The Impact of Wholesale Price Caps on Forward Contracting

by

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Abstract

Increasing (or eliminating) the caps on short-term wholesale prices is generally thought to promote long-term forward contracting for electricity. We find that a higher price cap typically enhances the incentives of electricity buyers (e.g., load-serving entities) to undertake forward contracting. However, a higher cap can diminish the incentives of electricity generators to engage in forward contracting. Consequently, higher wholesale price caps can reduce industry forward contracting.

Keywords: wholesale price caps; forward contracting; electricity markets

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1 Introduction

To protect consumers against price shocks, the maximum price that can be charged for electricity is explicitly limited in many restructured wholesale markets.[1] Wholesale price caps are often criticized, in part on the grounds that they reduce incentives to secure electricity via long-term "forward" contracts.[2] Forward contracting is valued for at least three reasons. First, like wholesale price caps, forward contracting can help to counteract short-term price volatility. Such volatility seems likely to increase over time as larger portions of electricity supply are generated by intermittent renewable resources (Newbery et al., 2018; Joskow, 2019; Wolak, 2022). Second, expanded forward contracting can encourage generators to increase the amount of electricity they supply in wholesale markets, thereby reducing wholesale prices (Allaz and Vila, 1993). Third, forward contracting can encourage expanded investment in generation capacity. It can do so by allowing generators to secure in advance a stable revenue stream for the output that will ultimately be produced by the expanded capacity.[3]

The reason why wholesale price caps are commonly believed to reduce incentives for forward contracting is straightforward. If buyers are protected against high prices in the wholesale market, they will be less inclined to sign forward contracts that offer protection against high wholesale prices for electricity. Wolak (2021, pp. 86-87) observes that in the presence of a "limited prospect of very high prices because of [price] caps, retailers may decide not to sign fixed-price forward contracts. ... The lower the [price] cap, the greater is the likelihood that the retailer will delay its electricity purchases to the short-term market." Similarly, Mays and Jenkins (2022, p. 2) suggest that "[w]ithout the threat of high prices, consumers of energy have insufficient incentive to enter forward contracts with generators."

The purpose of the present research is to assess this common wisdom formally, explicitly accounting for the strategic decisions of generators and the endogeneity of short-term and

[1]To illustrate, the prevailing cap on the wholesale price of electricity is $999.99 per megawatt hour (MWh) in Alberta, Canada (Brown and Olmstead, 2017) and $5,000 per MWh in Texas (Smith, 2022). A lower cap is imposed in Texas if electricity generators are deemed to have secured sufficient profit from an extended period of high wholesale prices (University of Texas, 2021).

[2]Wholesale price caps are also criticized because they limit the profit that generators secure during periods of particularly high demand, and can thereby reduce investment in generator capacity (Joskow, 2008; Hogan, 2017).

[3]In part to facilitate entry by new generators, Australia requires large buyers of electricity to secure a significant portion of their anticipated demand via long-term forward contracts (Australian Government, 2019).

long-term electricity prices. We find that, although the common wisdom has considerable merit, it does not fully capture the effects of wholesale price caps on incentives for forward contracting for two reasons. First, although a higher wholesale price cap typically enhances an electricity buyer's incentive for forward contracting, it does not necessarily do so. Second, and of greater empirical relevance, a higher price cap can reduce the incentives of generators to undertake forward contracting.

A higher price cap ($\overline{w}$) can either enhance or diminish a generator's incentive for forward contracting because an increase in $\overline{w}$ entails both a *revenue enhancement effect* that promotes forward contracting and a *regime shifting effect* that can diminish incentives for forward contracting. The revenue enhancement effect arises because an increase in $\overline{w}$ increases the payment (i.e., the wholesale price) a generator receives for each unit of electricity it sells when the price cap binds. The increased wholesale price enhances a generator's incentive to increase its equilibrium output, which it achieves by expanding its forward contracting.[4] Thus, the revenue enhancement effect of an increase in $\overline{w}$ strengthens a generator's incentive to undertake forward contracting.

The regime shifting effect arises because an increase in $\overline{w}$ reduces the likelihood that the cap binds. Consequently, an increase in $\overline{w}$ can reduce a generator's incentive for forward contracting if the rate at which expanded forward contracting enhances a generator's profit is lower when the price cap does not bind than when it binds. This outcome can arise under plausible conditions, in part because expanded forward contracting reduces the equilibrium wholesale price when the price cap does not bind, but does not alter the wholesale price when the cap binds.[5]

To determine whether the regime shifting effect can ever outweigh the revenue enhancement effect, causing forward contracting to decline as $\overline{w}$ increases, we examine equilibrium outcomes in a stylized setting that reflects selected elements of actual electricity markets. We find that when generators choose their preferred levels of forward contracting (non-cooperatively) in this setting, the equilibrium level of aggregate forward contracting often declines as the wholesale price cap increases.

Our analysis contributes to the extensive formal literature on forward contracting, which

[4]Allaz and Vila (1993) show that expanded forward contracting endows a generator with a credible commitment to expand its output in the wholesale market.

[5]Expanded forward contracting increases equilibrium output, which reduces the wholesale price when the price cap does not bind.

establishes that forward contracting can induce generators to compete aggressively and expand their outputs in wholesale markets. Allaz and Vila (1993)'s seminal work documents these effects of forward contracting in a non-repeated setting with Cournot competition and publicly-observed levels of forward contracting. Subsequent studies consider alternative forms of competition (Holmberg, 2011; Holmberg and Willems, 2015), repeated interactions (Liski and Montero, 2006; Green and Le Coq, 2010), and settings where prevailing levels of forward contracting are not observed publicly (Hughes and Kao, 1997). Empirical studies also document the competition-enhancing effects of forward contracts (e.g., Wolak, 2000; Bushnell et al., 2008; van Eijkel et al., 2016). This extensive formal literature largely abstracts from the impact of wholesale price caps on forward contracting.[6]

This literature focuses on settings in which generators dictate the levels of forward contracting. In practice, large buyers of electricity are key counterparties in forward market transactions. Like our analysis, a few studies consider settings where buyers exercise some control over the extent of forward contracting (Anderson and Hu, 2008; Schneider, 2020; Brown and Sappington, 2023a, 2023b). However, these studies do not consider the effects of wholesale price caps on equilibrium levels of forward contracting.

A distinct strand of the literature analyzes the effects of price caps in oligopoly markets, but does not consider forward contracting. Studies in this literature analyze the impact of price caps on output, consumer surplus, and welfare in settings with uncertain demand (Earle et al., 2007; Grimm and Zottl, 2010; Reynolds and Rietzke, 2018). Other studies identify conditions under which a wholesale price cap reduces incentives for capacity investment (Fabra et al., 2011; Zottl, 2011).

We contribute to these strands of the literature by examining how wholesale price caps affect the incentives of both generators and large buyers of electricity to undertake forward contracting. We do so in a stylized model with linear demand and costs, a uniform density for demand uncertainty, and a regulated load serving entity that is required to serve the realized demand of its customers. We identify conditions under which the aforementioned common wisdom – that wholesale price caps reduce buyers' incentives for forward contracting

[6] Holmberg (2011) analyzes a model in which firms choose forward contracts before competing via supply functions in the wholesale market. The author conjectures (p. 187) that "reducing price caps ... would stimulate strategic [forward] contracting." However, he does not investigate this issue formally. Yao et al. (2007) develop an optimization program in which generators choose forward contracts and engage in Cournot competition. A wholesale price cap reduces the generators' incentives for forward contracting in the numerical example the authors analyze.

– prevails.[7] We further demonstrate that the common wisdom about buyers' incentives does not readily extend to generators' incentives. Consequently, any attempt to assess how a change in a prevailing wholesale price cap will affect industry forward contracting should consider both the relative influence of buyers and sellers in determining the levels of forward contracting and the distinct ways in which a price cap affects their incentives for forward contracting.

The ensuing analysis proceeds as follows. Section 2 describes the key elements of our model. Section 3 examines the electricity buyer's incentives for forward contracting. Section 4 analyzes electricity generators' incentives for forward contracting. Section 5 provides concluding observations. The Appendix provides the proofs of all formal conclusions.

2 Model Elements

A large buyer of electricity (e.g., a load serving entity) is required to deliver all the electricity its retail customers demand. Realized retail demand is $\overline{Q}+\eta$, where $\overline{Q}$ is expected demand and $\eta \in [\,\underline{\eta}, \overline{\eta}\,]$ is the realization of a mean-zero random variable. Thus, retail demand is stochastic and perfectly price inelastic. Retail demand is always positive, so $\overline{Q} + \underline{\eta} > 0$.

Commercial and industrial customers also consume electricity, which they purchase in the wholesale market at unit price w. Their demand for electricity is $Q^I(w) = a^I - b^I w$, where a^I and b^I are positive constants. Aggregate demand for electricity – the sum of retail demand and industrial demand – is:

$$Q(\cdot) \;=\; a^I - b^I w + \overline{Q} + \eta \,. \tag{1}$$

The corresponding inverse demand curve is:

$$w(\cdot) \;=\; a + \varepsilon - b\,Q \quad \text{where} \;\; a = \frac{a^I + \overline{Q}}{b^I}, \;\; \varepsilon = \frac{\eta}{b^I}, \;\text{and}\;\; b = \frac{1}{b^I}\,. \tag{2}$$

$h(\varepsilon)$ is the density function for the random demand parameter $\varepsilon \in [\,-\overline{\varepsilon}, \overline{\varepsilon}\,]$. $H(\varepsilon)$ is the corresponding distribution function.

Electricity is supplied by $n \geq 2$ generators, $G1, ..., Gn$. Gi's cost of producing q units of output is $c_i\, q$, where $c_i > 0$ is a parameter, for $i = 1, ..., n$.

The large buyer (B) can purchase electricity in the wholesale market and/or procure

[7] We show that a buyer's incentive for forward contracting depends in part on the nature of the retail price regulation it faces. This incentive typically declines as the regulated retail price tracks the expected wholesale price more closely.

electricity via a long-term forward contract. A forward contract between B and generator Gi obligates Gi to ensure that B ultimately can purchase a unit of electricity in the wholesale market at net price p^f. This net price is the difference between the realized wholesale price, w, and the compensation that Gi delivers to B, which is $w - p^f$.[8] To illustrate, when Gi and B sign F_i forward contracts and the realized wholesale price is $w > p^f$, Gi must pay B the amount $F_i \left[w - p^f \right]$ to ensure that B's net cost of securing the F_i units of electricity is $F_i \left[w - \left(w - p^f \right) \right] = p^f F_i$.[9] For expositional ease, the ensuing discussion will refer to p^f as the *price* of a forward contract.

We adopt the standard assumption in the literature that the price of a forward contract, p^f, is equal to the expected wholesale price of electricity, $E\{w(\varepsilon)\}$, where $w(\varepsilon)$ is the wholesale price that prevails when realized retail demand is $\overline{Q} + \varepsilon$. This equality will prevail in the following two stage game. In the first stage, either the buyer or the generators specify the number of forward contracts that will be signed.[10] In the second stage, risk-neutral, non-strategic actors (e.g., financial traders) with rational expectations bid for the contracts. The bidding ensures that the price of a forward contract reflects its expected value, an outcome that is anticipated in the first stage of the game.[11]

If Gi signs F_i forward contracts with B, then Gi's profit when it produces q_i units of output and wholesale price w prevails is:

$$\pi_i^G = w \left[q_i - F_i \right] + p^f F_i - c_i \, q_i \, . \tag{3}$$

Equation (3) reflects the fact that Gi sells $q_i - F_i$ units of output in the wholesale market at unit price w and effectively sells F_i units of output via forward contract at unit price p^f.

When realized retail demand is $\overline{Q} + b^I \varepsilon$ and the wholesale price is w, B's profit is:

$$\pi^B(\varepsilon) = R(\varepsilon) - w \left[\overline{Q} + b^I \varepsilon - F \right] - p^f F - K \, , \tag{4}$$

[8]Thus, we consider fixed-price, fixed-quantity financial forward contracts that are settled at the prevailing wholesale price. Such contracts are common in electricity markets.

[9]If $w < p^f$ after Gi and B sign F_i forward contracts, then B pays Gi the amount $F_i \left[p^f - w \right]$. This payment ensures that B's net cost of securing the F_i units of electricity is $w \, F_i + F_i \left[p^f - w \right] = p^f F_i$.

[10]When the generators choose the levels of forward contracting, they do so simultaneously and non-collusively.

[11]See Allaz and Vila (1993, Appendix A) for a formal proof of this conclusion in a setting where generators determine the levels of forward contracting in the first stage of the game. Holmberg and Willems (2015) explain why the assumption that forward markets are efficient in this sense often constitutes a reasonable caricature of electricity forward markets. The concluding discussion considers alternative processes for determining the prices of forward contracts.

where $R(\varepsilon)$ denotes B's revenue when ε is realized,[12] $F \equiv \sum_{j=1}^{n} F_j$, and $K \geq 0$ denotes the (fixed) cost that B incurs in addition to electricity procurement costs. Equation (4) reflects the fact that B purchases $\overline{Q} + b^I \varepsilon - F$ units of electricity at unit price w in the wholesale market and procures F units of electricity at unit price p^f via forward contract.

$\overline{w}$ denotes the wholesale price cap, which is the highest wholesale price that is permitted. The wholesale price that prevails is the minimum of $\overline{w}$ and the wholesale price that equates the aggregate demand and the aggregate supply of electricity. We assume $\overline{w}$ > maximum $\{c_1, ..., c_n\}$.

The timing in the model is as follows. After the regulator specifies the wholesale price cap $\overline{w}$ and the retail revenue function $R(\cdot)$, the levels of forward contracting are determined.[13] Then the realization of ε is observed publicly. Next, the generators choose their outputs, simultaneously and non-cooperatively. After the wholesale price is determined, industrial customers decide how much electricity to purchase in the wholesale market. Finally, the terms of all forward contracts are fulfilled and B purchases the amount of electricity required to satisfy the realized demand of its retail customers.

Before examining the levels of forward contracting that arise in equilibrium, it is helpful to examine the wholesale price and the generators' outputs that arise in equilibrium, given the prevailing levels of forward contracting. Lemma 1 characterizes Gi's equilibrium output ($q_i(\varepsilon)$) when generator Gj signs F_j forward contracts with B ($j = 1, ..., n$) and when realized retail demand is $\overline{Q} + b^I \varepsilon$. Lemma 1 also characterizes the corresponding equilibrium wholesale price ($w(\varepsilon)$). The lemma refers to $\widehat{\varepsilon} \in (\underline{\varepsilon}, \overline{\varepsilon})$, which is the smallest realization of ε for which the wholesale price cap binds.[14] The lemma also refers to $C \equiv \sum_{j=1}^{n} c_j$, $C_{-i} \equiv \sum_{j=1, j \neq i}^{n} c_j$, $F \equiv \sum_{j=1}^{n} F_j$, and $F_{-i} \equiv \sum_{j=1, j \neq i}^{n} F_j$.

Lemma 1. *In equilibrium, given* ε *and* $F_1, ..., F_n$*:*

$$\textit{For all } \varepsilon < \widehat{\varepsilon} \equiv [n+1]\,\overline{w} - [a + C - b\,F]: \tag{5}$$

$$w(\varepsilon) = \frac{1}{n+1}[a + \varepsilon + C - b\,F] \quad \textit{and} \tag{6}$$

[12]For example, as explained further below, the regulator might specify a unit price, r, that retail customers must pay for electricity. In this case, $R(\varepsilon) = r\,[\,\overline{Q} + b^I \varepsilon\,]$.

[13]The values of $\overline{w}$ and $R(\cdot)$ are taken as given. The regulator's choice of these values is not modeled formally.

[14]The maintained assumption that $\widehat{\varepsilon} \in (\underline{\varepsilon}, \overline{\varepsilon})$ rules out uninteresting settings in which the price cap never binds or binds for all realizations of ε.

$$q_i(\varepsilon) \;=\; \frac{a + \varepsilon + C_{-i} - n\,c_i + b\,n\,F_i - b\,F_{-i}}{b\,[\,n+1\,]} \;\; for \;\; i \,=\, 1,...,n\,. \tag{7}$$

$$For\ all\ \varepsilon \,\geq\, \widehat{\varepsilon}: \quad w(\varepsilon) \;=\; \overline{w} \;\; and \;\; \sum_{i=1}^{n} q_i(\varepsilon) \;=\; \frac{1}{b}\,[\,a + \varepsilon - \overline{w}\,]\,. \tag{8}$$

Equation (7) implies that when $\varepsilon < \widehat{\varepsilon}$ (so the price cap does not bind), Gi's equilibrium output increases and the equilibrium outputs of rival generators decline as Gi's forward contracting (F_i) increases. Gi's increased forward contracting reduces the amount of electricity it sells at the prevailing wholesale price, w. This reduced exposure to the decline in w caused by an increase in output implies that Gi's profit increases more rapidly with its output, which induces Gi to increase its output. The corresponding reduction in w induces rival generators to reduce their equilibrium outputs (Allaz and Vila, 1993). On balance, industry output increases, so the wholesale price declines as a generator increases its forward contracting. (See equation (6).) The reduced wholesale price causes the set of the highest ε realizations for which the price cap binds to contract, i.e., $\widehat{\varepsilon}$ increases. (See equation (5).)

Equation (8) implies that multiple equilibria arise when $\varepsilon \geq \widehat{\varepsilon}$, as in Buehler et al. (2010). When the price cap binds, the wholesale price does not decline as a generator increases its output. Consequently, Gi's profit increases (at the rate $\overline{w} - c_i > 0$) as q_i increases, so each generator finds it profitable to serve the entire excess demand or any portion thereof. Therefore, when $\varepsilon \geq \widehat{\varepsilon}$ so the price cap binds, any set of outputs that sum to the realized aggregate demand when $w = \overline{w}$ constitute equilibrium outputs.[15]

It remains to characterize the levels of forward contracting that arise in equilibrium. These levels vary according to whether the extent of forward contracting is determined by the buyer or by generators. Section 3 examines the levels of forward contracting preferred by the buyer (B). Section 4 examines the corresponding equilibrium levels that will be chosen (non-cooperatively) by generators.

[15] For any set of such outputs, no generator has an incentive to reduce its output unilaterally because $\frac{\partial \pi_i^G}{\partial q_i} > 0$. Furthermore, no generator can increase its output unilaterally because, by construction, the inital set of outputs meet prevailing demand exactly.

3 The Buyer's Preferred Level of Forward Contracting

To determine B's preferred levels of forward contracting, consider the setting where B sells electricity to its retail customers at fixed unit price

$$r \;=\; \gamma\, r_0 + [\, 1 - \gamma \,]\, E\{\, w(\varepsilon) \} \tag{9}$$

where $\gamma \in (\, 0, 1\,]$ and $r_0 > 0$ are parameters. r_0 is assumed to be large enough to ensure that B secures nonnegative expected profit in equilibrium.[16] The formulation in equation (9) allows the regulated unit price to increase as the equilibrium expected wholesale price increases, as will be the case if the regulator adjusts the retail price to reflect anticipated impacts of forward contracting on the expected wholesale price. The formulation also encompasses the setting where the retail price of electricity is set at a fixed level (r_0) that does not vary with the anticipated wholesale price.[17]

Equation (4) implies that B's profit when ε is realized is:

$$\pi^B(\varepsilon) \;=\; r\, \left[\, \overline{Q} + b^I \varepsilon \,\right] - w(\varepsilon) \left[\, \overline{Q} + b^I \varepsilon - F \,\right] - p^f F - K\,. \tag{10}$$

Equation (10) implies that, because $p^f = E\{\, w(\varepsilon)\, \}$, B's expected profit is:

$$E\{\pi^B(\varepsilon)\} \;=\; E\{\, [\, r - w(\varepsilon)\,] \left[\, \overline{Q} + b^I \varepsilon \,\right] \} - K\,. \tag{11}$$

Equation (11) implies that B's expected profit can be written as the sum of: (i) $E\{\pi^{BD}(\varepsilon)\}$, B's expected profit from serving expected (or "deterministic") demand, $\overline{Q}$; and (ii) $E\{\pi^{BS}(\varepsilon)\}$, B's expected profit from serving stochastic demand, $b^I \varepsilon$.[18] Formally:

$$E\{\pi^B(\varepsilon)\} \;=\; E\{\pi^{BD}(\varepsilon)\} + E\{\pi^{BS}(\varepsilon)\} - K, \;\; \text{where}$$

$$E\{\pi^{BD}(\varepsilon)\} \;=\; E\left\{ [\, r - w(\varepsilon)\,]\, \overline{Q} \right\} \;\; \text{and} \;\; E\{\pi^{BS}(\varepsilon)\} \;=\; E\left\{ [\, r - w(\varepsilon)\,]\, b^I \varepsilon \right\}. \tag{12}$$

Lemma 2 characterizes the impact of expanded forward contracting on B's expected profit and its components.

Lemma 2. *Expanded forward contracting: (i) increases $E\{\pi^{BD}(\varepsilon)\}$; (ii) reduces $E\{\pi^{BS}(\varepsilon)\}$; and (iii) increases $E\{\pi^B(\varepsilon)\}$ if γ is sufficiently close to 1. More precisely, for each $i =$*

[16] This will be the case if r_0 is sufficiently large to ensure that B's expected profit is nonnegative even in the absence of forward contracting.

[17] For expositional ease, the ensuing analysis focuses on the case where $\gamma > 0$, so the retail price does not track the expected wholesale price exactly. The case where $\gamma = 0$ is dicussed in footnote 24 below.

[18] The labels "deterministic" and "stochastic" should not be taken literally. Expected retail demand, $\overline{Q}$, is not certain to arise, and it is the entire retail demand, $\overline{Q} + b^I \varepsilon$, that is stochastic.

$1, ..., n$:

$$\frac{\partial E\{\pi^{BD}(\varepsilon)\}}{\partial F_i} = \frac{\gamma\, b\, \overline{Q}\, [\, \widehat{\varepsilon} + \overline{\varepsilon}\,]}{2\, \overline{\varepsilon}\, [\, n+1\,]} > 0\,; \quad \frac{\partial E\{\pi^{BS}(\varepsilon)\}}{\partial F_i} = -\frac{(\, \overline{\varepsilon}\,) - (\, \widehat{\varepsilon}\,)}{4\, \overline{\varepsilon}\, [\, n+1\,]} < 0\,; \textit{ and}$$

$$\frac{\partial E\{\pi^{B}(\varepsilon)\}}{\partial F_i} = \frac{\widehat{\varepsilon} + \overline{\varepsilon}}{4\, \overline{\varepsilon}\, [\, n+1\,]} \left[\, 2\, \gamma\, b\, \overline{Q} - (\, \overline{\varepsilon} - \widehat{\varepsilon}\,)\, \right] \gtreqless 0 \;\Leftrightarrow\; \gamma \gtreqless \frac{\overline{\varepsilon} - \widehat{\varepsilon}}{2\, b\, \overline{Q}}\,. \qquad (13)$$

An increase in F_i increases $E\{\pi^{BD}(\varepsilon)\}$ by increasing B's expected profit margin, $E\{r - w(\cdot)\}$. The higher expected profit margin arises because an increase in F_i reduces the expected wholesale price more rapidly than it reduces the retail price.[19]

An increase in F_i reduces $E\{\pi^{BS}(\varepsilon)\}$ for two reasons. First, the increase in F_i increases B's profit margin, $r - w(\varepsilon)$, whenever the price cap does not bind. This is the case because $w(\varepsilon)$ declines more rapidly than r declines as F_i increases.[20] The increased margin is applied primarily (if not entirely) to negative stochastic demand, so $E\{\pi^{BS}(\varepsilon)\}$ declines.[21] Second, the increase in F_i reduces B's profit margin, $r - \overline{w}$, when the price cap binds. This is the case because r declines (when $\gamma < 1$) but $w(\varepsilon) = \overline{w}$ does not change. Because this reduced margin is applied primarily (if not entirely) to positive stochastic demand, $E\{\pi^{BS}(\varepsilon)\}$ declines.[22]

When γ is close to 1, the retail price is relatively insensitive to the expected wholesale price. Consequently, the predominant effect of an increase in F_i is to increase $E\{\pi^{BD}(\varepsilon)\}$ by increasing B's expected profit margin.[23] Therefore, $E\{\pi^{B}(\varepsilon)\}$ increases when γ is sufficiently close to 1. When γ is sufficiently small, an increase in F_i reduces r at nearly the same rate it reduces $E\{w(\varepsilon)\}$. The resulting limited impact on B's expected profit margin implies that the increase in F_i has little impact on $E\{\pi^{BD}(\varepsilon)\}$. Therefore, the predominant effect of an

[19] Equation (6) implies that $\frac{\partial E\{w(\varepsilon)\}}{\partial F_i} < 0$. Therefore, equation (9) implies that $\frac{\partial E\{r\}}{\partial F_i} < 0$ and $\left| \frac{\partial E\{r\}}{\partial F_i} \right| = [\, 1 - \gamma\,] \left| \frac{\partial E\{w(\varepsilon)\}}{\partial F_i} \right| < \left| \frac{\partial E\{w(\varepsilon)\}}{\partial F_i} \right|$.

[20] Equation (6) implies that $\left| \frac{\partial w(\varepsilon)}{\partial F_i} \right| = \frac{b}{n+1}$ when $\varepsilon \in [\, \underline{\varepsilon}, \widehat{\varepsilon}\,)$. Equation (33) in the proof of Lemma 2 in the Appendix reports that $\left| \frac{\partial E\{w(\varepsilon)\}}{\partial F_i} \right| = \frac{b}{n+1} \left[\, \frac{\widehat{\varepsilon} + \overline{\varepsilon}}{2\, \overline{\varepsilon}}\, \right] < \frac{b}{n+1}$. $\left| \frac{\partial E\{w(\varepsilon)\}}{\partial F_i} \right| < \left. \frac{\partial w(\varepsilon)}{\partial F_i} \right|_{\varepsilon < \widehat{\varepsilon}}$ because $w(\varepsilon)$ does not decline as F_i increases when $\varepsilon \in (\, \widehat{\varepsilon}, \overline{\varepsilon}\,]$.

[21] When stochastic demand is negative, demand is relatively small. The price cap does not bind for the smaller demand realizations.

[22] When stochastic demand is positive, demand is relatively large. The price cap binds for the largest demand realizations.

[23] From equation (9), $\frac{\partial E\{\, r - w(\varepsilon)\}}{\partial F_i} = [\, 1 - \gamma\,] \frac{\partial E\{w(\varepsilon)\}}{\partial F_i} - \frac{\partial E\{w(\varepsilon)\}}{\partial F_i} = -\gamma \frac{\partial E\{w(\varepsilon)\}}{\partial F_i} > 0$. The inequality holds because equation (6) implies that $\frac{\partial E\{w(\varepsilon)\}}{\partial F_i} < 0$.

increase in F_i is to reduce $E\{\pi^{BS}(\varepsilon)\}$, so $E\{\pi^{B}(\varepsilon)\}$ declines.[24]

For expositional convenience, we will refer to $\frac{\partial E\{\pi^B(\varepsilon)\}}{\partial F_i}$ as B's incentive for forward contracting. Proposition 1 explains how changes in the level of the wholesale price cap affect this incentive.

Proposition 1. *As the price cap increases: (i)* $\frac{\partial E\left\{\pi^{BD}(\varepsilon)\right\}}{\partial F_i}$ *increases; and (ii) B's incentive for forward contracting increases if* $\widehat{\varepsilon} > 0$ *or* γ *is sufficiently close to* 1*. Formally, for each* $i = 1, ..., n$*:*

$$\frac{\partial}{\partial \overline{w}} \left(\frac{\partial E\left\{\pi^{BD}(\varepsilon)\right\}}{\partial F_i} \right) = \frac{\gamma\, b\, \overline{Q}}{2\, \overline{\varepsilon}} > 0 \quad \textit{and}$$

$$\frac{\partial}{\partial \overline{w}} \left(\frac{\partial E\left\{\pi^{B}(\varepsilon)\right\}}{\partial F_i} \right) = \frac{\gamma\, b\, \overline{Q} + \widehat{\varepsilon}}{2\, \overline{\varepsilon}} \gtreqless 0 \quad \Leftrightarrow \quad \widehat{\varepsilon} \gtreqless -\gamma\, b\, \overline{Q}\,. \tag{14}$$

Proposition 1 indicates that an increase in $\overline{w}$ always increases $\frac{\partial E\{\pi^{BD}(\varepsilon)\}}{\partial F_i}$ for two reasons. First, $\widehat{\varepsilon}$ increases as $\overline{w}$ increases. The increase in $\widehat{\varepsilon}$ expands the $[\,\underline{\varepsilon}, \widehat{\varepsilon}\,]$ region in which an increase in F_i increases B's profit margin by reducing $w(\varepsilon)$ more rapidly than it reduces r. Second, the increase in $\widehat{\varepsilon}$ reduces the $(\widehat{\varepsilon}, \overline{\varepsilon}\,]$ region in which an increase in F_i reduces B's profit margin by reducing r (when $\gamma < 1$) without altering $w(\cdot)$.

Proposition 1 also reports that high values of γ increase the rate at which an increase in $\overline{w}$ enhances B's incentive for forward contracting. The larger is γ, the less sensitive is r to $E\{w(\varepsilon)\}$, and thus the more rapidly B's expected profit margin increases as F_i increases. When γ is sufficiently close to 1 (so $r \approx r_0$), an increase in $\overline{w}$ always enhances B's incentive for forward contracting (i.e., $\frac{\partial}{\partial \overline{w}} \left(\frac{\partial E\{\pi^B(\varepsilon)\}}{\partial F_i} \right) > 0$ because $\lim_{\gamma \to 1} \left\{ \gamma\, b\, \overline{Q} + \widehat{\varepsilon} \right\} = b\, \overline{Q} + \widehat{\varepsilon} > b\, \overline{Q} - \overline{\varepsilon} > 0$).

Proposition 1 also identifies conditions under which an increase in $\overline{w}$ could, in principle, reduce B's incentive for forward contracting. This inverse relationship can prevail when γ is relatively small (so an increase in F_i reduces r relatively rapidly) and when the price

[24]The conclusions reported in Lemma 2 (and in Proposition 1 below) also hold when $\gamma = 0$ if equation (9) is replaced by $r = R_0 + \gamma\, r_0 + [\,1 - \gamma\,]\, E\{\, w(\varepsilon)\}$, where $R_0 > 0$ is a lump-sum component of the retail price that is sufficiently large to ensure B's expected profit is nonnegative even when $\gamma = 0$. Lemma 2 implies that increased forward contracting reduces B's expected profit in this setting (i.e., $\frac{\partial E\{\pi^B(\varepsilon)\}}{\partial F_i} < 0$), so B prefers not to engage in forward contracting. Increased forward contracting reduces B's expected profit when $\gamma = 0$ in part because B secures no direct benefit from a lower expected wholesale price (i.e., $\frac{\partial E\{\pi^{BD}(\varepsilon)\}}{\partial F_i} = 0$) when the retail price declines at exactly the same rate the wholesale price declines as forward contracting increases.

cap binds for an extensive set of demand realizations. Specifically, the cap must bind even when realized demand is less than its expected value (because $\widehat{\varepsilon} < 0$). This condition is not empirically relevant. However, if a situation were to arise in which γ is close to 0 and $\widehat{\varepsilon} < 0$, an increase in $\overline{w}$ could reduce $\frac{\partial E\{\pi^B(\varepsilon)\}}{\partial F_i}$. It could do so primarily by expanding the $[\,\underline{\varepsilon}, \widehat{\varepsilon}\,]$ region and thereby reducing expected stochastic demand in this region $(\int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} b^I \varepsilon \, dH(\varepsilon))$ when $\widehat{\varepsilon} < 0$.

In summary, Proposition 1 implies that an increase in the wholesale price cap typically increases B's incentive for forward contracting in settings that arise in practice.

4 Generators' Preferred Levels of Forward Contracting

In practice, a buyer of electricity typically does not dictate prevailing levels of forward contracting unilaterally. Therefore, it is important to consider how a binding wholesale price cap affects generators' incentives for forward contracting. To do so, let $\overline{q}_i(\varepsilon)$ denote Gi's output when $\varepsilon > \widehat{\varepsilon}$ is realized (so the price cap binds), and let $q_i(\varepsilon)$ denote Gi's output when $\varepsilon \leq \widehat{\varepsilon}$ is realized. Equation (3) implies that, because $p^f = E\{w(\varepsilon)\}$, Gi's expected profit is:

$$E\{\,\pi_i^G(\varepsilon)\,\} \;=\; \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} [\,w(\varepsilon) - c_i\,]\, q_i(\varepsilon)\, dH(\varepsilon) + \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} [\,\overline{w} - c_i\,]\, \overline{q}_i(\varepsilon)\, dH(\varepsilon)\,. \tag{15}$$

Equation (15) implies that the rate at which Gi's expected profit increases as its forward contracting increases is:

$$\frac{\partial E\{\,\pi_i^G(\varepsilon)\,\}}{\partial F_i} \;=\; \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \left\{ [\,w(\varepsilon) - c_i\,]\, \frac{\partial q_i(\varepsilon)}{\partial F_i} + q_i(\varepsilon)\, \frac{\partial w(\varepsilon)}{\partial F_i} \right\} dH(\varepsilon)$$

$$+ \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} [\,\overline{w} - c_i\,]\, \frac{\partial\, \overline{q}_i(\varepsilon)}{\partial F_i}\, dH(\varepsilon). \tag{16}$$

Equation (16) implies that the impact of a change in $\overline{w}$ on Gi's incentive for forward contracting (i.e., on $\frac{\partial E\{\pi_i^G(\varepsilon)\}}{\partial F_i}$) is:

$$\frac{\partial}{\partial\,\overline{w}} \left(\frac{\partial E\,\{\pi_i^G(\varepsilon)\}}{\partial F_i} \right) \;=\; \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} \frac{\partial\, \overline{q}_i(\varepsilon)}{\partial F_i}\, dH(\varepsilon)$$

$$+ \frac{\partial \widehat{\varepsilon}}{\partial\,\overline{w}} \left\{ [\,w(\widehat{\varepsilon}) - c_i\,]\, \frac{\partial q_i(\varepsilon)}{\partial F_i}\bigg|_{\varepsilon=\widehat{\varepsilon}} + q_i(\widehat{\varepsilon})\, \frac{\partial w(\varepsilon)}{\partial F_i}\bigg|_{\varepsilon=\widehat{\varepsilon}} - [\,\overline{w} - c_i\,]\, \frac{\partial\, \overline{q}_i(\varepsilon)}{\partial F_i}\bigg|_{\varepsilon=\widehat{\varepsilon}} \right\} h(\widehat{\varepsilon}). \tag{17}$$

Equation (17) identifies two effects of an increase in the price cap, $\overline{w}$, on Gi's incentive for forward contracting: a revenue enhancement effect and a regime shifting effect. The integral term in equation (17) reflects the revenue enhancement effect, which arises when $\varepsilon > \widehat{\varepsilon}$, so the price cap binds. When $\varepsilon > \widehat{\varepsilon}$, an increase in $\overline{w}$ increases the rate at which Gi's revenue increases as increased forward contracting increases Gi's equilibrium output. The enhanced revenue increases Gi's incentive for forward contracting.

The second of the two terms in equation (17) captures the regime shifting effect. This effect arises because an increase in $\overline{w}$ reduces the likelihood that the price cap binds and increases the likelihood that the cap does not bind. The regime shifting effect diminishes (enhances) Gi's incentive for forward contracting if the rate at which Gi's profit increases with its forward contracting is higher (lower) when the price cap binds than when the cap does not bind.

To determine whether $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ is higher when the price cap binds or when it does not bind, it is necessary to determine the rate at which Gi's equilibrium output changes as F_i changes when the price cap binds. Recall that multiple equilibria arise when the cap binds. Therefore, we must introduce an assumption about how $\Delta(\varepsilon) \equiv Q(\overline{w}, \varepsilon) - Q(\overline{w}, \widehat{\varepsilon})$, the incremental aggregate demand that arises as ε increases above $\widehat{\varepsilon}$ when $w(\varepsilon) = \overline{w}$, is allocated among the generators. For analytic tractability, the following assumption is maintained throughout the ensuing discussion in the text:

$$\overline{q}_i(\varepsilon) \;=\; q_i(\widehat{\varepsilon}) + \alpha_i \left[Q(\overline{w}, \varepsilon) - Q(\overline{w}, \widehat{\varepsilon}) \right] \quad \text{for all } \; \varepsilon \in [\, \widehat{\varepsilon}, \overline{\varepsilon} \,], \tag{18}$$

where $\alpha_i \in [\,0, 1\,]$ is a constant for all $i = 1, ..., n$, and $\sum_{j=1}^{n} \alpha_j = 1$. The formulation in equation (18) permits any allocation of $\Delta(\varepsilon)$ among generators that is not affected by the prevailing levels of forward contracting. For example, $\Delta(\cdot)$ might be allocated equally among the n generators, so $\alpha_i = \frac{1}{n}$. Alternatively, $\Delta(\cdot)$ might be allocated equally among (only) the generators known to employ the least-cost technology.[25] Part C of the Appendix demonstrates that the key qualitative conclusions drawn below are not an artifact of the simplifying assumption that the generators' outputs and levels of forward contracting do not affect the allocation of $\Delta(\cdot)$.

Lemma 3 examines how a binding price cap affects the impact of expanded forward

[25] Formally, suppose generators $G1, ..., Gm$ employ the least-cost technology and generators $Gm+1, ..., Gn$ employ different technologies (where $m < n$). Then the allocation rule would be $\alpha_i = \frac{1}{m}$ for $i \in \{1, ..., m\}$ and $\alpha_i = 0$ for $i \in \{m+1, ..., n\}$.

contracting on a generator's profit.

Lemma 3. *Expanded forward contracting increases a generator's equilibrium profit more rapidly when the price cap binds than when it does not bind if the generator's share of* $\Delta(\varepsilon)$ *is not too pronounced. Formally,* $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ *is higher when* $\varepsilon \in (\,\widehat{\varepsilon}, \overline{\varepsilon}\,]$ *than when* $\varepsilon \in [\,\underline{\varepsilon}, \widehat{\varepsilon}\,)$ *if* $\alpha_i \in [\,0, \frac{2}{n+1}\,)$.

The conclusion in Lemma 3 reflects the following considerations. An increase in F_i reduces the wholesale price when the price cap does not bind (recall equation (6)), but does not alter the wholesale price when the cap binds. Therefore, an increase in F_i will increase $\pi_i^G(\varepsilon)$ more rapidly when the price cap binds than when it does not bind as long as $q_i(\varepsilon)$ does not increase much more rapidly than $\overline{q}_i(\varepsilon)$ increases as F_i increases.

Equation (7) implies that the rate at which $q_i(\varepsilon)$ increases as F_i increases is $\frac{\partial q_i(\varepsilon)}{\partial F_i} = \frac{n}{n+1}$. Equation (18) implies that the rate at which $\overline{q}_i(\varepsilon)$ increases with F_i has two components. First, $q_i(\widehat{\varepsilon})$ increases as F_i increases because a generator's equilibrium output increases as its forward contracting increases for all $\varepsilon \in [\,\underline{\varepsilon}, \widehat{\varepsilon}\,)$ (recall equation (7)). Second, $\Delta(\varepsilon)$ declines as F_i increases. The decline in $\Delta(\cdot)$ reflects the increase in $Q(\overline{w}, \widehat{\varepsilon})$ induced by an increase in forward contracting. The increase in $Q(\overline{w}, \widehat{\varepsilon})$ arises because $\widehat{\varepsilon}$ increases as expanded forward contracting increases industry output. The increased output reduces the wholesale price, which increases $\widehat{\varepsilon}$, the smallest ε for which the price cap binds. It is apparent from equation (18) that the decline in $\Delta(\varepsilon)$ reduces $\overline{q}_i(\varepsilon)$ relatively slowly when α_i is small. Consequently, if α_i is sufficiently small, $q_i(\varepsilon)$ does not increase much more rapidly than $\overline{q}_i(\varepsilon)$ increases as F_i increases. Consequently, $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ is higher when the price cap binds than when it does not bind in this case.

If $\pi_i^G(\varepsilon)$ increases more rapidly with F_i when the price cap binds than when it does not bind, then the regime switching effect causes $\frac{\partial E\{\,\pi_i^G(\varepsilon)\,\}}{\partial F_i}$ to decline as $\overline{w}$ increases. In contrast, the revenue enhancement effect causes $\frac{\partial E\{\,\pi_i^G(\varepsilon)\,\}}{\partial F_i}$ to increase as $\overline{w}$ increases. These observations underlie Proposition 2.

Proposition 2. *An increase in* $\overline{w}$ *can either enhance or diminish a generator's incentive for forward contracting because* $\frac{\partial}{\partial \overline{w}} \left(\frac{\partial E\{\,\pi_i^G(\varepsilon)\,\}}{\partial F_i} \right) = \frac{\Omega}{2\,\overline{\varepsilon}}$*, where*

$$\Omega \;\equiv\; [\,1-\alpha_i\,]\,[\,a+\overline{\varepsilon}+C_{-i}-n\,c_i-b\,F\,]-b\,F_i+[\,\overline{w}-c_i\,]\,[\,(\,n+1\,)\,(\,2\,\alpha_i-1\,)-2\,]\,. \quad (19)$$

The following corollary to Proposition 2 identifies factors that enhance or diminish the impact of an increase in $\overline{w}$ on Gi's incentive for forward contracting.

Corollary 1. *Suppose* $\alpha_i \in [0,1)$. *Then the rate at which an increase in* $\overline{w}$ *enhances a generator's incentive for forward contract* ($\frac{\partial}{\partial \overline{w}} \left(\frac{\partial E\{\pi_i^G(\varepsilon)\}}{\partial F_i} \right)$) *increases: (i) as* a^I, $\overline{Q}$, *or* C_{-i} *increases; (ii) as* c_i *increases if* $\alpha_i < \frac{3}{2+n}$; *and (iii) as* $\overline{w}$ *declines if* $\alpha_i < \frac{n+3}{2[n+1]}$.

Condition (i) in Corollary 1 reports that an increase in a^I, $\overline{Q}$, or C_{-i} increases the rate at which an increase in $\overline{w}$ enhances Gi's incentive for forward contracting. Expanded demand (a^I or $\overline{Q}$) or higher costs of rival generators promote higher wholesale prices, thereby increasing the range of demand realizations for which the price cap binds. (Recall equations (5) and (6).) The expected impact of the revenue enhancement effect increases as the price cap becomes more likely to bind, so an increase in $\overline{w}$ becomes more likely to enhance Gi's incentive for forward contracting.[26]

The equilibrium wholesale price also increases as c_i increases. Therefore, an increase in c_i will also increase $\frac{\partial}{\partial \overline{w}} \left(\frac{\partial E\{\pi_i^G(\varepsilon)\}}{\partial F_i} \right)$ as long as the increase in c_i does not substantially enhance a regime switching effect that reduces $\frac{\partial E\{\pi_i^G(\varepsilon)\}}{\partial F_i}$. As condition (ii) in Corollary 1 indicates, this will be the case if α_i is sufficiently small, because $\frac{\partial q_i(\varepsilon)}{\partial F_i}$ and $\frac{\partial \overline{q}_i(\varepsilon)}{\partial F_i}$ are not too disparate when α_i is small. Consequently, an increase in c_i reduces $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ at relatively comparable rates when the price cap binds and when it does not bind.[27]

Condition (iii) in Corollary 1 holds because the price cap becomes less likely to bind as $\overline{w}$ increases. The associated reduction in the expected impact of the revenue enhancement effect reduces the rate at which an increase in $\overline{w}$ increases $\frac{\partial E\{\pi_i^G(\varepsilon)\}}{\partial F_i}$ when α_i is sufficiently small (so the regime switching effect promotes a reduction in $\frac{\partial E\{\pi_i^G(\varepsilon)\}}{\partial F_i}$ as $\overline{w}$ increases).[28]

Proposition 2 also permits the specification of conditions under which an increase in $\overline{w}$ enhances a generator's incentive for forward contracting, as Corollary 2 reports.

[26] Equation (17) and equation (54) in the proof of Proposition 2 in the Appendix demonstrate that the revenue enhancement effect of an increase in $\overline{w}$ is $\frac{1}{2\,\overline{\varepsilon}}[1-\alpha_i][\,\overline{\varepsilon} - (n+1)\,\overline{w} + a + C - b\,F]$. This term is increasing in $a = b\left[a^I + \overline{Q}\right]$ and $C = C_{-i} + c_i$.

[27] $\frac{\partial \hat{\varepsilon}}{\partial \overline{w}} = n+1$ from equation (5). Therefore, equation (17) and equation (53) in the proof of Proposition 2 in the Appendix imply that the regime switching effect of an increase in $\overline{w}$ is $\frac{1}{2\,\overline{\varepsilon}}\{[\,\overline{w} - c_i][\alpha_i(n+1) - 2] - b\,F_i\}$. The magnitude of this effect increases with α_i, holding F_i constant.

[28] The expressions in the two preceding footnotes reveal that an increase in $\overline{w}$ reduces the revenue enhancement effect at the rate $\frac{1}{2\,\overline{\varepsilon}}[1-\alpha_i][n+1]$ and increases the regime shifting effect at the rate $\frac{1}{2\,\overline{\varepsilon}}[\alpha_i(n+1) - 2]$. The former rate exceeds the latter rate if and only if $[1-\alpha_i][n+1] > \alpha_i[n+1] - 2$ $\Leftrightarrow$ $\alpha_i < \frac{n+3}{2[n+1]}$.

Corollary 2. *An increase in the price cap enhances a generator's incentive for forward contracting (so* $\frac{\partial}{\partial \overline{w}}\left(\frac{\partial E\{\pi_i^G(\varepsilon)\}}{\partial F_i}\right) > 0$ *) if* α_i *is sufficiently close to* 1 *and* F_i *is sufficiently small.*

Corollary 2 reflects the following considerations. When α_i is close to 1, an increase in F_i has little impact on Gi's output when the price cap binds.[29] Consequently, $\frac{\partial \pi_i^G}{\partial F_i}$ is relatively small when the price cap binds. Therefore, the regime switching effect of an increase in $\overline{w}$ complements the revenue enhancement effect as long as F_i is sufficiently small (which ensures $\frac{\partial \pi_i^G}{\partial F_i} > 0$ when the price cap does not bind). The two effects together ensure that an increase in $\overline{w}$ increases Gi's incentive for forward contracting.

Corollary 3 provides an additional conclusion when the n generators are symmetric.

Corollary 3. *Suppose the generators are symmetric, so* $\alpha_i = \frac{1}{n}$ *and* $c_i = c$ *for* $i = 1, ..., n$*. Then an increase in* $\overline{w}$ *reduces each generator's incentive for forward contracting (i.e.,* $\frac{\partial}{\partial \overline{w}}\left(\frac{\partial E\{\pi_i^G(\varepsilon)\}}{\partial F_i}\right) < 0$ *) if* $\overline{w}$ *is sufficiently large, i.e., if* $\overline{w} > \frac{1}{n+2}[\,a + \overline{\varepsilon} + (n+1)\,c\,]$.

Corollary 3 reflects the fact that $\alpha_i < \frac{2}{n+1}$ when $\alpha_i = \frac{1}{n}$.[30] Therefore, $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ is higher when the price cap binds than when it does not bind. (Recall Lemma 3.) Consequently, the regime switching effect of an increase in $\overline{w}$ reduces Gi's incentive for forward contracting. Corollary 3 reports that this effect outweighs the countervailing revenue enhancement effect of an increase in $\overline{w}$ (which is only relevant when the price cap binds) if $\overline{w}$ is relatively high, so the price cap binds relatively infrequently.[31]

Corollary 3 establishes that an increase in $\overline{w}$ can reduce a generator's incentive for forward contracting, holding constant the forward contracting of other generators. We now consider how an increase in $\overline{w}$ affects the aggregate equilibrium level of forward contracting undertaken by all generators. To do so, we examine numerical solutions to our model for selected parameter values.

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[29] $q_i(\widehat{\varepsilon})$ increases at the same rate that $\Delta(\varepsilon)$ declines as F_i increases. ($\frac{\partial q_i(\widehat{\varepsilon})}{\partial F_i} = -\frac{\partial \Delta(\varepsilon)}{\partial F_i} = 1$ from equations (5) and (7).) Consequently, $q_i(\widehat{\varepsilon})$ increases at nearly the same rate that $\alpha_i\,\Delta(\varepsilon)$ declines as F_i increases when α_i is close to 1. Therefore, equation (18) implies that $\lim_{\alpha_i \to 1} \frac{\partial\, \overline{q}_i(\varepsilon)}{\partial F_i} = 0$.

[30] $\frac{1}{n} < \frac{2}{n+1} \Leftrightarrow 2\,n > n+1 \Leftrightarrow n > 1$.

[31] It is readily shown that if $\alpha_i = \frac{1}{n}$ for $i = 1, ..., n$ but c_i can vary across generators, then $\frac{\partial}{\partial \overline{w}}\left(\frac{\partial E\{\pi_i^G(\varepsilon)\}}{\partial F_i}\right) < 0$ if $\overline{w} > \frac{1}{n+2}[\,a + \overline{\varepsilon} + C_{-i} + 2\,c_i\,]$.

We select initial parameter values to reflect elements of actual electricity markets. However, the ensuing analysis does not necessarily reflect activities in any particular market because our model does not capture all relevant elements of actual electricity markets. In particular, for analytic tractability, our model abstracts from the sharply rising marginal cost a generator effectively experiences as its output approaches capacity. Our model of Cournot competition also does not account for the activities of fringe generators and must-run generation (e.g., wind and cogeneration). Furthermore, we take all demand realizations to be equally likely, whereas extreme deviations from expected demand typically are relatively unlikely in practice.

With this caveat in mind, we proceed to establish baseline parameter values. (Part B of the Appendix demonstrates that the key qualitative conclusions drawn below persist as baseline parameter values change.) The wholesale price cap is initially taken to be $1,000$ (so $\overline{w} = 1,000$), reflecting its value in Alberta, Canada.[32] We initially consider an expected wholesale price of 35 and assume that aggregate expected demand at this price is $8,500$. Formally:

$$a^I - 35\, b^I + \overline{Q} \; = \; 8,500\,. \tag{20}$$

This expected wholesale price approximates the \$33.92 quantity-weighted average wholesale price in the eight major U.S. electricity hubs in 2020.[33] The identified aggregate expected demand reflects the $8,487.88$ MWh average hourly electricity consumption in a U.S. state in 2020.[34]

We initially take the ratio of expected residential electricity consumption to industrial electricity consumption at the expected wholesale price to be 0.65, reflecting the corresponding ratio in the U.S. in 2020.[35] Therefore:

$$\frac{\overline{Q}}{a^I - 35\, b^I} \; = \; 0.65\,. \tag{21}$$

Each generator's marginal cost of production is initially assumed to be 25 (so $c_i = 25$ for $i = 1, ..., n$), reflecting the \$24.55 average cost of generating electricity in the U.S. in 2020

[32] As noted in the Introduction, the wholesale price cap in Alberta is \$999.99 per MWh (Brown and Olmstead, 2017).

[33] U.S. Energy Information Administration (https://www.eia.gov/electricity/wholesale/#history).

[34] Total U.S. electricity consumption in 2020 was $3,717,674$ thousand megawatthours. Dividing this number by the $8,760$ hours in a year, multiplying by 1,000 to convert to MWhs, and dividing by the 50 U.S. states provides an average state hourly consumption of $8,487.84$ MWhs (https://www.eia.gov/electricity/annual/html/epa_02_08.html).

[35] The ratio of electricity purchased by residential consumers to electricity purchased by commercial and industrial consumers in the U.S. in 2020 was $\frac{1,464,605}{1,287,440\,+\,959,082} \approx 0.65$ (https://www.eia.gov/electricity/annual/html/epa_02_08.html).

using natural gas.[36] We also initially assume there are five generators (so $n = 5$) to reflect a setting with a moderate level of competition among generators.

We initially set $b^I = 0.4$ and assume the maximum variation in residential demand is 50% of expected residential demand (i.e., $\overline{\eta} = \frac{1}{2}\,\overline{Q}$). This calibration helps to ensure the price cap binds for some, but not all, demand realizations, given the equilibrium levels of forward contracting in the model. Equations (20) and (21) imply that $a^I = 5,165.52$ and $\overline{Q} = 3,348.48$ (and so $\overline{\eta} = \frac{1}{2}\,\overline{Q} = 1,674.24$) when $b^I = 0.4$.[37]

Table 1 summarizes these baseline parameter values.[38]

a^I	$\overline{Q}$	b^I	$\overline{\eta}$	c_i	n	$\overline{w}$
$5,165.52$	$3,348.48$	0.4	$1,674.24$	25	5	$1,000$

Table 1. Baseline Parameter Values

The middle row of data in Table 2 presents equilibrium outcomes in our model for the baseline parameter values. The first and third rows of data report the corresponding outcomes when the price cap ($\overline{w}$) is reduced and increased by 20%, respectively, holding all other parameter values at their baseline levels. Table 2 reports each generator's level of forward contracting (F_i), expected output ($E\{q_i\}$), and expected profit ($E\{\pi_i^G\}$). The table also reports the expected wholesale price ($E\{w\}$) and the smallest realization of ε for which the wholesale price cap binds ($\widehat{\varepsilon}$).[39]

[36] U.S. Energy Information Administration (https://www.eia.gov/electricity/annual/html/epa_08_04.html).

[37] $a^I - 35\,b^I = 8,500 - \overline{Q}$, from equation (20). Therefore, $\overline{Q} = 0.65\left[\,8,500 - \overline{Q}\,\right] \Rightarrow \overline{Q} = 3,348.48$, from equation (21). Consequently, $a^I = 8,500 - 3,348.48 + 0.4\,[\,35\,] = 5,165.52$, from equation (20).

[38] These baseline parameter values imply that $\overline{\varepsilon} = \frac{\overline{\eta}}{b^I} = \frac{1,674.24}{0.4} = 4,185.6$.

[39] The entries in Table 2 are rounded.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	$1,415.2$	$1,660.9$	$875,523$	523.9	$1,080.5$
$1,000$	$1,393.3$	$1,653.3$	$1,036,418$	618.3	$2,006.3$
$1,200$	$1,365.5$	$1,646.3$	$1,182,306$	706.0	$2,859.1$

Table 2. Equilibrium Outcomes as $\overline{w}$ Varies

Table 2 reports that each generator reduces its equilibrium level of forward contracting as the wholesale price cap is increased. The reduced forward contracting and the higher price cap increase the expected wholesale price and expected generator profit, while reducing expected generator output.[40] The numerical solutions presented in Part B of the Appendix indicate that corresponding effects of an increased wholesale price cap arise as parameter values diverge from their baseline levels.

For the reasons explained above, the outcomes reported in Table 2 and in the Appendix do not imply that a higher wholesale price cap will always (or even often) reduce the levels of forward contracting preferred by generators in practice. However, the outcomes do suggest that this potential preference merits consideration when attempting to predict the effect of a change in the level of a wholesale price cap on industry forward contracting.

5 Conclusions

We have examined how a cap on the wholesale price of electricity affects incentives for forward contracting, explicitly accounting for the effects of forward contracting on short-term and long-term electricity prices. Our findings support the common wisdom that a price cap reduces a buyer's incentive for forward contracting. When the maximum price it can face in the wholesale market declines, a buyer derives less benefit from securing electricity at the expected wholesale price rather than facing the actual wholesale price.

In contrast, a binding wholesale price cap can increase a generator's incentive for forward contracting. Consequently, an increase in a prevailing price cap can reduce a generator's incentive for forward contracting. This is the case in part because a higher price cap renders the cap less likely to bind, thereby increasing the likelihood that expanded forward

[40] The relatively high expected wholesale prices reported in Table 2 reflect the aforementioned special features of our model. A model that included a fringe of competitive generators, must-run generation, sharply rising marginal cost near capacity, and infrequent demand realizations well above expected demand would permit substantially lower expected wholesale prices while allowing the price cap to bind for some, but not all, demand realizations.

contracting will reduce a generator's profit by reducing the prevailing wholesale price. This regime shifting effect of an increase in $\overline{w}$ can induce generators to prefer reduced levels of forward contracting.

Our findings suggest that policymakers should consider the incentives of both buyers and generators when attempting to assess the likely impact of a change in the prevailing level of a wholesale price cap on industry forward contracting. The common wisdom that a higher price cap will enhance a buyer's incentive for forward contracting does not necessarily extend to generators. Consequently, the ultimate impact of a higher price cap on industry forward contracting likely will depend in part on the details of the buyer-generator negotiations that determine the prevailing levels of forward contracting.

Future research should model these negotiations formally, allowing buyers and generators to bargain over both the number and the prices of forward contracts.[41] To facilitate comparison with other studies in the literature, we adopted the common assumption that the price of a forward contract (p^f) is the expected wholesale price of electricity ($E\{w(\varepsilon)\}$). Negotiated prices would change the details of our analysis. However, such prices seem unlikely to alter our main qualitative findings because the key considerations that underlie our findings (e.g., forward contracts reduce the expected wholesale price and thereby reduce the likelihood that the wholesale price cap binds) persist when p^f differs from $E\{w(\varepsilon)\}$. Non-constant marginal costs and non-uniform distributions of demand uncertainty also seem unlikely to alter our primary qualitative conclusions because these changes similarly will not alter the key forces that underlie the conclusions.[42]

A broader investigation of whether a binding wholesale price cap is likely to diminish generators' incentives to sign forward contracts in practice would be valuable. We found that these reduced incentives can arise in one particular environment. Corresponding investigations

[41] Anderson and Hu (2008) analyze a model in which a buyer proposes a level of forward contracting and associated compensation. The generator can either accept the buyer's proposal or decline to engage in forward contracting altogether. More generally, bilateral contracting between pairs of buyers and generators warrant consideration.

[42] To facilitate a tractable analysis, we have assumed that generators choose output levels. Models in which generators choose supply functions introduce considerable analytic complexity, including multiple equilibria. (See Klemperer and Meyer (1989), Green (1999), Newbery (1998), Holmberg (2011), and Holmberg and Willems (2015), for example.) Holmberg (2011) identifies conditions under which a generator's forward contracting induces its rival to shift its supply function inward. The author also notes that a more binding wholesale price cap can render the specified conditions more likely to arise. However, Holmberg (2011) does not undertake a systematic examination of the impacts of a wholesale price cap on incentives for forward contracting. Such an examination awaits future research.

across a broad spectrum of environments that prevail in practice would be valuable.

Future research might also analyze the effects of risk aversion. Buyer risk aversion introduces an additional consideration that runs counter to the conventional wisdom. A higher price cap can reduce the variance of a buyer's profit by reducing the buyer's profit margin when the price cap binds.[43] The lower variance, in turn, can diminish the risk-reducing benefit a risk averse buyer derives from forward contracting.[44] Consequently, in principle, if this effect were sufficiently pronounced, an increase in $\overline{w}$ could induce a sufficiently risk averse buyer to prefer a reduced level of forward contracting. Numerical solutions suggest this outcome is relatively unlikely.[45] However, this effect merits consideration in a comprehensive assessment of the impact of a wholesale price cap on incentives for forward contracting.[46]

[43]Brown and Sappington (2023b) examine how forward contracting affects the variance of buyer profit and generator profit in a model with no wholesale price cap.

[44]In contrast, a higher price cap can increase a generator's profit margin when the price cap binds, thereby increasing the variance of the generator's profit. The increased variance can induce a risk averse generator to prefer a higher level of (risk-reducing) forward contracting.

[45]We have extended our analysis to allow the buyer and the generators to have mean-variance preferences (e.g., Rolfo, 1980; Sargent, 1987, pp. 154-5). In the presence of such risk aversion, tractable analytic characterizations of equilibrium outcomes are difficult to derive. However, numerical solutions suggest that the buyer's expected utility often increases systematically as its forward contracting increases for any price cap that binds for some, but not all, demand realizations. Thus, a risk averse buyer often prefers the highest feasible levels of forward contracting, regardless of the level of the price cap.

[46]As noted above, a higher price cap can increase the variance of a generator's profit and thereby enhance a risk averse generator's incentive to undertake forward contracting. However, our numerical solutions reveal that when risk averse generators choose their preferred levels of forward contracting (non-cooperatively), a higher price cap can induce lower equilibrium levels of forward contracting, just as it can when generators are risk neutral.

Appendix

Part A of this Appendix provides the proofs of the formal conclusions in the text. Part B presents additional numerical solutions. Part C considers an alternative rule for allocating $\Delta(\varepsilon)$.

A. Proofs of Formal Conclusions in the Text.

Proof of Lemma 1. (3) implies that when ε is realized, Gi's problem is:

$$\underset{q_i \geq 0}{Maximize} \;\; \pi_i^G(\varepsilon) \;=\; w(\varepsilon)\,[\,q_i - F_i\,] + p^f\, F_i - c_i\, q_i\,. \tag{22}$$

(22) implies that the necessary condition for an interior maximum is:

$$\frac{\partial \pi_i^G(\varepsilon)}{\partial q_i} \;=\; w(\varepsilon) + [\,q_i - F_i\,]\,\frac{\partial w(\cdot)}{\partial Q} - c_i \;=\; 0\,. \tag{23}$$

(2) and (23) imply that Gi's profit-maximizing choice of $q_i > 0$ is determined by:

$$a + \varepsilon - b\,[\,q_i + Q_{-i}\,] - b\,[\,q_i - F_i\,] - c_i \;=\; 0$$

$$\Rightarrow \;\; 2\,b\,q_i \;=\; a + \varepsilon - b\,Q_{-i} + b\,F_i - c_i$$

$$\Rightarrow \;\; q_i \;=\; \frac{1}{2\,b}\,[\,a + \varepsilon - c_i + b\,F_i\,] - \frac{1}{2}\,Q_{-i} \tag{24}$$

where $Q_{-i} \equiv \sum_{\substack{j=1 \\ j \neq i}}^{n} q_j$. (24) implies that in equilibrium:

$$\begin{aligned}
Q_{-i} \;&=\; \sum_{\substack{j=1 \\ j \neq i}}^{n} \left[\, \frac{1}{2\,b}\,(\,a + \varepsilon - c_j + b\,F_j\,) - \frac{1}{2}\,Q_{-j} \right] \\
&=\; \frac{n-1}{2\,b}\,[\,a + \varepsilon\,] - \frac{1}{2\,b} \sum_{\substack{j=1 \\ j \neq i}}^{n} c_j + \frac{1}{2} \sum_{\substack{j=1 \\ j \neq i}}^{n} F_j - \frac{1}{2} \sum_{\substack{j=1 \\ j \neq i}}^{n} Q_{-j} \\
&=\; \frac{n-1}{2\,b}\,[\,a + \varepsilon\,] - \frac{1}{2\,b}\,C_{-i} + \frac{1}{2}\,F_{-i} \\
&\qquad - \frac{1}{2}\,[\,Q_{-1} + \ldots + Q_{-(i-1)} + Q_{-(i+1)} + \ldots + Q_{-n}\,] \\
&=\; \frac{n-1}{2\,b}\,[\,a + \varepsilon\,] - \frac{1}{2\,b}\,C_{-i} + \frac{1}{2}\,F_{-i} - \frac{1}{2}\,[\,(n-1)\,q_i + (n-2)\,Q_{-i}\,]\,.
\end{aligned} \tag{25}$$

(25) implies:

$$Q_{-i}\left[1+\frac{n-2}{2}\right] \;=\; \frac{n-1}{2\,b}\,[\,a+\varepsilon\,]-\frac{1}{2\,b}\,C_{-\,i}+\frac{1}{2}\;F_{-i}-\left[\frac{n-1}{2}\right]q_i$$

$$\Rightarrow \quad Q_{-i}\left[\frac{n}{2}\right] \;=\; \frac{n-1}{2\,b}\,[\,a+\varepsilon\,]-\frac{1}{2\,b}\,C_{-\,i}+\frac{1}{2}\;F_{-i}-\left[\frac{n-1}{2}\right]q_i$$

$$\Rightarrow \quad Q_{-i} \;=\; \frac{n-1}{b\,n}\,[\,a+\varepsilon\,]-\frac{1}{b\,n}\,C_{-\,i}+\frac{1}{n}\;F_{-i}-\left[\frac{n-1}{n}\right]q_i\,. \tag{26}$$

(24) and (26) imply that in equilibrium:

$$q_i \;=\; \frac{1}{2\,b}\,[\,a+\varepsilon\,]-\frac{1}{2\,b}\,c_i+\frac{1}{2}\,F_i-\frac{[\,n-1\,]\,[\,a+\varepsilon\,]}{2\,b\,n}+\frac{1}{2\,b\,n}\,C_{-\,i}-\frac{1}{2\,n}\;F_{-i}+\left[\frac{n-1}{2\,n}\right]q_i$$

$$\Rightarrow \quad q_i\left[1-\frac{n-1}{2\,n}\right] \;=\; \frac{n-(\,n-1\,)}{2\,b\,n}\,[\,a+\varepsilon\,]+\frac{1}{2\,b\,n}\,[\,C_{-\,i}-n\,c_i\,]+\frac{1}{2}\;F_i-\frac{1}{2\,n}\;F_{-i}$$

$$\Rightarrow \quad q_i\left[\frac{n+1}{2\,n}\right] \;=\; \frac{1}{2\,b\,n}\,[\,a+\varepsilon\,]+\frac{1}{2\,b\,n}\,[\,C_{-\,i}-n\,c_i\,]+\frac{1}{2}\;F_i-\frac{1}{2\,n}\;F_{-i}$$

$$\Rightarrow \quad q_i(\varepsilon) \;=\; \frac{1}{b\,[\,n+1\,]}\,[\,a+\varepsilon\,]+\frac{1}{b\,[\,n+1\,]}\,[\,C_{-\,i}-n\,c_i\,]+\frac{n}{n+1}\;F_i-\frac{1}{n+1}\;F_{-i}$$

$$= \;\frac{a+\varepsilon+C_{-\,i}-n\,c_i+b\,n\,F_i-b\,F_{-i}}{b\,[\,n+1\,]}\,. \tag{27}$$

Observe that:

$$\sum_{i=1}^{n}(\,C_{-\,i}-n\,c_i\,) \;=\; [\,n-1\,]\sum_{i=1}^{n}c_i-n\sum_{i=1}^{n}c_i \;=\; -\sum_{i=1}^{n}c_i\,. \tag{28}$$

Furthermore, because $\sum\limits_{i=1}^{n}F_{-i} \;=\; [\,n-1\,]\sum\limits_{i=1}^{n}F_i$:

$$\sum_{i=1}^{n}(\,b\,n\,F_i-b\,F_{-i}\,) \;=\; b\,n\sum_{i=1}^{n}F_i-b\,[\,n-1\,]\sum_{i=1}^{n}F_i \;=\; b\sum_{i=1}^{n}F_i\,. \tag{29}$$

(27), (28), and (29) imply that in equilibrium:

$$Q(\varepsilon) \;=\; \sum_{i=1}^{n}q_i(\varepsilon) \;=\; \frac{n\,[\,a+\varepsilon\,]-\sum\limits_{i=1}^{n}c_i+b\sum\limits_{i=1}^{n}F_i}{b\,[\,n+1\,]}\,. \tag{30}$$

(2) and (30) imply:

$$w(\varepsilon) \;=\; a+\varepsilon - \frac{n\,[\,a+\varepsilon\,] - \sum\limits_{i=1}^{n} c_i + b \sum\limits_{i=1}^{n} F_i}{n+1}$$

$$= \frac{[\,n+1\,]\,[\,a+\varepsilon\,] - n\,[\,a+\varepsilon\,] + \sum\limits_{i=1}^{n} c_i - b \sum\limits_{i=1}^{n} F_i}{n+1} \;=\; \frac{a+\varepsilon + \sum\limits_{i=1}^{n} c_i - b \sum\limits_{i=1}^{n} F_i}{n+1} \tag{31}$$

$$\Rightarrow\;\; p^f \;=\; E\{w(\varepsilon)\} \;=\; \frac{a + E\{\varepsilon\} + \sum\limits_{i=1}^{n} c_i - b \sum\limits_{i=1}^{n} F_i}{n+1}\,. \tag{32}$$

(31) implies that $w(\varepsilon)$ is strictly increasing in ε. Therefore, the price cap binds and $w = \overline{w}$ for all $\varepsilon > \widehat{\varepsilon}$ where:

$$w(\widehat{\varepsilon}) \;=\; \frac{1}{n+1}\left[a + \widehat{\varepsilon} + \sum_{i=1}^{n} c_i - b \sum_{i=1}^{n} F_i\right] \;=\; \overline{w}$$

$$\Leftrightarrow\;\; a + \widehat{\varepsilon} + \sum_{i=1}^{n} c_i - b \sum_{i=1}^{n} F_i \;=\; [\,n+1\,]\,\overline{w}$$

$$\Leftrightarrow\;\; \widehat{\varepsilon} \;=\; [\,n+1\,]\,\overline{w} - \left[a + \sum_{i=1}^{n} c_i - b \sum_{i=1}^{n} F_i\right].$$

(2) implies that when $\varepsilon > \widehat{\varepsilon}$:

$$\overline{w} \;=\; a + \varepsilon - b\,Q(\varepsilon) \;\;\Rightarrow\;\; \sum_{i=1}^{n} q_i(\varepsilon) \;=\; \frac{1}{b}\,[\,a + \varepsilon - \overline{w}\,]\,. \;\; \blacksquare$$

<u>Proof of Lemma 2</u>. (5) and (6) imply:

$$E\{\,w(\varepsilon)\,\} \;=\; \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \left(\frac{a+\varepsilon+C-b\,F}{n+1}\right)\frac{d\varepsilon}{2\,\overline{\varepsilon}} + \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} \overline{w}\,\frac{d\varepsilon}{2\,\overline{\varepsilon}}$$

$$\Rightarrow\;\; \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial F_i} \;=\; -\frac{b}{n+1}\left[\frac{\widehat{\varepsilon}+\overline{\varepsilon}}{2\,\overline{\varepsilon}}\right] + \frac{\partial\widehat{\varepsilon}}{\partial F_i}\left[\frac{1}{2\,\overline{\varepsilon}}\right][\,w(\widehat{\varepsilon}) - \overline{w}\,]$$

$$= -\frac{b}{n+1}\left[\frac{\widehat{\varepsilon}+\overline{\varepsilon}}{2\,\overline{\varepsilon}}\right]. \tag{33}$$

(6) implies:

$$E\{\,\varepsilon\,w(\varepsilon)\,\} \;=\; \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \frac{\varepsilon}{n+1}\,[\,a+\varepsilon+C-b\,F\,]\;dH(\varepsilon) + \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} \overline{w}\;\varepsilon\;dH(\varepsilon)$$

$$= \frac{a + C - b\,F}{2\,\overline{\varepsilon}\,[\,n+1\,]} \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \varepsilon\; d\varepsilon + \frac{1}{2\,\overline{\varepsilon}\,[\,n+1\,]} \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \varepsilon^2\, d\varepsilon + \frac{\overline{w}}{2\,\overline{\varepsilon}} \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} \varepsilon\; d\varepsilon$$

$$= \frac{a + C - b\,F}{4\,\overline{\varepsilon}\,[\,n+1\,]} \left[\,(\widehat{\varepsilon})^2 - (\underline{\varepsilon})^2\,\right] + \frac{1}{6\,\overline{\varepsilon}\,[\,n+1\,]} \left[\,(\widehat{\varepsilon})^3 - (\underline{\varepsilon})^3\,\right] + \frac{\overline{w}}{4\,\overline{\varepsilon}} \left[\,(\overline{\varepsilon})^2 - (\widehat{\varepsilon})^2\,\right]$$

$$= -\frac{a + C - b\,F}{4\,\overline{\varepsilon}\,[\,n+1\,]} \left[\,(\overline{\varepsilon})^2 - (\widehat{\varepsilon})^2\,\right] + \frac{1}{6\,\overline{\varepsilon}\,[\,n+1\,]} \left[\,(\overline{\varepsilon})^3 + (\widehat{\varepsilon})^3\,\right] + \frac{\overline{w}}{4\,\overline{\varepsilon}} \left[\,(\overline{\varepsilon})^2 - (\widehat{\varepsilon})^2\,\right]. \tag{34}$$

(5) and (34) imply:

$$\frac{\partial E\{\,\varepsilon\; w(\varepsilon)\,\}}{\partial F_i} = \frac{b\left[\,(\overline{\varepsilon})^2 - (\widehat{\varepsilon})^2\,\right]}{4\,\overline{\varepsilon}\,[\,n+1\,]} + \frac{\partial \widehat{\varepsilon}}{\partial F_i} \left[\frac{1}{2\,\overline{\varepsilon}}\right] \left[\frac{\widehat{\varepsilon}}{n+1}\,(\,a + C - b\,F\,) - \overline{w}\,\widehat{\varepsilon} + \frac{(\widehat{\varepsilon})^2}{n+1}\right]$$

$$= \frac{b\left[\,(\overline{\varepsilon})^2 - (\widehat{\varepsilon})^2\,\right]}{4\,\overline{\varepsilon}\,[\,n+1\,]} + \frac{b}{2\,\overline{\varepsilon}} \left[\widehat{\varepsilon}\left(-\frac{\widehat{\varepsilon}}{n+1}\right) + \frac{(\widehat{\varepsilon})^2}{n+1}\right] = \frac{b\left[\,(\overline{\varepsilon})^2 - (\widehat{\varepsilon})^2\,\right]}{4\,\overline{\varepsilon}\,[\,n+1\,]}. \tag{35}$$

(12) implies:

$$E\{\pi^{BD}(\varepsilon)\} = \gamma\,[\,r_0 - E\{\,w(\varepsilon)\,\}\,]\;\overline{Q}\,. \tag{36}$$

(33) and (36) imply:

$$\frac{\partial E\{\pi^{BD}(\varepsilon)\}}{\partial F_i} = -\,\gamma\;\overline{Q}\;\left[-\,\frac{b}{n+1}\left(\frac{\widehat{\varepsilon} + \overline{\varepsilon}}{2\,\overline{\varepsilon}}\right)\right] = \frac{\gamma\, b\;\overline{Q}}{2\;\overline{\varepsilon}\;[\,n+1\,]}\,[\,\widehat{\varepsilon} + \overline{\varepsilon}\,]. \tag{37}$$

(12) implies:

$$E\{\pi^{BS}(\varepsilon)\} = \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \left\{\,[\,\gamma\, r_0 + (\,1-\gamma\,)\,E\{\,w(\varepsilon)\,\} - w(\varepsilon)\,]\; b^I \varepsilon\,\right\}\; dH(\varepsilon)$$

$$+ \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} \left\{\,[\,\gamma\, r_0 + (\,1-\gamma\,)\,E\{\,w(\varepsilon)\,\} - \overline{w}\,]\; b^I \varepsilon\,\right\}\; dH(\varepsilon)\,. \tag{38}$$

(5), (12), and (33) imply that because $w(\widehat{\varepsilon}) = \overline{w}$:

$$\frac{\partial E\{\pi^{BS}(\varepsilon)\}}{\partial F_i} = \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \left\{\frac{\partial}{\partial F_i}\,[\,\gamma\, r_0 + (\,1-\gamma\,)\,E\{\,w(\varepsilon)\,\} - w(\varepsilon)\,]\; b^I \varepsilon\right\} dH(\varepsilon)$$

$$+ \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} \left\{[\,1-\gamma\,]\;\frac{\partial E\{\,w(\varepsilon)\,\}}{\partial F_i}\; b^I \varepsilon\right\} dH(\varepsilon)$$

$$= \left[-\,(1-\gamma)\;\frac{b\,(\,\widehat{\varepsilon} + \overline{\varepsilon}\,)}{2\;\overline{\varepsilon}\;(n+1)} + \frac{b}{n+1}\right] \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} b^I \varepsilon\, dH(\varepsilon)$$

$$- \frac{b\,[\,1-\gamma\,]}{n+1} \left[\frac{\widehat{\varepsilon} + \overline{\varepsilon}}{2\,\overline{\varepsilon}} \right] \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} b^I \varepsilon \; dH(\varepsilon)$$

$$= \; - \frac{b}{2\,\overline{\varepsilon}\,[\,n+1\,]} \, [\,(1-\gamma)\,(\,\widehat{\varepsilon} + \overline{\varepsilon}\,) - 2\,\overline{\varepsilon}\,] \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} b^I \varepsilon \, dH(\varepsilon)$$

$$- \frac{b\,[\,1-\gamma\,]}{n+1} \left[\frac{\widehat{\varepsilon} + \overline{\varepsilon}}{2\,\overline{\varepsilon}} \right] \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} b^I \varepsilon \; dH(\varepsilon)$$

$$= \; \frac{b}{2\,\overline{\varepsilon}\,[\,n+1\,]} \, [\,\overline{\varepsilon} - \widehat{\varepsilon} + \gamma\,(\,\widehat{\varepsilon} + \overline{\varepsilon}\,)\,] \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} b^I \varepsilon \, dH(\varepsilon)$$

$$- \frac{b\,[\,1-\gamma\,]}{n+1} \left[\frac{\widehat{\varepsilon} + \overline{\varepsilon}}{2\,\overline{\varepsilon}} \right] \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} b^I \varepsilon \; dH(\varepsilon)$$

$$= \; \frac{1}{4\,(\,\overline{\varepsilon}\,)^2\,[\,n+1\,]} \, [\,\overline{\varepsilon} - \widehat{\varepsilon} + \gamma\,(\,\widehat{\varepsilon} + \overline{\varepsilon}\,)\,] \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \varepsilon \; d\varepsilon - \frac{1-\gamma}{2\,\overline{\varepsilon}\,[\,n+1\,]} \left[\frac{\widehat{\varepsilon} + \overline{\varepsilon}}{2\,\overline{\varepsilon}} \right] \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} \varepsilon \; d\varepsilon$$

$$= \; \frac{1}{8\,(\,\overline{\varepsilon}\,)^2\,[\,n+1\,]} \, [\,\overline{\varepsilon} - \widehat{\varepsilon} + \gamma\,(\,\widehat{\varepsilon} + \overline{\varepsilon}\,)\,] \, [\,(\widehat{\varepsilon})^2 - (-\overline{\varepsilon})^2\,]$$

$$- \frac{1-\gamma}{8\,(\,\overline{\varepsilon}\,)^2\,[\,n+1\,]} \, [\,\widehat{\varepsilon} + \overline{\varepsilon}\,] \, [\,(\,\overline{\varepsilon}\,)^2 - (\,\widehat{\varepsilon}\,)^2\,]$$

$$= \; - \frac{(\,\overline{\varepsilon}\,)^2 - (\widehat{\varepsilon})^2}{8\,(\,\overline{\varepsilon}\,)^2\,[\,n+1\,]} \, [\,\overline{\varepsilon} - \widehat{\varepsilon} + \gamma\,(\,\widehat{\varepsilon} + \overline{\varepsilon}\,)\,] - \frac{1-\gamma}{8\,(\,\overline{\varepsilon}\,)^2\,[\,n+1\,]} \, [\,\widehat{\varepsilon} + \overline{\varepsilon}\,] \, [\,(\,\overline{\varepsilon}\,)^2 - (\,\widehat{\varepsilon}\,)^2\,]$$

$$= \; - \frac{(\,\overline{\varepsilon}\,)^2 - (\widehat{\varepsilon})^2}{8\,(\,\overline{\varepsilon}\,)^2\,[\,n+1\,]} \, [\,\overline{\varepsilon} - \widehat{\varepsilon} + \gamma\,(\,\widehat{\varepsilon} + \overline{\varepsilon}\,) + (\,1-\gamma\,)\,(\,\widehat{\varepsilon} + \overline{\varepsilon}\,)\,]$$

$$= \; - \frac{(\,\overline{\varepsilon}\,)^2 - (\widehat{\varepsilon})^2}{8\,(\,\overline{\varepsilon}\,)^2\,[\,n+1\,]} \, [\,\overline{\varepsilon} - \widehat{\varepsilon} + \widehat{\varepsilon} + \overline{\varepsilon}\,] \; = \; - \frac{(\,\overline{\varepsilon}\,)^2 - (\,\widehat{\varepsilon}\,)^2}{4\,\overline{\varepsilon}\,[\,n+1\,]} \; < \; 0 \, . \tag{39}$$

(12), (37), and (39) imply:

$$\frac{\partial E\{\pi^B(\varepsilon)\}}{\partial F_i} \; = \; \frac{1}{4\,\overline{\varepsilon}\,[\,n+1\,]} \, [\,\overline{\varepsilon} + \widehat{\varepsilon}\,] \left[\, 2\,\gamma\,b\,\overline{Q} \, - (\,\overline{\varepsilon} \, - \, \widehat{\varepsilon}\,) \right]$$

$$\stackrel{s}{=} \; \left[\, 2\,\gamma\,b\,\overline{Q} \, - \overline{\varepsilon} \, + \, \widehat{\varepsilon} \,\right] \; \equiv \; \varphi(\gamma) \, . \tag{40}$$

Observe that:

$$\varphi(0) \; = \; -\,\overline{\varepsilon} \, + \, \widehat{\varepsilon} \; < \; 0 \, ; \;\; \text{and}$$

$$\varphi(1) \; = \; 2\,b\,\overline{Q} \, - \overline{\varepsilon} \, + \, \widehat{\varepsilon} \; > \; 2 \left[\, b\,\overline{Q} \, - \overline{\varepsilon} \,\right] \; > \; 0 \, . \tag{41}$$

The last inequality in (41) holds because:

$$\overline{Q} + b^I \underline{\varepsilon} > 0 \quad \Rightarrow \quad \overline{Q} - b^I \overline{\varepsilon} \ > \ 0 \quad \Rightarrow \quad b\,\overline{Q} \ > \ \overline{\varepsilon}\,. \tag{42}$$

The last conclusion in the lemma follows from (40) and (41). ■

Proof of Proposition 1. (5) and (37) imply:

$$\frac{\partial}{\partial\,\overline{w}} \left(\frac{\partial E\left\{\pi^{BD}(\varepsilon)\right\}}{\partial F_i} \right) \ = \ \frac{\gamma\, b\, \overline{Q}}{2\,\overline{\varepsilon}\,[\,n+1\,]}\,[\,n+1\,] \ = \ \frac{\gamma\, b\, \overline{Q}}{2\,\overline{\varepsilon}}\,. \tag{43}$$

(5) and (39) imply:

$$\frac{\partial}{\partial\,\overline{w}} \left(\frac{\partial E\left\{\pi^{BS}(\varepsilon)\right\}}{\partial F_i} \right) \ = \ \frac{2\,\widehat{\varepsilon}\,[\,n+1\,]}{4\,\overline{\varepsilon}\,[\,n+1\,]} \ = \ \frac{\widehat{\varepsilon}}{2\,\overline{\varepsilon}}\,. \tag{44}$$

The last equality in (14) follows from (12), (43), and (44).

$b\,\overline{Q} \ > \ \widehat{\varepsilon}$ from (42). Therefore, the last expression in (14) is: (i) positive if $\widehat{\varepsilon} > 0$ or γ is sufficiently close to 1; and (ii) negative if $\widehat{\varepsilon} < 0$ and γ is sufficiently close to 0. ■

Proof of Lemma 3. (1) and (2) imply that for $\varepsilon \in [\,\widehat{\varepsilon}, \overline{\varepsilon}\,]$:

$$Q(\overline{w}, \varepsilon) - Q(\overline{w}, \widehat{\varepsilon}) \ = \ \frac{1}{b}\,[\,\varepsilon - \widehat{\varepsilon}\,]\,. \tag{45}$$

(5), (7), (18), and (45) imply that for $\varepsilon \in [\,\widehat{\varepsilon}, \overline{\varepsilon}\,]$:

$$\begin{aligned} \overline{q}_i(\varepsilon) \ &= \ q_i(\widehat{\varepsilon}) + \frac{\alpha_i}{b}\,[\,\varepsilon - \widehat{\varepsilon}\,] \\ \Rightarrow \quad \frac{\partial\,\overline{q}_i(\varepsilon)}{\partial F_i} \ &= \ \frac{b + b\,n}{b\,[\,n+1\,]} + \frac{\alpha_i}{b}\,[\,-\,b\,] \ = \ 1 - \alpha_i\,. \end{aligned} \tag{46}$$

(6), (7), and (16) imply that for $\varepsilon \in [\,\underline{\varepsilon}, \widehat{\varepsilon})$:

$$\begin{aligned} \frac{\partial \pi_i^G(\varepsilon)}{\partial F_i} \ &= \ [\,w(\varepsilon) - c_i\,]\,\frac{\partial q_i(\varepsilon)}{\partial F_i} + q_i(\varepsilon)\,\frac{\partial w(\varepsilon)}{\partial F_i} \\ &= \ \frac{1}{n+1}\,[\,a + \varepsilon + C - b\,F - (n+1)\,c_i\,]\,\frac{n}{n+1} \\ &\quad + \ \frac{1}{b\,[\,n+1\,]}\,[\,a + \varepsilon + C_{-i} - n\,c_i + b\,n\,F_i - b\,F_{-i}\,]\left[-\,\frac{b}{n+1}\right] \end{aligned}$$

$$= \frac{1}{[n+1]^2} \Big\{ n[a+\varepsilon+C_{-i}-n\,c_i-b\,F_i-b\,F_{-i}]$$

$$- [a+\varepsilon+C_{-i}-n\,c_i+b\,n\,F_i-b\,F_{-i}] \Big\}$$

$$= \frac{1}{[n+1]^2} \{ [n-1][a+\varepsilon+C_{-i}-n\,c_i-b\,F_{-i}] - 2\,b\,n\,F_i \}. \tag{47}$$

The expression in (47) is strictly increasing in ε. Therefore, (16), (46), and (47) imply that $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ is greater for $\varepsilon \in (\widehat{\varepsilon}, \overline{\varepsilon}\,]$ than for $\varepsilon \in [\,\underline{\varepsilon}, \widehat{\varepsilon})$ if:

$$[1-\alpha_i][\overline{w}-c_i] > \frac{1}{[n+1]^2} \{ [n-1][a+\widehat{\varepsilon}+C_{-i}-n\,c_i-b\,F_{-i}] - 2\,b\,n\,F_i \}$$

$$\Leftrightarrow \quad [1-\alpha_i][\overline{w}-c_i][n+1]^2$$

$$> [n-1][a+\widehat{\varepsilon}+C_{-i}-n\,c_i-b\,F_{-i}] - 2\,b\,n\,F_i\,. \tag{48}$$

(5) implies:

$$[n-1][a+\widehat{\varepsilon}+C_{-i}-n\,c_i-b\,F_{-i}] - 2\,b\,n\,F_i$$

$$= [n-1][a+(n+1)\,\overline{w} - a - C + b\,F + C_{-i} - n\,c_i - b\,F_{-i}] - 2\,b\,n\,F_i$$

$$= [n-1][(n+1)\,\overline{w} - (n+1)\,c_i + b\,F_i] - 2\,b\,n\,F_i$$

$$= [n-1][n+1][\overline{w} - c_i] - [n+1]\,b\,F_i\,. \tag{49}$$

(48) and (49) imply that $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ is greater for $\varepsilon \in (\widehat{\varepsilon}, \overline{\varepsilon}\,]$ than for $\varepsilon \in [\,\underline{\varepsilon}, \widehat{\varepsilon})$ if:

$$[1-\alpha_i][\overline{w}-c_i][n+1] > [n-1][\overline{w}-c_i] - b\,F_i$$

$$\Leftrightarrow \quad [\overline{w}-c_i][n+1-n+1-\alpha_i\,(n+1)] > -b\,F_i$$

$$\Leftrightarrow \quad [\overline{w}-c_i][2-\alpha_i\,(n+1)] > -b\,F_i\,. \tag{50}$$

The inequality in (50) holds for all $F_i \geq 0$ if $\alpha_i < \frac{2}{n+1}$. ■

Proof of Proposition 2. (6), (7), (16), and (46) imply:

$$\Psi \equiv [\,w(\widehat{\varepsilon}) - c_i\,] \left. \frac{\partial q_i(\varepsilon)}{\partial F_i} \right|_{\varepsilon=\widehat{\varepsilon}} + q_i(\widehat{\varepsilon}) \left. \frac{\partial w(\varepsilon)}{\partial F_i} \right|_{\varepsilon=\widehat{\varepsilon}} - [\,\overline{w} - c_i\,] \left. \frac{\partial \overline{q}_i(\varepsilon)}{\partial F_i} \right|_{\varepsilon=\widehat{\varepsilon}}$$

$$= [\,\overline{w} - c_i\,] \frac{n}{n+1} + \frac{a+\widehat{\varepsilon}+C_{-i}-n\,c_i+b\,n\,F_i-b\,F_{-i}}{b[n+1]} \left[-\frac{b}{n+1} \right]$$

$$- \; [\, \overline{w} - c_i \,] \, [\, 1 - \alpha_i \,]$$

$$= \; \frac{\overline{w} - c_i}{n+1} \, [\, n - (n+1)\,(1-\alpha_i)\,] - \frac{1}{[\, n+1 \,]^2} \, [\, a + \widehat{\varepsilon} + C_{-\,i} - n\, c_i + b\, n\, F_i - b\, F_{-i} \,] \,. \quad (51)$$

(5) implies:

$$a + \widehat{\varepsilon} + C_{-\,i} - n\, c_i + b\, n\, F_i - b\, F_{-i}$$

$$= \; a + [\, n+1 \,] \, \overline{w} - a - C + b\, F + C_{-\,i} - n\, c_i + b\, n\, F_i - b\, F_{-i}$$

$$= \; [\, n+1 \,] \, \overline{w} - [\, n+1 \,] \, c_i + [\, n+1 \,] \, b\, F_i \; = \; [\, n+1 \,] \, [\, \overline{w} - c_i + b\, F_i \,] \,. \quad (52)$$

(51) and (52) imply:

$$\Psi \; = \; \frac{\overline{w} - c_i}{n+1} \, [\, n - (n+1)\,(1-\alpha_i)\,] - \frac{1}{n+1} \, [\, \overline{w} - c_i + b\, F_i \,]$$

$$= \; \frac{1}{n+1} \, \{\, [\, \overline{w} - c_i \,] \, [\, n - (n+1) + \alpha_i\,(n+1) - 1 \,] - b\, F_i \,\}$$

$$= \; \frac{1}{n+1} \, \{\, [\, \overline{w} - c_i \,] \, [\, \alpha_i\,(n+1) - 2 \,] - b\, F_i \,\} \,. \quad (53)$$

(5) and (46) imply:

$$\int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} \frac{\partial\, \overline{q}_i(\varepsilon)}{\partial F_i} \, dH(\varepsilon) \; = \; \frac{1}{2\, \overline{\varepsilon}} \, [\, 1 - \alpha_i \,] \, [\, \overline{\varepsilon} - \widehat{\varepsilon} \,]$$

$$= \; \frac{1}{2\, \overline{\varepsilon}} \, [\, 1 - \alpha_i \,] \, [\, \overline{\varepsilon} - (n+1) \, \overline{w} + a + C - b\, F \,] \,. \quad (54)$$

(5) implies that $\frac{\partial \widehat{\varepsilon}}{\partial \overline{w}} = n + 1$. Therefore, (17), (53), and (54) imply:

$$\frac{\partial}{\partial\, \overline{w}} \left(\frac{dE\{\, \pi_i^G(\varepsilon) \,\}}{dF_i} \right) \; = \; \frac{1}{2\, \overline{\varepsilon}} \, [\, 1 - \alpha_i \,] \, [\, \overline{\varepsilon} - (n+1) \, \overline{w} + a + C - b\, F \,]$$

$$+ \; \frac{\overline{w} - c_i}{2\, \overline{\varepsilon}} \, [\, \alpha_i\,(n+1) - 2 \,] - \frac{b\, F_i}{2\, \overline{\varepsilon}}$$

$$= \; \frac{1}{2\, \overline{\varepsilon}} \left\{ \, [\, 1 - \alpha_i \,] \, [\, \overline{\varepsilon} - (n+1) \, \overline{w} + a + C - b\, F \,] \right.$$

$$\left. + \, [\, \overline{w} - c_i \,] \, [\, \alpha_i\,(n+1) - 2 \,] - b\, F_i \right\}$$

$$= \; \frac{1}{2\, \overline{\varepsilon}} \left\{ \, [\, 1 - \alpha_i \,] \, [\, \overline{\varepsilon} - (n+1)\,(\, \overline{w} - c_i) - (n+1)\, c_i + a + C - b\, F \,] \right.$$

$$+ \left[\overline{w} - c_i \right] \left[\alpha_i (n+1) - 2 \right] - b F_i \Bigg\}$$

$$= \frac{1}{2\,\overline{\varepsilon}} \Bigg\{ \left[\overline{w} - c_i \right] \left[\alpha_i (n+1) - 2 - (1-\alpha_i)(n+1) \right]$$

$$+ \left[1 - \alpha_i \right] \left[a + \overline{\varepsilon} + C - (n+1) c_i - b F \right] - b F_i \Bigg\}. \tag{55}$$

Observe that:

$$\alpha_i [n+1] - 2 - [1-\alpha_i][n+1] = [n+1][\alpha_i - (1-\alpha_i)] - 2$$

$$= [n+1][2\alpha_i - 1] - 2\,. \tag{56}$$

(55) and (56) imply that (19) holds. ■

Proof of Corollary 1. (19) implies that for $x \in \left\{ a^I, \overline{Q}, c_i, C_{-i}, \overline{w} \right\}$, $\frac{\partial}{\partial x} \left\{ \frac{\partial}{\partial \overline{w}} \left(\frac{\partial E\{ \pi_i^G(\varepsilon) \}}{\partial F_i} \right) \right\}$ $\overset{s}{=} \frac{\partial \Omega}{\partial x}$. Recall from (2) that $a = b \left[a^I + \overline{Q} \right]$. Therefore, (19) implies that for $\alpha_i \in [0,1)$:

$$\frac{\partial \Omega}{\partial a^I} > 0\,; \quad \frac{\partial \Omega}{\partial \overline{Q}} > 0\,; \text{ and } \frac{\partial \Omega}{\partial C_{-i}} > 0\,.$$

Furthermore:

$$\frac{\partial \Omega}{\partial c_i} = -n[1-\alpha_i] + 2 - [n+1][2\alpha_i - 1]$$

$$= -n + \alpha_i n + 2 - 2\alpha_i n + n - 2\alpha_i + 1$$

$$= 3 - \alpha_i n - 2\alpha_i = 3 - \alpha_i [n+2] > 0 \Leftrightarrow \alpha_i < \frac{3}{2+n}\,.$$

(19) also implies:

$$\frac{\partial \Omega}{\partial \overline{w}} = [n+1][2\alpha_i - 1] - 2 < 0 \Leftrightarrow [n+1][2\alpha_i - 1] < 2$$

$$\Leftrightarrow 2\alpha_i < 1 + \frac{2}{n+1} \Leftrightarrow \alpha_i < \frac{n+3}{2[n+1]}\,. \quad ■$$

Proof of Corollary 2. (19) implies that as $\alpha_i \to 1$:

$$\Omega \to -b F_i + \left[\overline{w} - c_i \right][n-1]\,. \tag{57}$$

The conclusion in the Corollary follows immediately from (19) and (57). ■

Proof of Corollary 3. (19) implies that $\frac{\partial}{\partial \overline{w}} \left(\frac{dE\{ \pi_i^G(\varepsilon) \}}{dF_i} \right) < 0$ for all $F_j \geq 0$ ($j = 1, ..., n$) under the specified conditions if:

$$\left[1 - \frac{1}{n}\right] [\, a + \overline{\varepsilon} + (n-1)\, c - n\, c \,] - [\, \overline{w} - c \,] \left[2 + (\, n+1\,) \left(1 - \frac{2}{n}\right)\right] \;<\; 0$$

$$\Leftrightarrow \quad [\, n-1\,] [\, a + \overline{\varepsilon} - c\,] - [\, \overline{w} - c\,] [\, 2\,n + (\, n+1\,)(\, n-2\,)\,] \;<\; 0$$

$$\Leftrightarrow \quad [\, n-1\,] [\, a + \overline{\varepsilon} - c\,] \;<\; [\, \overline{w} - c\,] \left[\, 2\,n + n^2 - n - 2 \,\right]$$

$$\Leftrightarrow \quad [\, n-1\,] [\, a + \overline{\varepsilon} - c\,] \;<\; [\, \overline{w} - c\,] \left[\, n^2 + n - 2 \,\right]$$

$$\Leftrightarrow \quad [\, n-1\,] [\, a + \overline{\varepsilon} - c\,] \;<\; [\, \overline{w} - c\,] [\, n-1\,] [\, n+2\,]$$

$$\Leftrightarrow \quad a + \overline{\varepsilon} - c \;<\; [\, n+2\,] [\, \overline{w} - c\,] \quad \Leftrightarrow \quad a + \overline{\varepsilon} + [\, n+1\,]\, c \;<\; [\, n+2\,]\, \overline{w}$$

$$\Leftrightarrow \quad \overline{w} \;>\; \frac{1}{n+2} [\, a + \overline{\varepsilon} + (n+1)\, c\,] . \quad \blacksquare$$

B. Additional Numerical Solutions.

Tables A1 – A12 report equilibrium outcomes corresponding to the outcomes reported in Table 2, with the exception that one baseline parameter value other than $\overline{w}$ is either increased or reduced by 20%. The title in each table identifies the value of the modified baseline parameter. All other parameter values remain at their baseline levels. Three values of $\overline{w}$ (800, 1,000, and 1,200) are considered in each setting.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,533.4	1,793.2	975,821	544.2	883.5
1,000	1,510.4	1,785.2	1,158,442	643.8	1,795.8
1,200	1,481.8	1,777.8	1,324,814	736.5	2,638.1

Table A1. Equilibrium Outcomes when $\overline{Q} = 4{,}018.18$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,297.9	1,528.7	777,761	501.8	1,288.0
1,000	1,277.0	1,521.6	917,916	590.7	2,226.5
1,200	1,250.0	1,515.0	1,044,307	673.2	3,089.3

Table A2. Equilibrium Outcomes when $\overline{Q} = 2{,}678.78$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,597.9	1,865.1	1,031,155	554.5	780.9
1,000	1,574.3	1,856.9	1,225,948	656.8	1,685.6
1,200	1,545.2	1,849.3	1,403,825	752.1	2,522.0

Table A3. Equilibrium Outcomes when $a^I = 6{,}198.62$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,234.6	1,457.1	725,943	488.8	1,405.2
1,000	1,214.2	1,450.2	855,286	574.8	2,350.3
1,200	1,187.6	1,443.8	971,535	654.3	3,218.1

Table A4. Equilibrium Outcomes when $a^I = 4,132.42$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,401.2	1,657.3	806,386	474.4	1,533.2
1,000	1,377.4	1,649.3	948,253	557.0	2,485.1
1,200	1,346.0	1,642.0	1,075,251	633.5	3,358.1

Table A5. Equilibrium Outcomes when $b^I = 0.48$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,434.9	1,665.5	960,892	582.5	488.7
1,000	1,414.5	1,658.5	1,147,258	692.8	1,369.8
1,200	1,390.0	1,651.9	1,318,518	795.7	2,187.6

Table A6. Equilibrium Outcomes when $b^I = 0.32$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,411.6	1,663.2	846,762	495.5	1,035.4
1,000	1,397.7	1,656.1	1,001,444	583.6	2,061.9
1,200	1,377.1	1,649.6	1,141,651	665.5	3,004.1

Table A7. Equilibrium Outcomes when $\overline{\eta} = 2,009.09$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,414.8	1,658.3	913,034	565.5	1,075.2
1,000	1,383.8	1,650.1	1,082,983	658.8	1,888.1
1,200	1,348.1	1,642.5	1,237,272	753.7	2,640.7

Table A8. Equilibrium Outcomes when $\overline{\eta} = 1,339.39$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,415.3	1,660.7	870,989	526.4	1,056.8
1,000	1,393.6	1,653.1	1,032,232	621.0	1,984.7
1,200	1,365.9	1,646.1	1,178,404	708.8	2,839.2

Table A9. Equilibrium Outcomes when $c_i = 30$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,415.1	1,661.1	880,049	521.5	1,104.1
1,000	1,393.0	1,653.5	1,040,598	615.7	2,027.7
1,200	1,365.1	1,646.5	1,186,202	703.2	2,878.9

Table A10. Equilibrium Outcomes when $c_i = 20$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,202.1	1,388.8	635,378	452.5	2,196.1
1,000	1,178.4	1,383.7	744,144	529.4	3,241.0
1,200	1,148.8	1,379.0	841,184	600.4	$\overline{\varepsilon}$

Table A11. Equilibrium Outcomes when $n = 6$.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,720.0	2,067.6	1,225,096	608.8	− 185.1
1,000	1,697.6	2,055.8	1,504,869	727.5	590.7
1,200	1,670.0	2,044.6	1,735,109	838.6	1,314.6

Table A12. Equilibrium Outcomes when $n = 4$.

In each of the settings in Tables A1 – A12, the generators reduce their forward contracting as the price cap increases. The entry in the last row and last column of Table 11 indicates that when $n = 6$, the relatively intense competition among generators ensures that wholesale prices are sufficiently low that the price cap $\overline{w} = 1,200$ never binds.

C. A Pro-rata Rule for Allocating $\Delta(\varepsilon)$.

We now establish that the qualitative conclusions reported in Table 2 persist when the fraction of $\Delta(\varepsilon) \equiv Q(\overline{w}, \varepsilon) - Q(\overline{w}, \widehat{\varepsilon})$ that is allocated to generator i is:

$$\alpha_i(\cdot) \; = \; \frac{q_i(\widehat{\varepsilon})}{Q(\widehat{\varepsilon})} \; \text{ for } \; i \; = \; 1, ..., n \, . \tag{58}$$

When the pro-rata allocation rule in (58) prevails, the fraction of $\Delta(\varepsilon)$ that a generator is allocated is the generator's share of total output when the price cap first binds (i.e., when $\varepsilon = \widehat{\varepsilon}$).

Lemma A1 and its corollary first establish a parallel to Lemma 3, identifying conditions under which expanded forward contracting increases a generator's equilibrium profit more rapidly when the wholesale price cap ($\overline{w}$) binds than when it does not bind. Proposition A1 then establishes a parallel to Proposition 2, characterizing the rate at which a generator's incentive for forward contracting ($\frac{dE\{\,\pi_i^G(\varepsilon)\,\}}{dF_i}$) increases as $\overline{w}$ increases when the pro-rata allocation rule in (58) prevails. Next, Proposition A2 provides a characterization of the equilibrium levels of forward contracting in a symmetric equilibrium when (58) prevails. Finally, Table A13 establishes that the qualitative conclusions reported in Table 2 continue to hold when the baseline parameters in Table 1 hold and the allocation rule in (58) prevails.

Lemma A1. *Suppose (58) holds and* $[\,n-1\,][\,\overline{w} + c_i\,] - 2\,C_{-i} \; > \; b\,[\,n-1\,]\,F_i - 2\,b\,F_{-i}$. *Then expanded forward contracting increases a generator's equilibrium profit more rapidly when the price cap binds than when it does not bind (i.e.,* $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ *is higher when* $\varepsilon \in (\,\widehat{\varepsilon}, \overline{\varepsilon}\,]$ *than when* $\varepsilon \in [\,\underline{\varepsilon}, \widehat{\varepsilon}\,)$ *for* $i = 1, ..., n$*).*

Proof. (5), (7), and (8) imply:

$$q_i(\widehat{\varepsilon}) \; = \; \frac{a + [\,n+1\,]\,\overline{w} - \; a - C + b\,F + C_{-\,i} - n\,c_i + b\,n\,F_i - b\,F_{-i}}{b\,[\,n+1\,]}$$

$$= \; \frac{[\,n+1\,]\,\overline{w} - c_i + b\,F_i - n\,c_i + b\,n\,F_i}{b\,[\,n+1\,]} \; = \; \frac{\overline{w} - c_i + b\,F_i}{b}\,; \tag{59}$$

$$Q(\widehat{\varepsilon}) \; = \; \frac{n\,\overline{w} - C + b\,F}{b}\,. \tag{60}$$

(59) and (60) imply:

$$\frac{q_i(\widehat{\varepsilon})}{Q(\widehat{\varepsilon})} \; = \; \frac{\overline{w} - c_i + b\,F_i}{n\,\overline{w} - C + b\,F}. \tag{61}$$

(58) and (61) imply:

$$\frac{\partial \alpha_i(\cdot)}{\partial F_i} \; = \; \frac{1}{[\,n\,\overline{w} - C + b\,F\,]^2}\,[\,b\,(n\,\overline{w} - C + b\,F) - b\,(\,\overline{w} - c_i + b\,F_i\,)\,]$$

$$= \frac{b}{[\,n\,\overline{w} - C + b\,F\,]^2}\,[\,(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}\,] \;>\; 0\,. \tag{62}$$

The final inequality in (62) holds because $[\,n-1\,]\,\overline{w} > C_{-i} = \sum_{\substack{j=1\\ j\neq i}}^{n} c_j$, since $\overline{w} > \max\{c_1, ..., c_n\}$, by assumption.

(5), (7), (45), (58), (59), (61), and (62) imply that for $\varepsilon \in (\,\widehat{\varepsilon}, \overline{\varepsilon}\,]$:

$$\overline{q}_i(\varepsilon) \;=\; q_i(\widehat{\varepsilon}) + \frac{\alpha_i(\cdot)}{b}\,[\,\varepsilon - \widehat{\varepsilon}\,]$$

$$\Rightarrow \quad \frac{\partial\,\overline{q}_i(\varepsilon)}{\partial F_i} \;=\; 1 + \frac{\alpha_i(\cdot)}{b}\,[\,-b\,] + \frac{1}{b}\,[\,\varepsilon - \widehat{\varepsilon}\,]\;\frac{\partial \alpha_i(\cdot)}{\partial F_i}$$

$$= 1 - \alpha_i(\cdot) + \frac{\varepsilon - \widehat{\varepsilon}}{[\,n\,\overline{w} - C + b\,F\,]^2}\,[\,(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}\,]$$

$$= 1 - \frac{\overline{w} - c_i + b\,F_i}{n\,\overline{w} - C + b\,F} + \frac{\varepsilon - \widehat{\varepsilon}}{[\,n\,\overline{w} - C + b\,F\,]^2}\,[\,(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}\,]\,. \tag{63}$$

(16) and (63) imply that for $\varepsilon \in (\,\widehat{\varepsilon}, \overline{\varepsilon}\,]$:

$$\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i} \;=\; [\,\overline{w} - c_i\,]\Bigg\{ 1 - \frac{\overline{w} - c_i + b\,F_i}{n\,\overline{w} - C + b\,F} + \frac{\varepsilon - \widehat{\varepsilon}}{[\,n\,\overline{w} - C + b\,F\,]^2}\,[\,(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}\,]\Bigg\}. \tag{64}$$

The expressions in (47) and (64) are both strictly increasing in ε. (Recall that $[\,n-1\,]\,\overline{w} > C_{-i} = \sum_{\substack{j=1\\ j\neq i}}^{n} c_j$ because $\overline{w} > \max\{c_1, ..., c_n\}$, by assumption.) Therefore, (5), (47), and (64) imply that $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ is greater for $\varepsilon \in (\,\widehat{\varepsilon}, \overline{\varepsilon}\,]$ than for $\varepsilon \in [\,\underline{\varepsilon}, \widehat{\varepsilon}\,)$ if:

$$[\,\overline{w} - c_i\,]\left\{ 1 - \frac{\overline{w} - c_i + b\,F_i}{n\,\overline{w} - C + b\,F} + \frac{\widehat{\varepsilon} - \widehat{\varepsilon}}{[\,n\,\overline{w} - C + b\,F\,]^2}\,[\,(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}\,] \right\}$$

$$> \frac{1}{[\,n+1\,]^2}\,\{\,[\,n-1\,]\,[\,a + \widehat{\varepsilon} + C_{-i} - n\,c_i - b\,F_{-i}\,] - 2\,b\,n\,F_i\,\}$$

$$\Leftrightarrow \quad [\,n+1\,]^2\,[\,\overline{w} - c_i\,]\left[1 - \frac{\overline{w} - c_i + b\,F_i}{n\,\overline{w} - C + b\,F} \right]$$

$$> [\,n-1\,]\,[\,a + \widehat{\varepsilon} + C_{-i} - n\,c_i - b\,F_{-i}\,] - 2\,b\,n\,F_i\,. \tag{65}$$

(49) and (65) imply that $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ is greater for $\varepsilon \in (\widehat{\varepsilon}, \overline{\varepsilon}\,]$ than for $\varepsilon \in [\,\underline{\varepsilon}, \widehat{\varepsilon})$ if:

$$[\,n+1\,]\,[\,\overline{w}-c_i\,]\left[1-\frac{\overline{w}-c_i+b\,F_i}{n\,\overline{w}-C+b\,F}\right] \;>\; [\,n-1\,]\,[\,\overline{w}-c_i\,]-b\,F_i$$

$$\Leftrightarrow\;\; [\,\overline{w}-c_i\,]\left[(\,n+1\,)\left(\frac{n\,\overline{w}-C+b\,F-\overline{w}+c_i-b\,F_i}{n\,\overline{w}-C+b\,F}\right)-(\,n-1\,)\right] \;>\; -\,b\,F_i$$

$$\Leftrightarrow\;\; [\,\overline{w}-c_i\,]\left[(\,n+1\,)\left(\frac{[\,n-1\,]\,\overline{w}-C_{-i}+b\,F_{-i}}{n\,\overline{w}-C+b\,F}\right)-(\,n-1\,)\right] \;>\; -\,b\,F_i\,. \tag{66}$$

Because $n\,\overline{w} > C$ by assumption, the inequality in (66) holds for all $F_i \geq 0$ if:

$$[\,n+1\,]\,\frac{[\,n-1\,]\,\overline{w}-C_{-i}+b\,F_{-i}}{n\,\overline{w}-C+b\,F} \;>\; n-1$$

$$\Leftrightarrow\;\; [\,n+1\,]\,[\,(\,n-1\,)\,\overline{w}-C_{-i}+b\,F_{-i}\,]-[\,n-1\,]\,[\,n\,\overline{w}-C+b\,F\,] \;>\; 0$$

$$\Leftrightarrow\;\; [\,n-1\,]\,\overline{w}\,[\,n+1-n\,]+[\,n-1\,]\,C-[\,n+1\,]\,C_{-i}$$
$$+\,[\,n+1\,]\,b\,F_{-i}-[\,n-1\,]\,b\,F \;>\; 0$$

$$\Leftrightarrow\;\; [\,n-1\,]\,\overline{w}\,+[\,n-1\,]\,[\,c_i+C_{-i}\,]-[\,n+1\,]\,C_{-i}$$
$$+\,[\,n+1\,]\,b\,F_{-i}-[\,n-1\,]\,b\,[\,F_i+F_{-i}]\;>\; 0$$

$$\Leftrightarrow\;\; [\,n-1\,]\,\overline{w}+[\,n-1\,]\,c_i-2\,C_{-i}+2\,b\,F_{-i}-b\,[\,n-1\,]\,F_i \;>\; 0$$

$$\Leftrightarrow\;\; [\,n-1\,]\,[\,\overline{w}\,+c_i\,]-2\,C_{-i} \;>\; b\,[\,n-1\,]\,F_i-2\,b\,F_{-i}\,.\;\blacksquare \tag{67}$$

Corollary. *Suppose (58) holds, $c_1 = ... = c_n$, and $F_1 = ... = F_n$. Then $\frac{\partial \pi_i^G(\varepsilon)}{\partial F_i}$ is higher when $\varepsilon \in (\,\widehat{\varepsilon}, \overline{\varepsilon}\,]$ than when $\varepsilon \in [\,\underline{\varepsilon}, \widehat{\varepsilon})$.*

Proof. Let $c = c_1 = ... = c_n$ and $\widetilde{F} = F_1 = ... = F_n$. Then (67) holds for all $\widetilde{F} \geq 0$ if:

$$[\,n-1\,]\,\overline{w}+[\,n-1\,]\,c-2\,[\,n-1\,]\,c \;>\; b\,[\,n-1\,]\,\widetilde{F}-2\,b\,[\,n-1\,]\,\widetilde{F}$$

$$\Leftrightarrow\;\; [\,n-1\,]\,[\,\overline{w}-c\,] \;>\; -\,b\,[\,n-1\,]\,\widetilde{F} \;\;\Leftrightarrow\;\; \overline{w}-c \;>\; -\,b\,[\,n-1\,]\,\widetilde{F}\,.$$

The last inequality holds here because $\overline{w} > c$, by assumption. $\blacksquare$

Proposition A1. *Suppose (58) holds. Then an increase in $\overline{w}$ can either enhance or diminish a generator's incentive for forward contracting because* $\frac{\partial}{\partial \overline{w}}\left(\frac{dE\{\pi_i^G(\varepsilon)\}}{dF_i}\right) = \frac{\Omega_{R2}}{2\,\overline{\varepsilon}}$,

$$\text{where} \quad \Omega_{R2} \equiv \left[\frac{(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}}{(n\,\overline{w} - C + b\,F)^2}\right]$$

$$\cdot \left[(\overline{\varepsilon} - \widehat{\varepsilon})\,(n\,\overline{w} - C + b\,F) + \frac{1}{2}(\overline{\varepsilon} - \widehat{\varepsilon})^2 - (\overline{w} - c_i)\,(n+1)\right]$$

$$+ [\,\overline{w} - c_i\,]\,[n-1]\,b\,F_i\,.$$

Proof. (6), (7), (59), and (63) imply:

$$[\,w(\widehat{\varepsilon}) - c_i\,]\,\frac{\partial q_i(\widehat{\varepsilon})}{\partial F_i} + q_i(\widehat{\varepsilon})\,\frac{\partial w(\widehat{\varepsilon})}{\partial F_i} - [\,\overline{w} - c_i\,]\,\frac{\partial\,\overline{q}_i(\widehat{\varepsilon})}{\partial F_i}$$

$$= [\,\overline{w} - c_i\,]\left[\frac{n}{n+1}\right] + \left[\frac{\overline{w} - c_i + b\,F_i}{b}\right]\left[-\frac{b}{n+1}\right]$$

$$-[\,\overline{w} - c_i\,]\left\{1 - \frac{\overline{w} - c_i + b\,F_i}{n\,\overline{w} - C + b\,F} + \frac{\widehat{\varepsilon} - \widehat{\varepsilon}}{[\,n\,\overline{w} - C + b\,F\,]^2}\,[\,(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}\,]\right\}$$

$$= [\,\overline{w} - c_i\,]\left[\frac{n-1}{n+1}\right] - b\,F_i\left[\frac{1}{n+1}\right] - [\,\overline{w} - c_i\,]\left[\frac{(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}}{n\,\overline{w} - C + b\,F}\right]. \qquad (68)$$

(5) and (63) imply:

$$\int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} \frac{\partial\,\overline{q}_i(\varepsilon)}{\partial F_i}\,dH(\varepsilon) = \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}}\left\{1 - \frac{\overline{w} - c_i + b\,F_i}{n\,\overline{w} - C + b\,F} + \frac{\varepsilon - \widehat{\varepsilon}}{[\,n\,\overline{w} - C + b\,F\,]^2}\,[\,(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}\,]\right\}dH(\varepsilon)$$

$$= \frac{1}{2\,\overline{\varepsilon}}\left[1 - \frac{\overline{w} - c_i + b\,F_i}{n\,\overline{w} - C + b\,F}\right][\,\overline{\varepsilon} - \widehat{\varepsilon}\,] + \frac{1}{2\,\overline{\varepsilon}}\left[\frac{(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}}{(n\,\overline{w} - C + b\,F)^2}\right]\int_{\widehat{\varepsilon}}^{\overline{\varepsilon}}[\,\varepsilon - \widehat{\varepsilon}\,]\,d\varepsilon$$

$$= \frac{1}{2\,\overline{\varepsilon}}\left[1 - \frac{\overline{w} - c_i + b\,F_i}{n\,\overline{w} - C + b\,F}\right][\,\overline{\varepsilon} - \widehat{\varepsilon}\,]$$

$$+ \frac{1}{2\,\overline{\varepsilon}}\left[\frac{(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}}{(n\,\overline{w} - C + b\,F)^2}\right]\left[\frac{1}{2}(\overline{\varepsilon})^2 - \overline{\varepsilon}\,\widehat{\varepsilon} - \left(\frac{1}{2}(\widehat{\varepsilon})^2 - (\widehat{\varepsilon})^2\right)\right]$$

$$= \frac{1}{2\,\overline{\varepsilon}}\left[1 - \frac{\overline{w} - c_i + b\,F_i}{n\,\overline{w} - C + b\,F}\right][\,\overline{\varepsilon} - \widehat{\varepsilon}\,] + \frac{1}{2\,\overline{\varepsilon}}\left[\frac{(n-1)\,\overline{w} - C_{-i} + b\,F_{-i}}{(n\,\overline{w} - C + b\,F)^2}\right]\left[\frac{1}{2}(\overline{\varepsilon})^2 - \overline{\varepsilon}\,\widehat{\varepsilon} + \frac{1}{2}(\widehat{\varepsilon})^2\right]$$

$$= \frac{1}{2\,\overline{\varepsilon}}\left[1-\frac{\overline{w}-c_i+b\,F_i}{n\,\overline{w}-C+b\,F}\right][\,\overline{\varepsilon}-\widehat{\varepsilon}\,]+\frac{1}{4\,\overline{\varepsilon}}\left[\frac{(n-1)\,\overline{w}-C_{-i}+b\,F_{-i}}{(n\,\overline{w}-C+b\,F)^2}\right][\,\overline{\varepsilon}-\widehat{\varepsilon}\,]^2. \qquad (69)$$

Because $\frac{\partial\widehat{\varepsilon}}{\partial\overline{w}} = n+1$ from (5), (17), (68), and (69) imply:

$$\frac{\partial}{\partial\,\overline{w}}\left(\frac{dE\{\,\pi_i^G(\varepsilon)\,\}}{dF_i}\right) = \frac{1}{2\,\overline{\varepsilon}}\left[1-\frac{\overline{w}-c_i+b\,F_i}{n\,\overline{w}-C+b\,F}\right][\,\overline{\varepsilon}-\widehat{\varepsilon}\,]$$

$$+\ \frac{1}{4\,\overline{\varepsilon}}\left[\frac{(n-1)\,\overline{w}-C_{-i}+b\,F_{-i}}{(n\,\overline{w}-C+b\,F)^2}\right][\,\overline{\varepsilon}-\widehat{\varepsilon}\,]^2$$

$$+\ \frac{1}{2\,\overline{\varepsilon}}\left\{[\,\overline{w}-c_i\,][n-1]-b\,F_i-[\,\overline{w}-c_i\,][\,n+1\,]\left[\frac{(\,n-1\,)\,\overline{w}-C_{-i}+b\,F_{-i}}{n\,\overline{w}-C+b\,F}\right]\right\}$$

$$= \frac{1}{2\,\overline{\varepsilon}}\left[\frac{n\,\overline{w}-C+b\,F-\overline{w}+c_i-b\,F_i}{n\,\overline{w}-C+b\,F}\right][\,\overline{\varepsilon}-\widehat{\varepsilon}\,]+\frac{1}{2\,\overline{\varepsilon}}[\,(\,\overline{w}-c_i\,)\,(n-1)-b\,F_i\,]$$

$$+\ \frac{1}{2\,\overline{\varepsilon}}\left[\frac{(n-1)\,\overline{w}-C_{-i}+b\,F_{-i}}{(n\,\overline{w}-C+b\,F)^2}\right]\left[\frac{1}{2}\,(\,\overline{\varepsilon}-\widehat{\varepsilon}\,)^2-(\,\overline{w}-c_i\,)\,(\,n+1\,)\right]$$

$$= \frac{1}{2\,\overline{\varepsilon}}\left[\frac{(\,n-1\,)\,\overline{w}-C_{-i}+b\,F_{-i}}{n\,\overline{w}-C+b\,F}\right][\,\overline{\varepsilon}-\widehat{\varepsilon}\,]+\frac{1}{2\,\overline{\varepsilon}}[\,(\,\overline{w}-c_i\,)\,(\,n-1\,)-b\,F_i\,]$$

$$+\ \frac{1}{2\,\overline{\varepsilon}}\left[\frac{(n-1)\,\overline{w}-C_{-i}+b\,F_{-i}}{(n\,\overline{w}-C+b\,F)^2}\right]\left[\frac{1}{2}\,(\,\overline{\varepsilon}-\widehat{\varepsilon}\,)^2-(\,\overline{w}-c_i\,)\,(\,n+1\,)\right]$$

$$= \frac{1}{2\,\overline{\varepsilon}}\left[\frac{(\,n-1\,)\,\overline{w}-C_{-i}+b\,F_{-i}}{(\,n\,\overline{w}-C+b\,F\,)^2}\right]$$

$$\cdot\left[(\,\overline{\varepsilon}-\widehat{\varepsilon}\,)\,(\,n\,\overline{w}-C+b\,F\,)+\frac{1}{2}\,(\,\overline{\varepsilon}-\widehat{\varepsilon}\,)^2-(\,\overline{w}-c_i\,)\,(\,n+1\,)\right]$$

$$+\ \frac{1}{2\,\overline{\varepsilon}}[\,(\,\overline{w}-c_i\,)\,(n-1)-b\,F_i\,]. \quad \blacksquare$$

Proposition A2. *Suppose (58) holds and* $c_1 = ... = c_n = c$. *Then in a symmetric equilibrium, generator i's level of forward contracting,* F_i^*, *is determined by*:

$$[\,n-1\,]\,[\,\widehat{\varepsilon}-\underline{\varepsilon}\,]\left\{a-c+\frac{1}{2}\,[\,\widehat{\varepsilon}+\underline{\varepsilon}\,]-b\,F_i^*\left[\frac{n^2+1}{n-1}\right]\right\}$$

$$+\,[\,n+1\,]^2\,[\,\overline{w}-c\,]\,[\,\overline{\varepsilon}-\widehat{\varepsilon}\,]\left[\frac{(\,n-1\,)\,(\,\overline{w}-c\,)+b\,(\,n-1\,)\,F_i^*}{n\,(\,\overline{w}-c\,)+b\,n\,F_i^*}\right]$$

$$\cdot \left[1 + \frac{\overline{\varepsilon} - \widehat{\varepsilon}}{2 \, [\, n \, (\, \overline{w} - c \,) + b \, n \, F_i^* \,]} \right] \; = \; 0 \, . \tag{70}$$

<u>Proof</u>. (16), (47), and (64) imply:

$$\frac{dE\{\, \pi_i^G(\varepsilon) \,\}}{dF_i} \; = \; \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \left\{ \frac{1}{[\, n+1 \,]^2} \, [\, (\, n-1 \,) \, (\, a + \varepsilon + C_{-i} - n \, c_i - b \, F_{-i} \,) - 2 \, b \, n \, F_i \,] \right\} dH(\varepsilon)$$

$$+ \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} [\, \overline{w} - c_i \,] \left\{ 1 - \frac{\overline{w} - c_i + b \, F_i}{n \, \overline{w} - C + b \, F} \right.$$

$$\left. + \frac{\varepsilon - \widehat{\varepsilon}}{[\, n \, \overline{w} - C + b \, F \,]^2} \, [\, (n-1) \; \overline{w} - C_{-i} + b \, F_{-i} \,] \right\} dH(\varepsilon) \; = \; 0$$

$$= \; \frac{1}{2 \, \overline{\varepsilon} \; [\, n+1 \,]^2} \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \{ \; [\, n-1 \,] \, [\, a + \varepsilon + C_{-i} - n \, c_i - b \, F_{-i} \,] - 2 \, b \, n \, F_i \, \} \; d\varepsilon$$

$$+ \; \frac{1}{2 \, \overline{\varepsilon}} \; [\, \overline{w} - c_i \,] \left[1 - \frac{\overline{w} - c_i + b \, F_i}{n \, \overline{w} - C + b \, F} \right] [\, \overline{\varepsilon} - \widehat{\varepsilon} \,]$$

$$+ \; [\, \overline{w} - c_i \,] \left[\frac{(n-1) \; \overline{w} - C_{-i} + b \, F_{-i}}{2 \, \overline{\varepsilon} \; [\, n \, \overline{w} - C + b \, F \,]^2} \right] \int_{\widehat{\varepsilon}}^{\overline{\varepsilon}} [\, \varepsilon - \widehat{\varepsilon} \,] \; dH(\varepsilon) \; = \; 0$$

$$\Leftrightarrow \;\; [\, n-1 \,] \, [\, a + C_{-i} - n \, c_i - b F_{-i} \,] \; [\, \widehat{\varepsilon} - \underline{\varepsilon} \,] - 2 \, b \, n \, F_i \, [\, \widehat{\varepsilon} - \underline{\varepsilon} \,] + \frac{1}{2} \; [\, n-1 \,] \; \left[\, (\widehat{\varepsilon})^2 - (\underline{\varepsilon})^2 \, \right]$$

$$+ \; [\, n+1 \,]^2 \; [\, \overline{w} - c_i \,] \left[\frac{n \, \overline{w} - C + b \, F - \overline{w} + c_i - b \, F_i}{n \, \overline{w} - C + b \, F} \right] [\, \overline{\varepsilon} - \widehat{\varepsilon} \,]$$

$$+ \; [\, n+1 \,]^2 \; [\, \overline{w} - c_i \,] \; \frac{[\, n-1 \,] \; \overline{w} - C_{-i} + b \, F_{-i}}{[\, n \, \overline{w} - C + b \, F \,]^2} \; \frac{1}{2} \, [\, \varepsilon \; - \; \widehat{\varepsilon} \,]^2 \Big|_{\widehat{\varepsilon}}^{\overline{\varepsilon}} \; = \; 0$$

$$\Leftrightarrow \;\; [\, n-1 \,] \, [\, a + C_{-i} - n \, c_i - b F_{-i} \,] \; [\, \widehat{\varepsilon} - \underline{\varepsilon} \,] - 2 \, b \, n \, F_i \, [\, \widehat{\varepsilon} - \underline{\varepsilon} \,] + \frac{1}{2} \; [\, n-1 \,] \; \left[\, (\widehat{\varepsilon})^2 - (\underline{\varepsilon})^2 \, \right]$$

$$+ \; [\, n+1 \,]^2 \; [\, \overline{w} - c_i \,] \, [\, \overline{\varepsilon} - \widehat{\varepsilon} \,] \left[\frac{(\, n-1 \,) \, \overline{w} - C_{-i} + b \, F_{-i}}{n \, \overline{w} - C + b \, F} \right]$$

$$\cdot \left[1 + \frac{\overline{\varepsilon} - \widehat{\varepsilon}}{2 \, (\, n \, \overline{w} - C + b \, F \,)} \right] \; = \; 0 \, . \tag{71}$$

When $c_1 \; = \; ... \; = \; c_n \; = \; c$ and $F_1 \; = \; ... \; = \; F_n \; = \; F_i^*$, (71) holds if and only if:

$$[\, n-1 \,] \, [\, a - c - b \, (\, n-1 \,) F_i^* \,] \, [\, \widehat{\varepsilon} - \underline{\varepsilon} \,] - 2 \, b \, n \, F_i^* \; [\, \widehat{\varepsilon} - \underline{\varepsilon} \,] + \frac{1}{2} \; [\, n-1 \,] \; \left[\, (\widehat{\varepsilon})^2 - (\underline{\varepsilon})^2 \, \right]$$

$$+ [\, n+1\,]^2 \; [\, \overline{w} - c\,]\, [\, \overline{\varepsilon} - \widehat{\varepsilon}\,] \left[\frac{(\, n-1\,)\,(\, \overline{w} - c\,) + b\,(\, n-1\,)\, F_i^*}{n\,(\, \overline{w} - c\,) + b\, n\, F_i^*} \right]$$

$$\cdot \left[1 + \frac{\overline{\varepsilon} - \widehat{\varepsilon}}{2\; [\, n\,(\, \overline{w} - c\,) + b\, n\, F_i^*\,]} \right] \;=\; 0\,. \tag{72}$$

Observe that:

$$[\, n-1\,]\, [\, a - c - b\,(\, n-1\,)\, F_i^*\,]\, [\, \widehat{\varepsilon} - \underline{\varepsilon}\,] - 2\, b\, n\, F_i^*\, [\, \widehat{\varepsilon} - \underline{\varepsilon}\,] + \frac{1}{2}\; [\, n-1\,] \; \left[\, (\widehat{\varepsilon})^2 - (\underline{\varepsilon})^2\, \right]$$

$$= \; [\, n-1\,]\, [\, \widehat{\varepsilon} - \underline{\varepsilon}\,] \left\{ a - c + \frac{1}{2}\, [\, \widehat{\varepsilon} + \underline{\varepsilon}\,] - b\, F_i^* \left[\frac{(n-1)^2 + 2\, n}{n-1} \right] \right\}$$

$$= \; [\, n-1\,]\, [\, \widehat{\varepsilon} - \underline{\varepsilon}\,] \left\{ a - c + \frac{1}{2}\, [\, \widehat{\varepsilon} + \underline{\varepsilon}\,] - b\, F_i^* \left[\frac{n^2 + 1}{n-1} \right] \right\}. \tag{73}$$

(71), (72), and (73) imply that in a symmetric equilibrium, F_i^* is determined by (70). ■

Finally, we employ (70) to determine the equilibrium levels of forward contracting in this symmetric setting when the baseline parameter values (in Table 1) prevail. Table A13 reports each generator's equilibrium level of forward contracting (F_i), expected output $E\{q_i\}$, and expected profit ($E\{\pi_i^G\}$). The table also reports the equilibrium expected wholesale price ($E\{w\}$) and the smallest ε realization for which the price cap binds ($\widehat{\varepsilon}$) when the baseline parameter values prevail.

$\overline{w}$	F_i	$E\{q_i\}$	$E\{\pi_i^G\}$	$E\{w\}$	$\widehat{\varepsilon}$
800	1,422.4	1,661.6	861,155	515.5	1,169.6
1,000	1,397.2	1,653.8	1,027,028	612.3	2,055.5
1,200	1,367.1	1,646.6	1,177,935	703.2	2,879.1

Table A13. Equilibrium Outcomes as $\overline{w}$ Varies

The entries in Table A13 indicate that the qualitative conclusions reported in Table 2 continue to hold when the pro-rata allocation rule in (58) prevails. In particular, each generator reduces its equilibrium level of forward contracting as the wholesale price cap increases.

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