1,000$ θ_t values $(\theta_{1t}, \theta_{2t}, ..., \theta_{Mt})$ and solve for the state-specific variables for each $t \in \{L, H\}$. Taking the non-stochastic variables $\{K_G, K_{Di}, K_{Dn}\}$ derived in Step 1, we solve for the fixed charge, utility of each consumer group, environmental losses, and welfare for each particular θ_{jt} draw. Then, we approximate the expected value of these variables by taking a weighted average of these M draws, where the weights reflect the probability density function that specifies the likelihood of a particular θ_{jt}-draw.

TOU Pricing and TOP DG Payments

Now consider the setting where the regulator can set: (i) TOU retail prices that do not vary across customers (r_t); (ii) a fixed retail charge (R); (iii) technology-specific, time-varying DG payments (w_{yt}); and (iv) technology-specific DG capacity payments (k_y). The values of w_{it} and k_i are not unique in his setting. We assume $w_{it} = r_t$ for $t \in \{L, H\}$.

The necessary conditions for a solution to the regulator's problem in this setting include:

$$- \sum_{j \in \{N,D\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[r_t - a_v - 2\, b_v\, Q_t^v(\cdot) - \frac{c_v}{K_G} - e_v \right] [\, \alpha_{Dt} + \alpha_{Nt} \,]\, dF_t(\theta_t) \;=\; 0$$

$$\Rightarrow \quad r_t \;=\; a_v + \frac{c_v}{K_G} + e_v + 2\, b_v \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t)\, ; \tag{46}$$

$$w_{nt} \;=\; a_n + 2\, b_n\, \frac{Q_t^n}{K_{Dn}}\, ; \tag{47}$$

$$\sum_{t \in \{\psi,H\}} w_{it}\, \theta_t^E + k_i \;=\; a_{Di} + 2\, b_{Di}\, K_{Di}\, ; \tag{48}$$

$$\sum_{t \in \{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} b_n \left(\frac{Q_t^n}{K_{Dn}} \right)^2 dF_t(\theta_t) + k_n \;=\; a_{Dn} + 2\, b_{Dn}\, K_{Dn}$$

$$\Rightarrow \quad \frac{b_n}{(K_{Dn})^2} \sum_{t \in \{\psi,H\}} [\, Q_t^n \,]^2 \, + k_n \;=\; a_{Dn} + 2\, b_{Dn}\, K_{Dn}\, ; \tag{49}$$

$$\sum_{t \in \{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \frac{c_v\, Q_t^v(\cdot)}{(K_G)^2}\, dF_t(\theta_t) \;=\; a_K + 2\, b_K\, K_G + a_T^G$$

$$\Rightarrow \quad \frac{c_v}{(K_G)^2} \sum_{t \in \{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t) \;=\; a_K + 2\, b_K\, K_G + a_T^G\, ; \tag{50}$$

$$k_i \;=\; \sum_{t \in \{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[a_v + 2\, b_v\, Q_t^v(\cdot) + \frac{c_v}{K_G} - w_{it} + e_v - e_i \right] \theta_t\; dF_t(\theta_t) - a_T^{Di}$$

$$= \sum_{t \in \{\psi,H\}} \left[\left(a_v + \frac{c_v}{K_G} - w_{it} + e_v - e_i \right) \theta_t^E + 2\, b_v \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\; \theta_t\; dF_t(\theta_t) \right] - a_T^{Di}\, ; \tag{51}$$

$$
\begin{aligned}
w_{nt} &= \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[a_v + 2\, b_v\, Q_t^v(\cdot) + \frac{c_v}{K_G} + e_v - e_n \right] dF_t(\theta_t) \\
&= a_v + \frac{c_v}{K_G} + e_v - e_n + 2\, b_v \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\; dF_t(\theta_t)\,; \quad \text{and} \qquad (52)
\end{aligned}
$$

$$
\begin{aligned}
k_n &= \sum_{t \in \{\, \psi, H\,\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[a_v + 2\, b_v\, Q_t^v(\cdot) + \frac{c_v}{K_G} - w_{nt} + e_v - e_n \right] \frac{\partial Q_t^n}{\partial K_{Dn}}\, dF_t(\theta_t) - a_T^{Dn} \\
&= \sum_{t \in \{\, \psi, H\,\}} \frac{w_{nt} - a_n}{2\, b_n} \left[a_v + \frac{c_v}{K_G} - w_{nt} + e_v - e_n + 2\, b_v \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t) \right] - a_T^{Dn}\,. \qquad (53)
\end{aligned}
$$

(46) – (53) characterize a system of 13 equations and 13 unknowns that can be solved using the PATH algorithm and numerical integration. $E\{Q_t^v(\cdot)\}$ and $E\{Q_t^v(\cdot)\, \theta_t\}$ are defined in (27) and (28).

Flat Retail Prices and DG Payments

Now consider the setting where the regulator can set: (i) retail prices that do not vary across customers or over time (r); (ii) a fixed retail charge (R); (iii) technology-specific DG payments (w_y); and (iv) technology-specific DG capacity payments (k_y). w_i and k_i are not unique in this setting. We assume $w_i = r$.

The necessary conditions for a solution to the regulator's problem in this setting include:

$$w_n = a_n + 2\, b_n \frac{Q_t^n}{K_{Dn}}\,; \tag{54}$$

$$\sum_{t \in \{\psi, H\}} w_i\, \theta_t^E + k_i = a_{Di} + 2\, b_{Di}\, K_{Di}\,; \tag{55}$$

$$\sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} b_n \left(\frac{Q_t^n}{K_{Dn}} \right)^2 dF_t(\theta_t) + k_n = a_{Dn} + 2\, b_{Dn}\, K_{Dn}$$

$$\Rightarrow \frac{b_n}{(K_{Dn})^2} \sum_{t \in \{\psi, H\}} [\, Q_t^n\,]^2 + k_n = a_{Dn} + 2\, b_{Dn}\, K_{Dn}\,; \tag{56}$$

$$\sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \frac{c_v\, Q_t^v(\cdot)}{(K_G)^2}\, dF_t(\theta_t) = a_K + 2\, b_K\, K_G + a_T^G$$

$$\Rightarrow \frac{c_v}{(K_G)^2} \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t) = a_K + 2\, b_K\, K_G + a_T^G\,; \tag{57}$$

$$k_i = \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[a_v + 2\, b_v\, Q_t^v(\cdot) + \frac{c_v}{K_G} - w_i + e_v - e_i \right] \theta_t\, dF_t(\theta_t) - a_T^{Di}$$

$$= \sum_{t \in \{\psi, H\}} \left[\left(a_v + \frac{c_v}{K_G} - w_i + e_v - e_i \right) \theta_t^E + 2\, b_v \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, \theta_t\, dF_t(\theta_t) \right] - a_T^{Di}\,; \tag{58}$$

$$w_n = \frac{1}{2} \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\frac{\partial C_t^G(\cdot)}{\partial Q_t^v} + \psi_{tv}(\cdot) - \psi_{tn}(\cdot) \right] dF_t(\theta_t)$$

$$= \left[\, a_v + \frac{c_v}{K_G} + e_v - e_n \,\right] + b_v \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t)\,; \tag{59}$$

$$
\begin{aligned}
k_n &= \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[a_v + 2\, b_v\, Q_t^v(\cdot) + \frac{c_v}{K_G} - w_n + e_v - e_n \right] \frac{\partial Q_t^n}{\partial K_{Dn}}\, dF_t(\theta_t) - a_T^{Dn} \\
&= \sum_{t \in \{\psi, H\}} \frac{w_n - a_n}{2\, b_n} \left[a_v + \frac{c_v}{K_G} - w_n + e_v - e_n + 2\, b_v \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t) \right] - a_T^{Dn}\,; \qquad (60)
\end{aligned}
$$

$$
\begin{aligned}
r &= \frac{\sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[a_v + 2\, b_v\, Q_t^v(\cdot) + \frac{c_v}{K_G} + e_v \right] [\, \alpha_{Dt} + \alpha_{Nt} \,]\, dF_t(\theta_t)}{\sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} [\, \alpha_{Dt} + \alpha_{Nt} \,]\, dF_t(\theta_t)} \\
&= a_v + \frac{c_v}{K_G} + e_v + b_v \sum_{t \in \{\psi, H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t)\,. \qquad (61)
\end{aligned}
$$

Equations (54) – (61) characterize a system of 10 equations and 10 unknowns that can be solved using the PATH algorithm and numerical integration. $E\{Q_t^v(\cdot)\}$ and $E\{Q_t^v(\cdot)\,\theta_t\}$ are defined in (27) and (28).

Flat Retail Prices and No DG Capacity Payments

Now consider the setting where the regulator can set: (i) retail prices that do not vary across customers or over time (r); (ii) a fixed retail charge (R); and (iii) technology-specific DG payments (w_y). The regulator cannot implement DG capacity payments.

The necessary conditions for a solution to the regulator's problem in this setting include:

$$w_n \;=\; a_n + 2\, b_n \frac{Q_t^n}{K_{Dn}} \, ; \tag{62}$$

$$\sum_{t \in \{\, \psi, H\,\}} w_i \, \theta_t^E \;=\; a_{Di} + 2\, b_{Di}\, K_{Di} \, ; \tag{63}$$

$$\sum_{t \in \{\, \psi, H\,\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} b_n \left(\frac{Q_t^n}{K_{Dn}} \right)^2 dF_t(\theta_t) \;=\; a_{Dn} + 2\, b_{Dn}\, K_{Dn}$$

$$\Rightarrow \quad \frac{b_n}{(K_{Dn})^2} \sum_{t \in \{\, \psi, H\,\}} [\, Q_t^n \,]^2 \;=\; a_{Dn} + 2\, b_{Dn}\, K_{Dn} \, ; \tag{64}$$

$$\sum_{t \in \{\, \psi, H\,\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \frac{c_v \, Q_t^v(\cdot)}{(K_G)^2} \, dF_t(\theta_t) \;=\; a_K + 2\, b_K\, K_G + a_T^G$$

$$\Rightarrow \quad \frac{c_v}{(K_G)^2} \sum_{t \in \{\, \psi, H\,\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t) \;=\; a_K + 2\, b_K\, K_G + a_T^G \, ; \tag{65}$$

$$r \;=\; \frac{\sum_{t \in \{\, \psi, H\,\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[a_v + 2\, b_v \, Q_t^v(\cdot) + \frac{c_v}{K_G} + e_v \right] [\, \alpha_{Dt} + \alpha_{Nt} \,]\, dF_t(\theta_t)}{\sum_{t \in \{\, \psi, H\,\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} [\, \alpha_{Dt} + \alpha_{Nt} \,]\, dF_t(\theta_t)}$$

$$= \; a_v + \frac{c_v}{K_G} + e_v + b_v \sum_{t \in \{\, \psi, H\,\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\, dF_t(\theta_t) \, . \tag{66}$$

$$\sum_{t \in \{\, \psi, H\,\}} w_i \, \theta_t^E \;=\; \sum_{t \in \{\, \psi, H\,\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[a_v + 2\, b_v \, Q_t^v(\cdot) + \frac{c_v}{K_G} + e_v - e_i \right] \theta_t \; dF_t(\theta_t) - a_T^{Di}$$

$$= \; \sum_{t \in \{\, \psi, H\,\}} \left\{ \left[a_v + \frac{c_v}{K_G} + e_v - e_i \right] \theta_t^E + 2\, b_v \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot) \; \theta_t \; dF_t(\theta_t) \right\} - a_T^{Di} \, . \tag{67}$$

$$
-\Big[\sum_{t \in \{ \psi, H \}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(w_n - a_v - 2\, b_v\, Q_t^v(\cdot) - \frac{c_v}{K_G} + e_n - e_v \right) \frac{\partial Q_t^n}{\partial K_{Dn}}\, dF_t(\theta_t) + a_T^{Dn} \Big] \frac{\partial K_{Dn}}{\partial w_n}
$$

$$
- \sum_{t \in \{ \psi, H \}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[w_n - a_v - 2\, b_v\, Q_t^v(\cdot) - \frac{c_v}{K_G} + e_n - e_v \right] \frac{\partial Q_t^n}{\partial w_n}\, dF_t(\theta_t) \;=\; 0\,. \tag{68}
$$

From (62):

$$
\frac{\partial Q_t^n}{\partial K_{Dn}} \;=\; \frac{1}{2\, b_n} \left[\, w_n - a_n \,\right] \;=\; \frac{Q_t^n}{K_{Dn}}\,. \tag{69}
$$

Differentiating (62) and (64) with respect to w_n provides:

$$
\frac{2\, b_n}{K_{Dn}}\, \frac{dQ_\psi^n}{dw_n} - \frac{2\, b_n\, Q_\psi^n}{(K_{Dn})^2}\, \frac{dK_{Dn}}{dw_n} \;=\; 1\,; \tag{70}
$$

$$
\frac{2\, b_n}{K_{Dn}}\, \frac{dQ_H^n}{dw_n} - \frac{2\, b_n\, Q_H^n}{(K_{Dn})^2}\, \frac{dK_{Dn}}{dw_n} \;=\; 1\,; \tag{71}
$$

$$
\frac{2\, b_n\, Q_\psi^n}{(K_{Dn})^2}\, \frac{dQ_\psi^n}{dw_n} + \frac{2\, b_n\, Q_H^n}{(K_{Dn})^2}\, \frac{dQ_H^n}{dw_n}
- \frac{2\, b_n\, \sum_{t\in\{L,H\}} (Q_t^n)^2}{(K_{Dn})^3}\, \frac{dK_{Dn}}{dw_n} - 2\, b_{Dn}\, \frac{dK_{Dn}}{dw_n} \;=\; 0\,. \tag{72}
$$

(70) – (72) can be rewritten as:

$$
A \begin{bmatrix} \frac{dQ_\psi^n}{dw_n} \\ \frac{dQ_H^n}{dw_n} \\ \frac{dK_{Dn}}{dw_n} \end{bmatrix} \;=\; \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \tag{73}
$$

$$
\text{where} \quad A \;=\; \begin{bmatrix} \frac{2\, b_n}{K_{Dn}} & 0 & -\frac{2\, b_n\, Q_\psi^n}{(K_{Dn})^2} \\ 0 & \frac{2\, b_n}{K_{Dn}} & -\frac{2\, b_n\, Q_H^n}{(K_{Dn})^2} \\ \frac{2\, b_n\, Q_\psi^n}{(K_{Dn})^2} & \frac{2\, b_n\, Q_H^n}{(K_{Dn})^2} & -\frac{2\, b_n}{(K_{Dn})^3} \sum_{t\in\{L,H\}} (Q_t^n)^2 - 2\, b_{Dn} \end{bmatrix} \tag{74}
$$

Observe that:

$$
|\, A \,| \;=\; \frac{2\, b_n}{K_{Dn}} \left[\frac{2\, b_n}{K_{Dn}} \left(-\frac{2\, b_n}{(K_{Dn})^3} \sum_{t\in\{L,H\}} (Q_t^n)^2 - 2\, b_{Dn} \right) - \frac{2\, b_n\, Q_H^n}{(K_{Dn})^2} \left(\frac{-\, 2\, b_n\, Q_H^n}{(K_{Dn})^2} \right) \right]
$$

$$- \frac{2\,b_n\,Q^n_\psi}{(K_{Dn})^2}\left[0 - \left(\frac{2\,b_n}{K_{Dn}}\right)\frac{2\,b_n\,Q^n_\psi}{(K_{Dn})^2}\right] \;=\; \frac{-\,8\,b_{Dn}\,(b_n)^2}{(K_{Dn})^2}\,. \tag{75}$$

(74), (75), and Cramer's Rule imply:

$$\frac{d\,Q^n_\psi}{dw_n} \;=\; \frac{1}{|\,A\,|}\begin{vmatrix} 1 & 0 & -\frac{2\,b_n\,Q^n_\psi}{(K_{Dn})^2} \\ 1 & \frac{2\,b_n}{K_{Dn}} & -\frac{2\,b_n\,Q^n_H}{(K_{Dn})^2} \\ 0 & \frac{2\,b_n\,Q^n_H}{(K_{Dn})^2} & -\frac{2\,b_n}{(K_{Dn})^3}\sum_{t\in\{L,H\}}(Q^n_t)^2 - 2\,b_{Dn} \end{vmatrix}$$

$$= \frac{1}{|\,A\,|}\left[\frac{-\,4\,(b_n)^2}{(K_{Dn})^4}\sum_{t\,\in\,\{\psi,H\}}(Q^n_t)^2 - \frac{4\,b_n\,b_{Dn}}{K_{Dn}} + \frac{4\,(b_n)^2(Q^n_H)^2}{(K_{Dn})^4} - \frac{4\,(b_n)^2\,Q^n_H\;Q^n_\psi}{(K_{Dn})^4}\right]. \tag{76}$$

$Q^n_H \;=\; Q^n_\psi$, from (62). Consequently, (75) and (76) imply:

$$\frac{d\,Q^n_\psi}{dw_n} \;=\; -\,\frac{(K_{Dn})^2}{8\,b_{Dn}\,(b_n)^2}\left[-\,\frac{4\,b_n}{K_{Dn}}\right]\left[\frac{b_n}{(K_{Dn})^3}\;2\,(Q^n_\psi)^2 + b_{Dn}\right]$$

$$= \frac{K_{Dn}}{2\,b_n\,b_{Dn}}\left[\frac{2\,b_n}{(K_{Dn})^3}\,(Q^n_\psi)^2 + b_{Dn}\right] \;=\; \frac{1}{b_{Dn}}\left(\frac{Q^n_\psi}{K_{Dn}}\right)^2 + \frac{K_{Dn}}{2\,b_n}\,. \tag{77}$$

Similarly, it is readily shown that:

$$\frac{d\,Q^n_H}{dw_n} \;=\; \frac{1}{b_{Dn}}\left(\frac{Q^n_H}{K_{Dn}}\right)^2 + \frac{K_{Dn}}{2\,b_n}\,. \tag{78}$$

(73), (74), and Cramer's Rule imply:

$$\frac{dK_{Dn}}{dw_n} \;=\; \frac{1}{|\,A\,|}\begin{vmatrix} \frac{2\,b_n}{K_{Dn}} & 0 & 1 \\ 0 & \frac{2\,b_n}{K_{Dn}} & 1 \\ \frac{2\,b_n\,Q^n_\psi}{(K_{Dn})^2} & \frac{2\,b_n\,Q^n_H}{(K_{Dn})^2} & 0 \end{vmatrix} \;=\; \frac{1}{|\,A\,|}\left[-\,\frac{4\,(b_n)^2\,Q^n_H}{(K_{Dn})^3} - \frac{4\,(b_n)^2\,Q^n_\psi}{(K_{Dn})^3}\right]. \tag{79}$$

$Q^n_H \;=\; Q^n_\psi$, from (62). Consequently, (75) and (79) imply:

$$\frac{dK_{Dn}}{dw_n} \;=\; -\,\frac{(K_{Dn})^2}{8\,b_{Dn}\,(b_n)^2}\left[-\,\frac{8\,(b_n)^2\;Q^n_H}{(K_{Dn})^3}\right] \;=\; \frac{Q^n_H}{b_{Dn}\,K_{Dn}}\,. \tag{80}$$

$Q^n_H \;=\; Q^n_\psi \;=\; Q^n$ from (62). Therefore, (69), (77), (78), and (80) imply that (68) can be written as:

$$
\begin{aligned}
&-\Bigg[\sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left(w_n - a_v - 2\,b_v\,Q_t^v(\cdot) - \frac{c_v}{K_G} + e_n - e_v \right) \frac{Q_t^n}{K_{Dn}}\,dF_t(\theta_t) + a_T^{Dn} \Bigg] \frac{Q_H^n}{b_{Dn}\,K_{Dn}} \\
&- \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[w_n - a_v - 2\,b_v\,Q_t^v(\cdot) - \frac{c_v}{K_G} + e_n - e_v \right] \\
&\qquad\qquad \cdot \left[\frac{1}{b_{Dn}} \left(\frac{Q_H^n}{K_{Dn}} \right)^2 + \frac{K_{Dn}}{2\,b_n} \right] dF_t(\theta_t) \;=\; 0
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\quad & - \Bigg[2 \left(w_n - a_v - \frac{c_v}{K_G} + e_n - e_v \right) \frac{Q^n}{K_{Dn}} \\
&\qquad - \frac{2\,b_v\,Q^n}{K_{Dn}} \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\,dF_t(\theta_t) + a_T^{Dn} \Bigg] \frac{Q^n}{b_{Dn}\,K_{Dn}} \\
& -2 \left[w_n - a_v - \frac{c_v}{K_G} + e_n - e_v \right] \left[\frac{1}{b_{Dn}} \left(\frac{Q^n}{K_{Dn}} \right)^2 + \frac{K_{Dn}}{2\,b_n} \right] \\
& + 2\,b_v \left[\frac{1}{b_{Dn}} \left(\frac{Q^n}{K_{Dn}} \right)^2 + \frac{K_{Dn}}{2\,b_n} \right] \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\; dF_t(\theta_t) \;=\; 0
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\quad & -\,2 \left[w_n - a_v - \frac{c_v}{K_G} + e_n - e_v \right] \frac{(Q^n)^2}{b_{Dn}(K_{Dn})^2} \\
& + \frac{2\,b_v}{b_{Dn}} \left(\frac{Q^n}{K_{Dn}} \right)^2 \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\,dF_t(\theta_t) - \frac{Q^n}{b_{Dn}\,K_{Dn}}\; a_T^{Dn} \\
& -2 \left[w_n - a_v - \frac{c_v}{K_G} + e_n - e_v \right] \left[\frac{1}{b_{Dn}} \left(\frac{Q^n}{K_{Dn}} \right)^2 + \frac{K_{Dn}}{2\,b_n} \right] \\
& + 2\,b_v \left[\frac{1}{b_{Dn}} \left(\frac{Q^n}{K_{Dn}} \right)^2 + \frac{K_{Dn}}{2\,b_n} \right] \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\; dF_t(\theta_t) \;=\; 0
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\quad & -\,2 \left[w_n - a_v - \frac{c_v}{K_G} + e_n - e_v \right] \left[\frac{2}{b_{Dn}} \left(\frac{Q^n}{K_{Dn}} \right)^2 + \frac{K_{Dn}}{2\,b_n} \right] - \frac{Q^n}{b_{Dn}\,K_{Dn}}\; a_T^{Dn} \\
& + 2\,b_v \left[\frac{2}{b_{Dn}} \left(\frac{Q^n}{K_{Dn}} \right)^2 + \frac{K_{Dn}}{2\,b_n} \right] \sum_{t\in\{\psi,H\}} \int_{\underline{\theta}_t}^{\overline{\theta}_t} Q_t^v(\cdot)\; dF_t(\theta_t) \;=\; 0\,. \qquad (81)
\end{aligned}
$$

(62) – (67) and (81) constitute a system of 8 equations and 8 unknowns that can be solved using the PATH algorithm and numerical integration. $E\{Q_t^v(\cdot)\}$ and $E\{Q_t^v(\cdot)\,\theta_t\}$ are defined in (27) and (28).

C. Additional Numerical Solutions.

We now present two sets of additional numerical solutions to complement those provided in the paper. The first set of numerical solutions consider outcomes in a "coal-intensive" setting which is designed to reflect a environment where S primarily employs coal-powered generating units to serve a relatively small market (as in Ohio, for instance). In this setting, $\phi_c = 0.9$, $\phi_g = 0.1$ (so $e_v = 35.692$), and the demand parameters are chosen to ensure that expected demand for consumer j in period t is $\eta_j\,\widehat{X}_t$,[4] where $\widehat{X}_\psi = 9,748$ and $\widehat{X}_H = 18,142$.[5]

S's capacity cost function, $C^K(K_G) = 16.1\,K_G + .000674\,(K_G)^2$, is specified to ensure the marginal cost of capacity required to generate a MWh of electricity is approximately 60.4 when $K_G = \widehat{K}_G = 32,854$. $\widehat{K}_G = 32,854$ MW reflects the level of centralized non-renewable generation capacity in Ohio in 2013 (EIA, 2015c). The estimated cost of capacity required to produce a MWh of electricity using a coal generating unit is \$60.4 (EIA, 2015a).

As in the baseline setting, we choose a_v, b_v, and c_v (the parameters of S's quadratic $C^G(\cdot)$ function) to reflect Bushnell (2007)'s estimates for the serving region of the PJM regional transmission organization, assuming that the welfare-maximizing level of S's capacity in the model is $\widehat{K}_G$. Doing so provides the cost function $C^G(Q^v) = 0.00313\,(Q^v)^2$.

The values of a_{Dy} and b_{Dy} for $y \in \{i, n\}$ are set as in the baseline setting, except that the benchmark level of DG-n capacity is reduced to $\widehat{K}_{Dn} = 3,837.35 = 0.1168 \cdot \widehat{K}_G$, reflecting the ratio of non-intermittent to centralized capacity ($\overline{K}_{Dn}/\overline{K}_G = 0.1168$) observed in the baseline setting.

Table A1 records outcomes under the optimal regulatory policy in this setting when retail charges (r, R) cannot vary with customer type, the prevailing demand period, or the realized state. The variables reported in Table A1 are defined in the paper.

[4] The demand parameters are also set to ensure the relevant elasticities of demand with respect to θ are 0.1 and the relevant price elasticities of demand are -0.25 at the relevant expected levels of price and demand.

[5] Annual system peak load in Ohio was 27,563 MWs in 2013 (Ohio PUC, 2015). $\widehat{X}_H$ is taken to be $0.66\,[27,563] = 18,192$ MWs, employing the 0.66 ratio of average peak load to system peak load in California in 2014. $\widehat{X}_L$ is taken to be $1.867\,[18,192] = 9,748$ MWs, using the 1.867 ratio of average peak to off-peak demand that prevailed in California in 2014.

Variable	Without DG Capacity Charges	With DG Capacity Charges
r	156.1	156.3
w_i	189.3	156.3
w_n	134.4	135.2
k_i	0	23.0
k_n	0	-2.4
K_{Di}	$5,176$	$5,193$
K_{Dn}	$3,657$	$3,585$
K_G	$29,116$	$29,142$
$E\{c^v\}$	132.7	132.7
π_{Di}	$70,455$	$70,903$
π_{Dn}	$92,984$	$89,326$
$E\{U^D\}$	$1,100,890$	$1,098,048$
$E\{U^N\}$	$8,437,104$	$8,440,412$
$E\{\psi\}$	$841,345$	$841,610$
$E\{W\}$	$8,696,649$	$8,696,849$

Table A1. Outcomes in the Coal Intensive Setting.

Table A1 reports that when the regulator can set DG capacity charges in the coal intensive setting, she optimally implements a capacity payment ($k_i > 0$) for the clean DG-i technology and a capacity tax ($k_n < 0$) for the less clean DG-n technology. This payment and tax induce increased investment in DG-i capacity and reduced investment in DG-n capacity. On balance, profit from DG operations declines and social losses from externalities increase slightly.

The second set of numerical solutions examine the optimal regulatory policy and industry outcomes in the baseline setting when the regulator sets DG capacity charges to maximize expected welfare. Table A2 compares outcomes for three cases: (i) when the retail price of electricity (r) cannot vary within or across time periods ("Flat"); (ii) when TOU retail pricing is implemented ("TOU"); and (iii) when state-specific retail pricing is implemented ("SS").[6] Because consumer D cannot control the level of output from the DG-i technology other than through his choice of capacity (K_{Di}), w_i and k_i are redundant instruments for the regulator when both are available. In this event, we assume the regulator implements net metering ($w_i = r$) for the DG-i technology.

[6] In each case, the regulator can set a per-unit retail charge (r) and a fixed retail charge (R). Reflecting common practice, these charges do not vary by customer type. The unit retail prices (r) and DG output payments (w) reported in the last column of Table A2 reflect expected prices and payments.

Variable	Flat	TOU	SS
r_ψ	148.4	122.7	122.3
r_H	148.4	172.5	172.6
$w_{i\psi}$	148.4	122.7	122.3
w_{iH}	148.4	172.5	172.6
$w_{n\psi}$	127.4	101.7	101.3
w_{nH}	127.4	151.5	151.6
k_i	21.5	5.6	-5.1
k_n	-2.4	-2.4	-2.4
K_{Di}	$3,869$	$2,922$	$2,301$
K_{Dn}	$6,010$	$6,564$	$6,964$
K_G	$66,831$	$66,362$	$66,266$
$E\{c^v\}$	191.5	189.3	187.4
π_{Di}	$39,368$	$22,446$	$14,034$
π_{Dn}	$113,125$	$134,940$	$152,012$
$E\{U^D\}$	$1,444,739$	$1,457,830$	$1,471,888$
$E\{U^N\}$	$11,630,212$	$11,703,983$	$11,752,576$
$E\{\psi\}$	$604,761$	$612,459$	$623,007$
$E\{W\}$	$12,470,189$	$12,549,354$	$12,601,456$

Table A2. The Effects of Retail Pricing Flexibility in the Baseline Setting.

Table A2 reports that when TOU and state-specific pricing are feasible, the regulator sets (expected) retail prices and DG output payments that are higher in the peak period than in the off-peak period. The optimal level of centralized capacity declines modestly and the regulator reduces k_i in order to (substantially) reduce investment in DG-i capacity. Expected losses from environmental externalities increase modestly, as does the expected utility of both consumers. Expected aggregate welfare increases slightly.

References Not Included in Paper

Ohio Public Utilities Commission. (2015). *Ohio Long Term Forecast of Energy Requirements.* A Report by the Staff of the Public Utilities Commission of Ohio. https://www.puco.ohio.gov/industry-information/statistical-reports/ohio-long-term-energy-forecast/ohio-ltfr-2014-2033.