Hotelling Competition with Avoidable Horizontal Product Differentiation

by

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Abstract

We extend the standard model of Hotelling competition to allow consumers to change the default horizontal characteristic of the product they purchase. The potential to change this characteristic can alter the nature of the prevailing equilibrium, as sellers focus more on attracting all potential consumers and less on serving only "close" consumers. This potential can also lead to the non-existence of equilibria. Substantial default-switching costs can reduce a product's appeal, and thereby reduce a seller's profit. However, these costs can also serve as a credible commitment to compete less vigorously for "distant" consumers, and thereby increase equilibrium seller profit (while reducing consumer welfare and total welfare).

Keywords: Hotelling competition; default-switching costs; avoidable horizontal product differentiation.

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1 Introduction.

Hotelling (1929)'s classic model provides the basis for many investigations of competition among sellers of differentiated products.[1] In this model, firms set prices and product characteristics, after which consumers choose their preferred products, taking prices and product characteristics as given.

In practice, consumers can sometimes alter characteristics of the products they purchase. For example, consumers generally can alter default settings on the smartphones they purchase. Historically, the default settings on iPhones have limited the ability of advertisers to track a user's online activities.[2] In contrast, the default settings on Android phones have often facilitated such tracking.[3] In both cases, though, the owner of a smartphone can spend the time and effort required to change the phone's default settings to reflect the owner's preference for more or less "privacy."[4] Consumers can similarly change the default operating system on some computers, change the appliances that are included in a purchased home or apartment, and add to or otherwise modify the set of video games that come pre-installed on video game consoles. In addition, consumers can sometimes purchase adapters (or otherwise modify equipment) to ensure that new equipment is compatible with existing equipment or that new devices can be powered by preferred energy sources.[5] Consumers may also undertake costly "localization," whereby individuals in one culture or region adapt for their own use products designed for use in other cultures or regions.[6]

The purpose of this research is to examine how the outcomes in Hotelling (1929)'s classic model of competition between two sellers of differentiated products change when consumers can alter default horizontal product characteristics. Specifically, we consider settings where, at personal cost, a consumer can switch the default horizontal characteristic of the product she purchases to the corresponding default characteristic of the rival seller's product. For example, the consumer might switch the default "privacy" setting on the iPhone she purchases

[1]For selective reviews of the literature on Hotelling competition, see Graitson (1982), Gabszewicz and Thisse (1992), and Biscaia and Mota (2013), for example. Also see Launhardt (1885)'s pioneering work on spatial competition. Ferreira (1998) compares the analyses of Launhardt (1885) and Hotelling (1929).

[2]See https://developer.apple.com/app-store/user-privacy-and-data-use.

[3]See Grant and *Bloomberg* (2021) and Samsung (2024), for example.

[4]Some consumers may prefer to conceal information about their online activities because they fear the information might be (mis)construed to reflect their personal needs, beliefs, or preferences. Other consumers may prefer to share this information with advertisers because the sharing can increase the likelihood of receiving targeted, customized advertisements that can facilitate informed purchasing decisions. See Tucker (2012), Taylor and Wagman (2014), and Acquisti et al. (2016), for example.

[5]Furthermore, a consumer who purchases an automobile might install additional equipment (e.g., a satellite radio) in the car or pay a third party to change the color of the vehicle.

[6]See Paranthaman (2025), for example.

to the default "disclosure" setting on an Android phone.

We find that the potential to change default product characteristics can alter the nature of the equilibrium that arises. Specifically, when default-switching costs are sufficiently small, sellers focus more on securing the patronage of all consumers and less on attracting only "close" consumers.[7] The resulting intense competition can cause the relatively strong seller to drive its weaker rival from the market even in the presence of pronounced (default) horizontal product differentiation.[8]

We also find that an equilibrium may not exist when consumers can change default horizontal product characteristics. This is the case, for instance, when the strong seller's default-switching cost is large and the weak seller's default-switching cost is small. No equilibrium in which both sellers attract consumers exists in this setting because when relatively high "market-sharing" prices initially prevail, the weak seller can increase its profit by reducing its price sufficiently to attract all consumers. Distant consumers are attracted by the reduced price because they only need to incur a small default-switching cost to ensure that the weak seller's product embodies their preferred horizontal characteristic. An equilibrium in which the strong seller attracts all consumers also does not exist in this setting. This is the case because the strong seller must charge a low price to attract distant consumers, who incur the large default-switching cost to ensure that the strong seller's product embodies their preferred horizontal characteristic. The strong seller finds it more profitable to set a relatively high price for its product and attract only close consumers (who choose not to alter the firm's horizontal product characteristic).

One might suspect that when an equilibrium exists, a seller's equilibrium profit declines as the personal cost that consumers must incur to change the default horizontal characteristic of the seller's product increases. As this cost increases, the product becomes less attractive to distant consumers, which reduces their willingness to pay for the product. This effect can indeed serve to reduce the seller's profit. However, a countervailing effect can arise that causes a seller's equilibrium profit to increase as its default-switching cost increases. As this cost increases, the seller must reduce the price it charges (to all consumers) to secure the patronage of distant consumers (who switch the horizontal characteristic if they purchase the seller's product).[9] Therefore, the seller finds it less profitable to compete for distant

[7]A consumer who is "close" to a seller is a consumer whose most preferred horizontal product characteristic is similar to the firm's default characteristic.

[8]The relatively strong seller operates with relatively low production costs and/or offers preferred vertical product characteristics.

[9]Each seller charges a uniform price for its product in our model, and information about the willingness to pay of individual consumers is not available. Consequently, the intensity of competition is affected primarily by the magnitudes of default-switching costs in our model. Other studies (e.g., Montes et al., 2019; Chen

consumers as its default-switching cost increases. Consequently, a higher default-switching cost can serve as a credible commitment to compete less vigorously for distant consumers, and to focus instead on attracting only close consumers (who do not change the default horizontal characteristic when they purchase the seller's product). The credible commitment to focus on attracting only close consumers (with a relatively high price) can induce accommodating behavior from the rival seller, thereby leading to increased profit for both sellers.[10] We find that higher default-switching costs tend to increase equilibrium industry profit when the competitive advantage of the strong seller is limited (so this seller secures relatively little profit when it competes to attract all consumers).[11]

Our analysis is most closely related to the literatures on Hotelling competition with endogenous seller locations and with endogenous consumer transportation costs. The former literature (e.g., d'Aspremont et al., 1979; Cremer et al., 1991; Brander and Spencer, 2015; Hinloopen and Martin, 2017; Liu et al., 2020; Ma et al., 2021) focuses on how sellers choose their locations on the Hotelling line (or circle) to enhance the profit they secure in the ensuing price competition. We take each seller's default location to be exogenous. However, we allow each seller to choose the cost its customers must incur to effectively change the seller's perceived location.

The latter literature (e.g., von Ungern-Sternberg, 1988; Ferreira and Thisse, 1996; Hendel and de Figueiredo, 1997; Troncoso-Valverde and Robert, 2004; Hou et al., 2013) allows sellers to alter the unit transportation cost that consumers incur. We take unit transportation costs to be exogenous and immutable. However, each seller can affect the personal cost that its customers must incur to reduce the distance they effectively travel to purchase the seller's product. Higher unit transportation costs and higher default-switching costs can both increase equilibrium profit by effectively enhancing horizontal product differentiation.[12,13] However, these two distinct cost increases affect consumer utility differently. As

et al., 2020; Bounie et al., 2021) examine how the intensity of competition is affected by the availability of information about the willingness to pay of individual consumers in settings where price discrimination is feasible (as it is in Thisse and Vives (1988), for example).

[10] When the strong seller serves all consumers, its equilibrium profit declines as its default-switching cost increases. Therefore, a higher default-switching cost can either increase or reduce a firm's equilibrium profit, so the relationship between a firm's default-switching cost and its equilibrium profit can be non-monotonic.

[11] As higher default-switching costs increase industry profit, they also reduce consumer welfare and total welfare.

[12] Hendel and de Figueiredo (1997) show that a seller's incentive to increase transportation costs can vary with the number of adjacent competitors. Each seller faces only a single adjacent competitor in our model of duopoly competition.

[13] Wilson (2010) identifies conditions under which a firm can enhance its profit by unilaterally increasing the time it takes for consumers to discover the price of the firm's product. The resulting increased search

the unit transportation cost increases, the disutility a consumer incurs when she purchases a seller's product increases linearly with the distance between the consumer's location and the seller's location. In contrast, as a seller's default-switching cost increases, the disutility a consumer incurs when she purchases the seller's product increases by a fixed amount (i.e., the magnitude of the cost increase) if and only if the consumer's location is sufficiently distant from the seller's location. These different effects of increases in the unit transportation cost and increases in default-switching costs affect the type of equilibria that arise and the conditions under which no equilibrium exists.

The ensuing analysis proceeds as follows. Section 2 describes our model. Section 3 characterizes equilibrium outcomes in two benchmark settings: one where default horizontal product characteristics cannot be altered (as in the standard Hotelling model) and one where consumers can change these characteristics costlessly. Section 4 presents our primary findings, characterizing the equilibria that arise in the setting of primary interest where default-switching costs are intermediate in magnitude. This section also identifies conditions under which no equilibrium exists. Section 5 extends the analysis to allow default-switching costs to be endogenous. Section 6 summarizes our key findings and suggests directions for future research. The Appendix provides the proofs of the formal conclusions in the text.

2 The Model

We analyze competition between two sellers of products that embody both horizontal and vertical characteristics. We model consumer preferences for the horizontal characteristics of the sellers' products in standard Hotelling fashion. Specifically, each seller (and its horizontal product characteristic) is located at one end of the unit interval: Firm 1 is located at 0, Firm 2 is located at 1. Potential consumers are distributed uniformly on the unit interval. The total mass of consumers is normalized to 1. If a consumer located at $x \in [0, 1]$ purchases Firm 1's (respectively, Firm 2's) product and does not change the product's default horizontal characteristic, the consumer incurs "transportation" cost $t x$ (respectively, $t [1 - x]$). $t > 0$ is a parameter that reflects the intensity of consumer preferences for the horizontal product characteristic.

We modify the standard Hotelling analysis by allowing for the possibility that a consumer might switch the default horizontal characteristic of the product she purchases to the default characteristic of the other seller's product. For example, as noted in the Introduction, the two

costs for relatively "impatient" consumers induce them to learn the rival's price first and renders them less likely to subsequently learn the firm's price, thereby softening competition for more patient consumers. The softened competition in Wilson's model gives rise to conditions under which, as in our model, a firm can enhance its profit by unilaterally suppressing the demand for its product by a subgroup of consumers.

firms might sell smartphones. The default setting on Firm 1's phone might limit disclosure of any identifying personal information, whereas the default setting on Firm 2's phone might facilitate such disclosure. In a setting like this, a consumer might purchase a smartphone from Firm 1 and change the default setting on the phone to facilitate the disclosure of personal information (to encourage the receipt of more relevant advertisements). $K_i \geq 0$ is the personal cost that a customer must incur to change the default horizontal characteristic of the product she purchases from Firm i to the default horizontal characteristic of Firm j's product ($j \neq i,\ i, j \in \{1, 2\}$). This cost might reflect, for example, the time and effort required to learn how to change the product's default setting, and then implement the change.

The vertical characteristics of the products that Firms 1 and 2 sell also can differ. In the foregoing smartphone example, the vertical characteristics might include the phone's processor speed, memory capacity, and battery life, for instance. G_i is the gross value that each consumer derives from a product supplied by Firm $i \in \{1, 2\}$. The utility that a consumer derives when she purchases Firm i's product is G_i, less any transportation and default-switching costs the consumer incurs, less p_i, which is the price that Firm i charges for its product. Consumers value at most one unit of the product. We assume that G_1 and G_2 are sufficiently large that every potential consumer purchases exactly one unit of the product in equilibrium.

Firm i's constant unit production cost is $c_i > 0$. Without essential loss of generality, we assume that $G_1 - c_1 > G_2 - c_2$. This "competitive advantage" for Firm 1 ensures that Firm 1 always sells its product to some consumers in equilibrium. Two types of equilibria can arise in the ensuing analysis. In a *monopoly equilibrium*, all consumers purchase Firm 1's product. In a *duopoly equilibrium*, some consumers purchase Firm 1's product and other consumers purchase Firm 2's product.[14]

The timing in the model is as follows. After G_i, c_i, K_i ($i \in \{1, 2\}$), and t are determined exogenously,[15] the two suppliers set their prices simultaneously and noncooperatively. Each consumer then decides whether to purchase Firm 1's product or Firm 2's product. Finally, each customer decides whether to retain the default horizontal characteristic of the product she has purchased or switch it to the default horizontal characteristic of the other supplier's product.

Each consumer purchases the product that ensures her the highest utility (gross value, less price, less relevant transportation and default-switching costs). The consumer located

[14] If $G_2 - c_2 \geq G_1 - c_1$, then all consumers might purchase Firm 2's product in equilibrium. We abstract from this possibility for expositional ease.

[15] The analysis in section 5 allows K_1 and K_2 to be endogenous.

at $x \in [0,1]$ purchases Firm 1's product rather than Firm 2's product if and only if:[16]

$$G_1 - p_1 - \min\{tx,\, t[1-x] + K_1\} \;\geq\; G_2 - p_2 - \min\{t[1-x],\, tx + K_2\}. \qquad (1)$$

Inequality (1) reflects the fact that, after purchasing Firm 1's product, the consumer located at x will switch the product's default characteristic if and only if the sum of the default-switching cost (K_1) and the consumer's transportation cost to Firm 2's location is less than the consumer's transportation cost to Firm 1's location, i.e.:[17]

$$K_1 + t[1-x] \;<\; tx \quad \Leftrightarrow \quad x \;>\; \frac{1}{2} + \frac{K_1}{2t}. \qquad (2)$$

Expression (2) implies that when $K_1 < t$, the (only) consumers who switch the default horizontal characteristic when they purchase Firm 1's product are those located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$, i.e., those located furthest from Firm 1 and closest to Firm 2.

Similarly, after purchasing Firm 2's product, the consumer located at x will switch the product's default horizontal characteristic to the default horizontal characteristic of Firm 1's product if and only if:

$$K_2 + tx \;<\; t[1-x] \quad \Leftrightarrow \quad x \;<\; \frac{1}{2} - \frac{K_2}{2t}. \qquad (3)$$

Expression (3) implies that when $K_2 < t$, the (only) consumers who switch the default horizontal characteristic of the product they purchase from Firm 2 are those located in $[0, \frac{1}{2} - \frac{K_2}{2t})$, i.e., those located furthest from Firm 2 and closest to Firm 1. Unless otherwise noted, the ensuing analysis considers settings in which $K_i \in (0, t)$ for $i \in \{1, 2\}$. These settings avoid the relatively uninteresting case in which default-switching costs exceed the transportation cost associated with traversing the entire unit interval. In this case, no consumer would ever change the default horizontal characteristic of the product she purchases.

3 Benchmark Settings

In this section, we briefly characterize equilibrium outcomes in two benchmark settings. The first benchmark reflects the standard Hotelling model in which all product characteristics (including the default horizontal characteristic) are immutable. The second benchmark reflects the other extreme in which each consumer can costlessly switch the default horizontal characteristic of the product she purchases to the default horizontal characteristic of the other seller's product.

The following lemmas refer to π_i, which denotes the profit of Firm $i \in \{1, 2\}$. The

[16] For expositional ease, we assume that when a consumer is indifferent between purchasing Firm 1's product and Firm 2's product, she purchases Firm 1's product.

[17] For expositional ease, we assume that when a consumer is indifferent between retaining and switching the default horizontal characteristic of the product she purchases, she retains the default characteristic.

lemmas also refer to $A \equiv \frac{1}{3}[G_1 - c_1 - (G_2 - c_2)]$, which is a measure of the magnitude of Firm 1's competitive advantage. Lemma 1 characterizes equilibrium outcomes when default product characteristics cannot be changed. Lemma 2 characterizes equilibrium outcomes when each consumer can costlessly switch the default horizontal characteristic of the product she purchases to the default horizontal characteristic of the other seller's product.[18]

Lemma 1. *Suppose $A < t$ and default product characteristics cannot be changed. Then the unique equilibrium is the duopoly equilibrium in which all consumers located in $[0, x_0]$ buy Firm 1's product, whereas all consumers located in $(x_0, 1]$ buy Firm 2's product, where $x_0 \equiv \frac{1}{2} + \frac{A}{2t} \in \left(\frac{1}{2}, 1\right)$. Furthermore, $p_1 = c_1 + t + A$, $p_2 = c_2 + t - A$, $\pi_1 = \frac{1}{2t}[t + A]^2$, and $\pi_2 = \frac{1}{2t}[t - A]^2$.*[19]

Lemma 1 considers the standard setting in which Firm 1's competitive advantage is limited relative to the unit transportation cost (i.e., $A < t$), so both firms attract consumers in equilibrium.[20] When all product characteristics are immutable in this setting, consumers located relatively close to Firm 1 (respectively, Firm 2) purchase Firm 1's (respectively, Firm 2's) product. Due to its competitive advantage, Firm 1 sells more units of its product and secures greater profit than does Firm 2 (i.e., $x_0 > \frac{1}{2}$ and $\pi_1 > \pi_2$). In standard fashion, equilibrium prices increase with own production costs and with transportation costs (i.e., $\frac{\partial p_i}{\partial c_i} > 0$ for $i \in \{1, 2\}$ and $\frac{\partial p_i}{\partial t} > 0$). Firm 1's price increases and Firm 2's price declines as Firm 1's competitive advantage increases (i.e., $\frac{\partial p_1}{\partial A} > 0$ and $\frac{\partial p_2}{\partial A} < 0$).

Lemma 2. *Suppose $K_1 = K_2 = 0$. Then the unique equilibrium is the monopoly equilibrium in which $p_1 = c_1 + 3A$, $p_2 = c_2$, $\pi_1 = 3A$, $\pi_2 = 0$, and (only) consumers located in $(\frac{1}{2}, 1]$ change the default horizontal characteristic of the product they purchase.*

When consumers can costlessly switch the default horizontal characteristic of the product they purchase, consumers who prefer the default characteristic of Firm i's product to the default characteristic of Firm j's product (costlessly) implement the former characteristic on the product they purchase. Consequently, each consumer effectively perceives the two

[18] A putative equilibrium is an equilibrium if neither firm can strictly increase its profit by unilaterally changing the price it sets in the putative equilibrium. We restrict attention to equilibria in which $p_i \geq c_i$ for $i \in \{1, 2\}$. Below-cost pricing is a weakly dominated strategy for a firm in the sense that such pricing cannot increase the firm's equilibrium profit above the level it achieves if it declines to operate, regardless of the price set by the rival firm.

[19] Lemma 1 reflects standard conclusions. See, for example, Belleflamme and Peitz (2015).

[20] If A were to exceed t, then all consumers would purchase Firm 1's product in equilibrium.

products to have the same horizontal product characteristic.[21] The effective absence of horizontal product differentiation leads to intense "winner-take-all" price competition. Firm 1 reduces its price to the highest level that allows Firm 1 to attract all consumers when Firm 2 sets the lowest price at which it could profitably serve all consumers (i.e., $p_2 = c_2$). Inequality (1) implies that this price for Firm 1 is determined by $G_1 - p_1 = G_2 - p_2$. Therefore, $p_1 = G_1 - G_2 + c_2 = c_1 + 3A$, which generates profit $p_1 - c_1 = 3A$ for Firm 1.

4 Outcomes with Exogenous Default-Switching Costs

We now characterize equilibrium outcomes in the setting of primary interest where $K_1 \in (0, t)$ and $K_2 \in (0, t)$, so: (i) it is costly for consumers to change the default horizontal characteristic of the product they purchase; but (ii) each firm's default-switching cost is less than the transportation cost of traversing the entire unit interval. Lemma 3 identifies the duopoly equilibrium that can arise in this setting. Lemma 4 identifies the corresponding monopoly equilibrium. The lemmas refer to the following critical values of default-switching costs for the firms' products.

Definitions. $K_{1a} \equiv \frac{1}{2}[3A - t]$; $K_{1b} \equiv 3A - \frac{1}{8t}[t + 3A]^2$; $\underline{K}_1 = \max\{K_{1a}, K_{1b}\}$;

$$\overline{K}_1 \equiv \frac{1}{2t}[t^2 + 2At - A^2]; \quad \overline{K}_2 \equiv \frac{1}{2t}[t^2 - 2At - A^2].$$ [22]

Lemma 3. *Suppose $A < t$ and $K_2 \geq \overline{K}_2$. Then the duopoly equilibrium identified in Lemma 1 exists if and only if $K_1 \geq \overline{K}_1$. At the unique such equilibrium, no customer changes the horizontal characteristic of the product she purchases.*

Lemma 3 indicates that when its default-switching cost (K_1) is sufficiently pronounced, Firm 1 finds it more profitable to set a relatively high price and attract only "close" consumers than to set the lower price required to attract all consumers (when $p_2 = c_2$). This is the case because it becomes more costly for Firm 1 to attract "distant" consumers as K_1 increases.[23] Because $K_1 < t$, consumers located relatively far from Firm 1 (i.e., relatively close to location 1) switch the default horizontal characteristic if they purchase Firm 1's

[21] For consumers located in $(0, 1)$, neither product embodies the horizontal characteristic that is ideal from the consumer's perspective. However, when it is costless to change default horizontal product characteristics, every consumer perceives the two products to effectively offer the same horizontal product characteristic, which is, of the two default characteristics offered by the sellers, the one the consumer prefers.

[22] $\overline{K}_1 > \underline{K}_1$ when $A < t$. See the proof of Observation A2 in the Appendix.

[23] Firm 1's "close" consumers are those located in $[0, \frac{1}{2} + \frac{K_1}{2t}]$, who will not change the default horizontal characteristic if they purchase Firm 1's product. Firm 1's "distant" consumers are those located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$, who will change the default horizontal characteristic if they purchase Firm 1's product.

product.[24] Consequently, to secure the patronage of such distant consumers, Firm 1 must reduce p_1 to compensate them for the default-switching cost they incur.[25] When K_1 is sufficiently large ($K_1 \geq \overline{K}_1$), Firm 1 finds it more profitable to set a relatively high price and sell its product only to relatively close consumers who do not switch the default horizontal characteristic when they purchase Firm 1's product.[26]

Lemma 3, along with Lemma 1, report that equilibrium profits do not vary as default-switching costs change in the duopoly equilibrium.[27] Because each firm only attracts relatively close consumers in this equilibrium, no consumer switches the default horizontal characteristic of the product she purchases. Consequently, the firms do not have to reduce their prices to ensure the continued patronage of their customers as default-switching costs increase.

Lemma 4. *The monopoly equilibrium exists if and only if $K_1 \leq \underline{K}_1$. At the unique monopoly equilibrium, $p_1 = c_1 + 3A - K_1$, $p_2 = c_2$, $\pi_1 = 3A - K_1 > 0$, and $\pi_2 = 0$. Furthermore, all consumers located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$ (and only these consumers) change the default horizontal characteristic of the product they purchase.*

Lemma 4 reports that when K_1 is sufficiently small ($K_1 \leq \underline{K}_1$), Firm 1 finds it more profitable to set the relatively low price required to attract all consumers (when $p_2 = c_2$) than to set a higher price that would only attract close consumers. Lemma 4 also reports that Firm 1's profit declines as K_1 increases in the identified monopoly equilibrium. As K_1 increases, Firm 1's most distant customers incur the higher default-switching cost. Consequently, Firm 1 must reduce p_1 to secure their patronage. The lower price (charged to all customers) reduces Firm 1's profit.

Proposition 1 explains further when the equilibria identified in Lemmas 3 and 4 prevail. The proposition also identifies conditions under which no equilibrium exists.

Proposition 1. *(i) The duopoly equilibrium identified in Lemma 3 is the unique equilibrium if Firm 1's competitive advantage is sufficiently limited (i.e., $A < t$) and default-switching*

[24] Specifically, as noted above, consumers located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$ switch the default horizontal characteristic if they purchase Firm 1's product.

[25] Because price discrimination is not feasible, Firm 1 must reduce the price it charges to all its customers when it reduces p_1 to compensate distant customers for the higher default-switching cost they incur.

[26] Similarly, when K_2 is sufficiently large ($K_2 > \overline{K}_2$), Firm 2 finds it more profitable (when $p_1 = c_1 + t + A$) to set a relatively high price ($p_2 = c_2 + t - A$) and sell its product only to relatively close consumers than to set a lower price that would induce all consumers to buy Firm 2's product.

[27] The duopoly equilibrium identified in Lemma 3 is the unique duopoly equilibrium. See Lemma A18 in the Appendix.

costs are sufficiently pronounced (i.e., $K_1 \geq \overline{K}_1$ and $K_2 \geq \overline{K}_2$). (ii) The monopoly equilibrium identified in Lemma 4 is the unique equilibrium if Firm 1's default-switching cost is sufficiently small (i.e., $K_1 \leq \underline{K}_1$). (iii) No equilibrium exists if: (a) Firm 1's competitive advantage and default-switching costs are sufficiently pronounced (i.e. $A > t$ and $K_1 > \underline{K}_1$); (b) Firm 1's competitive advantage is sufficiently limited and default-switching costs are intermediate in magnitude (i.e., $A \leq t$ and $K_1 \in \left(\underline{K}_1, \overline{K}_1\right)$); or (c) Firm 2's default-switching cost is sufficiently small and Firm 1's default-switching cost is sufficiently large (i.e., $K_2 < \overline{K}_2$ and $K_1 > \underline{K}_1$).

Conclusion (i) in Proposition 1 reflects two primary considerations. First, when Firm 1's competitive advantage is limited relative to the unit transportation cost, Firm 1 would face intense competition from a relatively strong rival if it sought to attract all consumers. Consequently, Firm 1's profit would be relatively small if it reduced p_1 sufficiently to attract all consumers (when $p_2 = c_2$). Firm 1 secures more profit in this case by setting a relatively high price and attracting only relatively close consumers.

Second, a large default-switching cost limits the profit a firm can secure if it successfully attracts all consumers. This is the case because the firm's most distant customers incur the relevant default-switching cost, so the firm must reduce its price sufficiently to compensate these consumers for incurring this cost. Consequently, when K_1 and K_2 are both sufficiently large, neither firm can increase its profit by reducing its price below the price it sets in the identified duopoly equilibrium to the level required to attract all consumers.

Conclusion (ii) in Proposition 1 reflects the fact that when K_1 is sufficiently small, Firm 1 can attract all consumers (when $p_2 = c_2$) even when it sets a moderately high price. Consequently, Firm 1 finds it more profitable to serve all consumers at a moderately high price than to serve only close consumers at a higher price.

Conclusion (iii)(a) in Proposition 1 reports that neither a duopoly equilibrium nor a monopoly equilibrium exists when Firm 1's competitive advantage (A) and default-switching cost (K_1) are sufficiently pronounced. No duopoly equilibrium exists when $A > t$ because Firm 2's competitive disadvantage is so pronounced that it would have to reduce p_2 below c_2 to attract any consumers if no consumer switched the default horizontal characteristic of the product she purchases (which is the case in a duopoly equilibrium).

No monopoly equilibrium exists when $K_1 \geq \overline{K}_1$ because when Firm 2 sets $p_2 = c_2$, Firm 1 secures more profit when it sets p_1 to attract most (but not all) consumers than when it sets p_1 to attract all consumers. To secure the patronage of all consumers, Firm 1 would have to set p_1 to ensure that even the most distant consumers – who incur default-switching cost $K_1 \geq \overline{K}_1$ – prefer to purchase Firm 1's product than to purchase Firm 2's product. When Firm 1's competitive advantage is sufficiently pronounced, Firm 1 can instead attract most consumers even when it sets a relatively high price if no customer incurs the default-switching cost. Consequently, when $p_2 = c_2$, Firm 1 maximizes its profit by setting p_1 at a relatively high level that attracts most consumers (none of whom change the default horizontal characteristic of Firm 1's product) but effectively cedes the most distant consumers to Firm 2.

Conclusion (iii)(b) in Proposition 1 reports that even when A is more limited, Firm 1 continues to prefer the duopoly outcome to the most profitable monopoly outcome (when $p_2 = c_2$) if $K_1 \geq \underline{K}_1$. Therefore, no monopoly equilibrium exists in this case. Furthermore, no duopoly equilibrium exists when $K_1 < \overline{K}_1$ because Firm 1 prefers to lower p_1 sufficiently to secure the patronage of all consumers when its default-switching cost is sufficiently small.

Conclusion (iii)(c) in Proposition 1 reports that no equilibrium exists when Firm 2's default-switching cost is sufficiently small and Firm 1's default-switching cost is sufficiently large (i.e., $K_2 < \overline{K}_2$ and $K_1 > \underline{K}_1$). No duopoly equilibrium exists in this case because Firm 2's relatively low default-switching cost ensures that (when $p_1 = c_1 + t + A$) Firm 2 can increase its profit above the level it secures in the putative duopoly equilibrium by unilaterally lowering p_2 to the level that attracts all consumers.[28] No monopoly equilibrium exists in this case because the relatively large default-switching cost (K_1) that Firm 1's distant customers incur require the firm to set a relatively low price to attract all consumers. Firm 1 finds it more profitable (when $p_2 = c_2$) to set a higher price and attract only relatively close consumers (who do not incur K_1 because they do not change the default horizontal characteristic when they purchase Firm 1's product).

It remains to determine how default-switching costs affect industry profit and consumer welfare. In principle, higher default-switching costs might either increase or reduce a firm's profit. A higher default-switching cost might reduce a firm's profit by reducing the attraction of the firm's product to distant consumers, thereby reducing their willingness to pay for the product. In contrast, a higher default-switching cost might enhance a firm's profit by acting as a credible commitment to compete less vigorously for distant consumers (because they become more costly to attract as the firm's default-switching cost increases). Such

[28]Recall that when the consumer located at 0 purchases Firm 2's product, she incurs default-switching cost K_2 and thereby avoids all transportation costs. Therefore, Firm 2's profit does not vary with t when Firm 2 sets p_2 to attract all consumers.

a commitment might induce corresponding accommodating behavior from the rival seller, thereby increasing a firm's equilibrium profit.

Proposition 2 (below) identifies conditions under which this latter consideration prevails, so both firms secure greater equilibrium profit in the presence of substantial default-switching costs than in their absence. Proposition 2 also reports that the increase in profit is outweighed by the associated reduction in consumer welfare, so total welfare declines. The proof of Proposition 2 employs the expressions for consumer welfare presented in Lemma 5. As is apparent from these expressions, consumer welfare is the aggregate utility of all consumers. Total welfare is the sum of consumer welfare and industry profit.

Lemma 5. *Consumer welfare in the duopoly equilibrium identified in Lemma 3 is:*[29]

$$W^{Cd} \equiv \int_0^{x_0} [\, G_1 - p_1 - t\,x \,]\, dx + \int_{x_0}^{1} [\, G_2 - p_2 - t\,(1 - x\,) \,]\, dx \;=\; G_2 - c_2 - \frac{5\,t}{4} + \frac{3\,A}{2} + \frac{A^2}{4\,t}\,.$$

Consumer welfare in the monopoly equilibrium identified in Lemma 2 is:[30,31]

$$W_0^{Cm} \equiv \int_0^{\frac{1}{2}} [\, G_1 - p_1 - t\,x \,]\, dx + \int_{\frac{1}{2}}^{1} [\, G_1 - p_1 - t\;(1 - x\,) \,]\, dx \;=\; G_2 - c_2 - \frac{t}{4}\,.$$

Proposition 2. *Suppose $A < \frac{t}{2+\sqrt{3}}$, $K_1 \geq \overline{K}_1$, and $K_2 \geq \overline{K}_2$.*[32] *Then Firm 1 and Firm 2 both secure more profit in the duopoly equilibrium identified in Lemma 3 (where $K_1 > 0$ and $K_2 > 0$) than in the monopoly equilibrium identified in Lemma 2 (where $K_1 = K_2 = 0$). Consumer welfare and total welfare are both lower in the duopoly equilibrium than in the monopoly equilibrium.*

[29]The expression for W^{Cd} reflects the fact that no consumer changes the default horizontal characteristic of the product she purchases in the duopoly equilibrium identified in Lemma 3. Recall from Lemma 1 that $x_0 = \frac{1}{2} + \frac{A}{2\,t}$

[30]The expression for W_0^{Cm} reflects the fact that (only) consumers located in $(\frac{1}{2}, 1]$ change the default horizontal characteristic of the product they purchase in the monopoly equilibrium identified in Lemma 2. Default-switching costs do not appear in the expression for W_0^{Cm} because $K_1 = K_2 = 0$ in this equilibrium.

[31]Consumer welfare in the monopoly equilibrium identified in Lemma 4 is $\int_0^{\frac{1}{2}+\frac{K_1}{2\,t}} [\, G_1 - p_1 - t\,x \,]\, dx + \int_{\frac{1}{2}+\frac{K_1}{2\,t}}^{1} [\, G_1 - p_1 - t\;(1 - x\,) - K_1 \,]\, dx = G_2 - c_2 - \frac{t}{4} + K_1 \left[\, \frac{1}{2} + \frac{K_1}{4\,t} \,\right]$. Observe that consumer welfare increases as K_1 increases in this equilibrium. The increase in consumer welfare reflects the price reduction Firm 1 must implement (for all customers) to offset the higher default-switching cost that customers located in $(\, \frac{1}{2} + \frac{K_1}{2\,t}, 1\,]$ incur in the monopoly equilibrium.

[32]These three conditions all hold simultaneously if $K_1 > A\left[\frac{5+3\sqrt{3}}{2+\sqrt{3}}\right]$, $K_2 > A\left[\frac{1+\sqrt{3}}{2+\sqrt{3}}\right]$, and $t \in \left(t_H, \min\{\,\bar{t}_1, \bar{t}_2\,\}\right]$, where $t_H = A\left[\,2+\sqrt{3}\,\right]$, $\bar{t}_1 \equiv K_1 - A + \sqrt{[\,K_1 - A\,]^2 + A^2}$ and $\bar{t}_2 \equiv A + K_2 + \sqrt{(A + K_2)^2 + A^2}$. See the proof of Proposition 2 in Chakravorty and Sappington (2025).

Proposition 2 reports that when Firm 1's competitive advantage is sufficiently limited (i.e., when $A < \frac{t}{2+\sqrt{3}}$), higher default-switching costs can increase the equilibrium profit of both firms. Firm 1's relatively limited competitive advantage ensures that Firm 1 secures relatively little profit when it reduces p_1 to the level that induces all consumers to buy Firm 1's product (when $p_2 = c_2$). Firm 1 (and Firm 2) can secure greater profit when relatively high default-switching costs lead both firms to focus more on attracting close consumers (who do not change the default horizontal characteristic of the product they purchase) and less on competing to attract all consumers.

Proposition 2 does not imply that a firm's equilibrium profit always increases as its default-switching cost increases. To the contrary, a firm's equilibrium profit can decline as its default-switching cost increases. This is the case, for instance, when K_1 is sufficiently small that the prevailing equilibrium is the monopoly equilibrium in which all consumers purchase the product from Firm 1. Firm 1's profit $(3\,A - K_1)$ declines as K_1 increases in this equilibrium (because Firm 1 must reduce the price it charges (to all customers) to compensate distant customers for the higher default-switching cost they incur). It is only when K_1 becomes so large that the duopoly equilibrium identified in Lemma 3 prevails (and when $K_2 \geq \overline{K}_2$) that Firm 1 can secure a higher profit than it secures in the monopoly equilibrium that prevails when $K_1 = 0$. These observations imply that Firm 1's profit can vary non-monotonically with K_1.

5 Outcomes with Endogenous Default-Switching Costs

Before concluding, we briefly extend the foregoing analysis to account for the possibility that default-switching costs might be endogenous. In practice, through product design or through the clarity of the instructions it provides, a seller might be able to facilitate (or hinder) its customers' efforts to switch the default horizontal characteristics of the firm's product. To account for this possibility, we now examine equilibrium outcomes in the *setting with endogenous* K, where the interaction between Firm 1 and Firm 2 proceeds in two stages. In the first stage, Firm 1 chooses K_1 and Firm 2 chooses K_2, simultaneously and noncooperatively, anticipating the prices that will be set in the second stage. In the second stage, Firm 1 chooses p_1 and Firm 2 chooses p_2, simultaneously and noncooperatively, taking as given the default-switching costs that were implemented in the first stage.[33] For simplicity, we abstract from any costs the firms might incur to affect the personal costs that consumers

[33]Formally, we consider subgame perfect equilibria (e.g., Selten, 1965; Fudenberg and Tirole, 1991, chapter 3).

must incur to change default horizontal product characteristics.

Proposition 3 presents our main finding in this setting.

Proposition 3. *In the setting with endogenous K: (i) If $A > \frac{t}{2+\sqrt{3}}$, then the only equilibria are monopoly equilibria in which $K_1 = 0$ and all consumers buy Firm 1's product. (ii) If $A < \frac{t}{2+\sqrt{3}}$, then duopoly equilibria in which $K_1 \geq \overline{K}_1$ and $K_2 \geq \overline{K}_2$ exist, as can monopoly equilibria in which $K_1 = 0$, $K_2 \in [0, \overline{K}_2)$, and all consumers buy Firm 1's product.*

Proposition 3 reports that when Firm 1's competitive advantage is sufficiently pronounced, the only equilibria that arise in the setting with endogenous K are monopoly equilibria in which $K_1 = 0$ and all consumers buy Firm 1's product. When A is sufficiently large ($A > \frac{t}{2+\sqrt{3}}$), Firm 1 anticipates greater profit when it eliminates its default-switching cost and competes to attract all consumers than when it sets a high default-switching cost that would effectively commit the firm to compete only for close consumers. This is the case regardless of the magnitude of Firm 2's default switching cost (K_2). Firm 1 always reduces K_1 to 0 when it anticipates competing to serve all consumers because the firm's equilibrium profit ($3A - K_1$) declines as K_1 increases.[34]

When its competitive advantage is limited ($A < \frac{t}{2+\sqrt{3}}$), Firm 1 recognizes that it would secure relatively little profit if it set a low default-switching cost and competed to attract all consumers. Consequently, if Firm 2 implements a relatively high default-switching cost ($K_2 > \overline{K}_2$), Firm 1 will also set a relatively high default-switching cost ($K_1 \geq \overline{K}_1$), thereby effectively committing itself to focus on attracting only relatively close consumers who will not change the default horizontal characteristic when they purchase Firm 1's product. In the presence of Firm 1's high default-switching cost ($K_1 \geq \overline{K}_1$), Firm 2 finds it most profitable to set a high default-switching cost ($K_2 \geq \overline{K}_2$), which ensures that a duopoly equilibrium ensues (in which Firm 2 secures strictly positive profit).

However, if Firm 2 sets a low default-switching cost ($K_2 < \overline{K}_2$), Firm 1 recognizes that the duopoly equilibrium identified in Lemma 3 will not ensue,[35] even if it sets K_1 above $\overline{K}_1$. Consequently, Firm 1 finds it most profitable to set $K_1 = 0$, anticipating that it will subsequently set $p_1 = c_1 + 3A$ to secure the patronage of all consumers (when Firm 2 sets $p_2 = c_2$). When Firm 1 sets $K_1 = 0$, Firm 2 has no strict incentive to set K_2 at or above $\overline{K}_2$ because Firm 2 recognizes that only monopoly equilibria arise when $K_1 = 0$.

[34] Recall that when they purchase Firm 1's product, consumers located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$ switch the product's default horizontal characteristic. Consequently, the larger is K_1, the more p_1 must be reduced to attract these distant consumers (when $p_2 = c_2$).

[35] Recall that this equilibrium is the unique duopoly equilibrium. See Lemma A18 in the Appendix.

Firm 1 and Firm 2 both secure less profit in the monopoly equilibrium than in the duopoly equilibrium when $A < \frac{t}{2+\sqrt{3}}$. Nevertheless, the monopoly equilibrium (with $K_1 = 0$) can arise if Firm 1 and Firm 2 cannot coordinate their choices of default-switching costs or otherwise ensure that both firms set relatively high default-switching costs.

6 Conclusions and Extensions

We have analyzed a streamlined model of Hotelling competition between two sellers of differentiated products. Our model differs from the standard model of Hotelling competition by allowing consumers to alter the default horizontal characteristic of the product they purchase. When consumers can costlessly change this characteristic, horizontal product differentiation is effectively eliminated. Relatively intense competition can ensue, giving rise to a monopoly equilibrium in which all consumers purchase the product from the strongest seller (Firm 1).

Non-trivial default-switching costs restore meaningful horizontal product differentiation. Although such costs can reduce a product's appeal, they can serve as a credible commitment to compete vigorously only for relatively close consumers. Such commitment can increase industry profit, especially when the strong seller's competitive advantage is limited. The diminished intensity of industry competition harms consumers and reduces total welfare.[36]

Our findings identify two distinct ways in which actions that reduce prevailing default-switching costs can enhance consumer welfare. First, the cost reductions can enable consumers to secure preferred product characteristics at lower personal cost. Second, the cost reductions can increase the intensity of industry competition by encouraging industry suppliers to compete to serve a broad spectrum of consumers, rather than focusing on attracting a relatively small number of consumers who value the firm's idiosyncratic product characteristics particularly highly.

Our model was intentionally streamlined to facilitate both a tractable analysis and a focus on the new considerations that arise when consumers can change the default horizontal characteristics of the products they purchase. Future research might consider several extensions of our model. For example, additional dimensions of consumer heterogeneity might be admitted. When consumers have different incomes or different innate valuations of vertical product characteristics, duopoly equilibria may arise even in the absence of default-switching costs. Duopoly equilibria may also be relatively likely to arise in the presence of additional dimensions of horizontal product differentiation.

Future research might also consider endogenous default horizontal product characteris-

[36]Thus, consumer welfare can be higher under monopoly than under duopoly.

tics, default-switching costs that vary across consumers, supplier costs of reducing customer default-switching costs, and repeated interactions among sellers and consumers.[37,38] These model extensions can help to assess the robustness of our findings, but seem unlikely to fundamentally alter our primary qualitative conclusions.

[37] Gehrig and Stenbacka (2004) and Rhodes (2014), among others, analyze repeated interactions between consumers and sellers in models of Hotelling competition.

[38] Future research might also consider settings where consumers can implement a wide range of horizontal product characteristics, not simply the two distinct default characteristics imbedded in the sellers' products.

Appendix

This Appendix outlines the proofs of the formal conclusions in the text. Chakravorty and Sappington (2025) provides more detailed proofs, including the proofs of Lemmas A1 – A17, which are employed to prove the formal conclusions in the text.

Lemma A1. If a consumer buys Firm 2's product, she will change its default horizontal characteristic if and only if she is located in $[\,0, \frac{1}{2} - \frac{K_2}{2t}\,)$.
Lemma A2. If a consumer buys Firm 1's product, she will change its default horizontal characteristic if and only if she is located in $(\,\frac{1}{2} + \frac{K_1}{2t}, 1\,]$.

Lemma A3. A consumer located in $\left[\,\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2} + \frac{K_1}{2t}\,\right]$ will not change the default horizontal characteristic of the product she purchases.

Lemma A4. Suppose a consumer located at $x_0 \in [\,\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2} + \frac{K_1}{2t}\,]$ is indifferent between buying Firm 1's product and buying Firm 2's product. Then: (i) all consumers located in $[0, x_0\,]$ will buy Firm 1's product; and (ii) all consumers located in $(\,x_0, 1\,]$ will buy Firm 2's product.

Lemma A5. Suppose a consumer located at $x_1 \in [\,0, \frac{1}{2} - \frac{K_2}{2t}\,)$ is indifferent between buying Firm 1's product and buying Firm 2's product. Then: (i) all consumers located in $[\,0, \frac{1}{2} - \frac{K_2}{2t}\,]$ are similarly indifferent; and (ii) all consumers located in $(\,\frac{1}{2} - \frac{K_2}{2t}, 1\,]$ will buy Firm 2's product.

Lemma A6. Suppose a consumer located at $x_2 \in (\,\frac{1}{2} + \frac{K_1}{2t}, 1\,]$ is indifferent between buying Firm 1's product and buying Firm 2's product. Then: (i) all consumers located in $[\,\frac{1}{2} + \frac{K_1}{2t}, 1\,]$ are similarly indifferent; and (ii) all consumers located in $[\,0, \frac{1}{2} + \frac{K_1}{2t}\,]$ will buy Firm 1's product.

Lemma A7. If $p_1 \geq p_2 + G_1 - G_2 + K_2$, then all consumers located in $[\,0, \frac{1}{2} - \frac{K_2}{2t}\,)$ (weakly) prefer to buy Firm 2's product than to buy Firm 1's product. The preference is strict if the inequality holds strictly.

Lemma A8. If $p_1 \geq p_2 + G_1 - G_2 + K_2$, then all consumers located in $[\,\frac{1}{2} - \frac{K_2}{2t}, 1\,]$ (weakly) prefer to buy Firm 2's product than to buy Firm 1's product. The preference is strict if the inequality holds strictly.

Lemma A9. If $p_2 \geq p_1 + G_2 - G_1 + K_1$, then all consumers located in $(\frac{1}{2} + \frac{K_1}{2t}, 1\,]$ (weakly) prefer to buy Firm 1's product than to buy Firm 2's product. The preference is strict if the inequality holds strictly.

Lemma A10. If $p_2 \geq p_1 + G_2 - G_1 + K_1$, then all consumers located in $[\,0, \frac{1}{2} + \frac{K_1}{2t}\,]$ (weakly) prefer to buy Firm 1's product than to buy Firm 2's product. The preference is strict if the inequality holds strictly.

Assumption 1. $K_1 \in [\,0,\,t\,)$, $K_2 \in [\,0,\,t\,)$, and $(K_1\,,\,K_2\,) \neq (0\,,\,0\,)$.

Lemma A11. Suppose Assumption 1 holds. Then in any equilibrium in which $p_1 - p_2 \in (G_1 - G_2 - K_1, G_1 - G_2 + K_2)$: (i) all consumers located in $[0, \frac{1}{2} - \frac{K_2}{2t}\,)$ strictly prefer to buy Firm 1's product than to buy Firm 2's product; (ii) all consumers located in $(\,\frac{1}{2} + \frac{K_1}{2t}, 1\,]$ strictly prefer to buy Firm 2's product than to buy Firm 1's product; and (iii) some consumer located in $[\,\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2} + \frac{K_1}{2t}\,]$ is indifferent between buying Firm 1's product and buying Firm 2's product.

Lemma A12. When Assumption 1 holds: (i) c_1 is the lowest price that Firm 1 can profitably charge when all consumers buy its product; and (ii) c_2 is the lowest price that Firm 2 can profitably charge when all consumers buy its product.

Lemma A13. Suppose Assumption 1 holds. Then an equilibrium does not exist in which $p_1 > p_2 + G_1 - G_2 + K_2$.

Lemma A14. Suppose Assumption 1 holds. Then an equilibrium does not exist in which $p_1 = p_2 + G_1 - G_2 + K_2$.

Lemma A15. Suppose Assumption 1 holds. Then an equilibrium does not exist in which $p_2 \geq p_1 + G_2 - G_1 + K_1$ and $p_2 \neq c_2$.

Lemma A16. Suppose Assumption 1 holds. Then an equilibrium does not exist in which $p_2 > p_1 + G_2 - G_1 + K_1$ and $p_2 = c_2$.

Lemma A17. Suppose Assumption 1 holds. Then in any duopoly equilibrium, there is a consumer located in $(\,0, 1\,)$ who is indifferent between buying Firm 1's product and buying Firm 2's product.

<u>Proof of Lemma 1</u>. The proof follows from Lemmas A1.1 – A1.5 (below).[39]

Lemma A1.1. When horizontal product characteristics cannot be changed: (i) all consumers buy Firm 1's product if $p_2 - p_1 \geq G_2 - G_1 + t\,$; and (ii) all consumers buy Firm 2's product if $p_2 - p_1 < G_2 - G_1 - t\,$.

[39] Lemmas A1.1 – A1.5 are proved in Chakravorty and Sappington (2025). The details are not presented here because they are straightforward and because Lemma 1 reflects standard conclusions.

Lemma A1.2. When horizontal product characteristics cannot be changed and $t > 3A$, no equilibrium exists in which all consumers buy the product from the same firm.

Lemma A1.3. When horizontal product characteristics cannot be changed and $p_2 - p_1 \in [G_2 - G_1 - t,\, G_2 - G_1 + t]$: (i) a consumer located at $x_0 \equiv \frac{1}{2t}[t + G_1 - G_2 + p_2 - p_1] \in [0,1]$ is indifferent between buying Firm 1's product and buying Firm 2's product; (ii) if $x_0 > 0$, all consumers located in $[0, x_0]$ buy Firm 1's product; and (iii) if $x_0 < 1$, all consumers located in $(x_0, 1]$ buy Firm 2's product.

Lemma A1.4. Suppose horizontal product characteristics cannot be changed and $t > 3A$. Then in equilibrium, there exists a $x_0 \in [0,1]$ such that: (i) a consumer located at x_0 is indifferent between buying Firm 1's product and buying Firm 2's product; (ii) all consumers located in $[0, x_0]$ buy Firm 1's product; and (iii) all consumers located in $(x_0, 1]$ buy Firm 2's product. Furthermore: $p_1 = c_1 + t + A$; $p_2 = c_2 + t - A$; $\pi_1 = \frac{1}{2t}[t + A]^2$; and $\pi_2 = \frac{1}{2t}[t - A]^2$.

Lemma A1.5. Suppose horizontal product characteristics cannot be changed and $t \in (A, 3A]$. Then in the unique equilibrium, both firms attract customers, Firm 1's profit is $\pi_1 = \frac{1}{2t}[t + A]^2 > 0$, and Firm 2's profit is $\pi_2 = \frac{1}{2t}[t - A]^2 > 0$. ■

<u>**Proof of Lemma 2**</u>. The proof follows directly from Lemmas A2.1 – A2.3 (below).

Lemma A2.1. Suppose $K_1 = K_2 = 0$. Then: (i) a consumer located in $[0, \frac{1}{2})$ will change the default horizontal characteristic of the product she purchases if and only if she purchases Firm 2's product; (ii) a consumer located in $(\frac{1}{2}, 1]$ will change the default horizontal characteristic of the product she purchases if and only if she purchases Firm 1's product; and (iii) a consumer located at $\frac{1}{2}$ will not change the default horizontal characteristic of the product she purchases.

Lemma A2.2. Suppose $K_1 = K_2 = 0$. Then: (i) all consumers buy Firm 1's product if $p_2 > p_1 + G_2 - G_1$; (ii) all consumers buy Firm 2's product if $p_2 < p_1 + G_2 - G_1$; and (iii) all consumers are indifferent between buying Firm 1's product and buying Firm 2's product if $p_2 = p_1 + G_2 - G_1$.

Lemma A2.3. Suppose $K_1 = K_2 = 0$. Then in equilibrium: (i) all consumers buy Firm 1's product at price $p_1 = c_2 + G_1 - G_2$; (ii) Firm 2's profit is 0; and (iii) Firm 1's profit is $3A = G_1 - c_1 - (G_2 - c_2)$.

Lemma A2.1 follows directly from the proofs of Lemmas A1 – A3. Lemma A2.2 follows from Lemma A2.1. Lemma A2.3 follows from Lemmas A2.1 and A2.2.[40] ■

Proof of Lemma 3. Lemma A17 implies that at a duopoly equilibrium, there is a consumer located at x_0 who is indifferent between buying Firm 1's product and Firm 2's product, where:

$$G_1 - t\,x_0 - p_1 \;=\; G_2 - t\,[\,1 - x_0\,] - p_2 \quad\Leftrightarrow\quad x_0 \;=\; \frac{1}{2\,t}\,[\,t + G_1 - G_2 + p_2 - p_1\,]\,. \tag{4}$$

(4), along with Lemmas A4 and A11, imply that in the equilibrium identified in Lemma 1, the profits of Firm 1 and Firm 2 are, respectively:

$$\pi_1 \;=\; [\,p_1 - c_1\,]\,\frac{1}{2\,t}\,[\,t + G_1 - G_2 + p_2 - p_1\,]\,; \tag{5}$$

$$\pi_2 \;=\; [\,p_2 - c_2\,]\,\frac{1}{2\,t}\,[\,t + G_2 - G_1 + p_1 - p_2\,]\,. \tag{6}$$

The unique value of p_1 that maximizes π_1 in (5) is given by:

$$\frac{\partial \pi_1}{\partial p_1} \;=\; 0 \quad\Leftrightarrow\quad p_1 \;=\; \frac{1}{2}\,[\,t + c_1 + G_1 - G_2 + p_2\,]\,. \tag{7}$$

The unique value of p_2 that maximizes π_2 in (6) is given by:

$$\frac{\partial \pi_2}{\partial p_2} \;=\; 0 \quad\Leftrightarrow\quad p_2 \;=\; \frac{1}{2}\,[\,t + c_2 + G_2 - G_1 + p_1\,]\,. \tag{8}$$

(7) and (8) imply that in equilibrium:

$$p_1 \;=\; c_1 + t + A \quad\text{and}\quad p_2 \;=\; c_2 + t - A\,. \tag{9}$$

(4) and (9) imply that the consumer who is indifferent between buying Firm 1's product and buying Firm 2's product is located at:

$$x_0 \;=\; \frac{1}{2\,t}\left[\,t + G_1 - G_2 + \frac{1}{3}\,(\,c_2 - c_1 + 2\,G_2 - 2\,G_1\,)\right] \;=\; \frac{1}{2} + \frac{A}{2\,t}\,. \tag{10}$$

(10) implies that $x_0 \in (\,\frac{1}{2} - \frac{K_2}{2\,t}, \frac{1}{2} + \frac{K_1}{2\,t}\,)$ when $K_1 \,\geq\, \overline{K}_1$. Therefore, no customer changes the default horizontal characteristic of the product she purchases (from Lemma A3).

(9) implies:

$$p_1 - p_2 \;\in\; (\,G_1 - G_2 - K_1,\, G_1 - G_2 + K_2\,)\,. \tag{11}$$

(9) and (10) imply:

$$\pi_1 \;=\; [\,p_1 - c_1\,]\,x_0 \;=\; [\,t + A\,]\left[\frac{t + A}{2\,t}\right] \;=\; \frac{1}{2\,t}\,[\,t + A\,]^2 \;>\; 0 \quad\text{and} \tag{12}$$

$$\pi_2 \;=\; [\,p_2 - c_2\,]\,[\,1 - x_0\,] \;=\; [\,t - A\,]\left[\frac{t - A}{2\,t}\right] \;=\; \frac{1}{2\,t}\,[\,t - A\,]^2 \;>\; 0\,. \tag{13}$$

The foregoing analysis and Lemma A11 imply that the identified putative equilibrium is unique among equilibria in which (11) holds. It remains to verify that when $K_1 \,\geq\, \overline{K}_1$,

[40] See Chakravorty and Sappington (2025) for details.

neither firm can strictly increase its profit by unilaterally changing its price so that (11) does not hold.[41] We first show this is the case for Firm 1.

Lemmas A7 and A8 imply that if Firm 1 sets $p_1 > p_2 + G_1 - G_2 + K_2$, then no consumers purchase Firm 1's product. Therefore, Firm 1's profit (0) is less than the profit specified in (12). If Firm 1 sets $p_1 = p_2 + G_1 - G_2 + K_2$ when p_2 is as specified in (9), then:

$$p_1 \; = \; p_2 + G_1 - G_2 + K_2 \; = \; c_2 + t - A + G_1 - G_2 + K_2 \; = \; c_1 + 2\,A + t + K_2$$

$$\Rightarrow \; p_1 - c_1 \; = \; 2\,A + t + K_2 \; > \; 0\,. \tag{14}$$

When $p_1 = p_2 + G_1 - G_2 + K_2$, Lemmas A7 and A8 imply that all consumers located in $[\,0, \frac{1}{2} - \frac{K_2}{2\,t}\,]$ buy Firm 1's product whereas all consumers located in $(\,\frac{1}{2} - \frac{K_2}{2\,t}, 1\,]$ buy Firm 2's product. Therefore, (14) implies that Firm 1's profit is:

$$\pi_{1D} \; = \; [\,p_1 - c_1\,] \left[\frac{1}{2} - \frac{K_2}{2\,t} \right] \; = \; [\,2\,A + t + K_2\,] \left[\frac{1}{2} - \frac{K_2}{2\,t} \right]. \tag{15}$$

(12) and (15) imply that Firm 1 cannot increase its profit by setting $p_1 = p_2 + G_1 - G_2 - K_1$ when p_2 is as specified in (9) because:

$$\pi_1 \; > \; \pi_{1D} \;\; \Leftrightarrow \;\; \frac{1}{2\,t}\,[\,t + A\,]^2 \; > \; [\,2\,A + t\, + K_2\,] \left[\frac{1}{2} - \frac{K_2}{2\,t} \right] \;\; \Leftrightarrow \;\; A^2 \; > \; -\,2\,A\,K_2 - (K_2)^2\,.$$

The last inequality here always holds because $A > 0$ and $K_2 \geq 0$.

Lemmas A9 and A10 imply that if Firm 1 sets $p_1 \leq p_2 + G_1 - G_2 - K_1$, then all consumers purchase Firm 1's product. Therefore, the maximum profit Firm 1 can secure by setting such a price when p_2 is as specified in (9) is:

$$\pi_{1D}^{'} \; = \; p_2 + G_1 - G_2 - K_1 - c_1 \; = \; 3\,A - A + t - K_1 \; = \; 2\,A + t - K_1\,. \tag{16}$$

(12) and (16) imply that Firm 1 cannot increase its profit by setting $p_1 \leq p_2 + G_1 - G_2 - K_1$ when p_2 is as specified in (9) when the maintained conditions hold because:

$$\pi_1 \; \geq \; \pi_{1D}^{'} \;\; \Leftrightarrow \;\; \frac{1}{2\,t}\,[\,t + A\,]^2 \; \geq \; 2\,A + t\, - K_1 \Leftrightarrow K_1 \; \geq \; \frac{1}{2\,t}\left[\,t^2 + 2\,A\,t - A^2\,\right] \; \equiv \; \overline{K}_1\,. \tag{17}$$

Now we show that Firm 2 cannot increase its profit by unilaterally changing its price so that (11) does not hold when p_1 is as specified in (9).

Lemmas A9 and A10 imply that if Firm 2 sets $p_2 > p_1 + G_2 - G_1 + K_1$, then no consumers purchase Firm 2's product, so Firm 2's profit (0) is no greater than the profit specified in (13).

If Firm 2 sets $p_2 = p_1 + G_2 - G_1 + K_1$ when p_1 is as specified in (9), then:

$$p_2 \; = \; c_1 + t + A + G_2 - G_1 + K_1 \; = \; -\,2\,A + t + K_1 + c_2 \; > \; c_2\,.$$

The last inequality holds here because $K_1 > A$ and because $t > A$, by assumption. Because

[41] Recall the maintained assumption that a putative equilibrium is an equilibrium if neither firm can strictly increase its profit by deviating unilaterally from the putative equilibrium.

$p_2 = p_1 + G_2 - G_1 + K_1 > c_2$ when p_1 is as specified in (9), the proof of Lemma A15[42] implies that Firm 2 can increase its profit by setting p_2 to ensure $p_1 - p_2 \in (G_1 - G_2 - K_1, G_1 - G_2 + K_2)$. Therefore, Firm 2 cannot increase its profit by setting $p_2 = p_1 + G_2 - G_1 + K_1$ when p_1 is as specified in (9).

Lemmas A7 and A8 imply that if Firm 2 sets $p_2 < p_1 + G_2 - G_1 - K_2$, then all consumers purchase Firm 2's product. Therefore, the maximum profit Firm 2 can secure by setting such a price when p_1 is as specified in (9) is nearly:

$$\pi_{2D} = p_1 + G_2 - G_1 - K_2 - c_2 = -2A + t - K_2 . \tag{18}$$

(13) and (18) imply that Firm 2 cannot increase its profit by setting $p_2 < p_1 + G_2 - G_1 - K_2$ when p_1 is as specified in (9) and the maintained conditions hold because:

$$\pi_2 \geq \pi_{2D} \Leftrightarrow \frac{1}{2t}[t - A]^2 \geq -2A + t - K_2 \Leftrightarrow K_2 \geq \frac{1}{2t}\left[t^2 - 2At - A^2\right] \equiv \overline{K}_2 . \tag{19}$$

If Firm 2 sets $p_2 = p_1 + G_2 - G_1 - K_2$ when p_1 is as specified in (9), Lemma A5 implies that all consumers located in $[0, \frac{1}{2} - \frac{K_2}{2t}]$ buy Firm 1's product, whereas all consumers located in $(\frac{1}{2} - \frac{K_2}{2t}, 1]$ buy Firm 2's product. Therefore, Firm 2's profit is:

$$\pi_{2D}^{'} = [p_2 - c_2]\left[\frac{1}{2} + \frac{K_2}{2t}\right] < p_2 - c_2 = \pi_{2D} < \frac{1}{2t}[t - A]^2 = \pi_2 . \tag{20}$$

The first inequality in (20) holds because $K_2 < t$ by assumption and because $p_2 - c_2$ must be strictly positive if Firm 2 is to secure positive profit in this case. The last inequality in (20) reflects (19). (13) and (20) imply that Firm 2 will not set $p_2 = p_1 + G_2 - G_1 - K_2$ when p_1 is as specified in (9).

It remains to show that the putative equilibrium identified above does not exist when $K_1 < \overline{K}_1 \equiv \frac{1}{2t}[t^2 + 2At - A^2]$. (12) establishes that Firm 1's profit is $\frac{1}{2t}[t + A]^2$ at this putative equilibrium. (16) implies that if Firm 1 reduces its price to ensure that all consumers purchase its product, it can secure profit $2A + t - K_1$. (17) implies:

$$2A + t - K_1 > \frac{1}{2t}[t + A]^2 \text{ if } K_1 < \overline{K}_1 .$$

Therefore, the putative equilibrium identified above is not an equilibrium when $K_1 < \overline{K}_1$. ■

Observation A1.[43]

$$\underline{K}_1 = \begin{cases} K_{1a} & \text{if } t < A \\ K_{1b} & \text{if } t \geq A . \end{cases}$$

<u>Proof of Lemma 4</u>. <u>Case (i)</u>. $t < A$ and $K_1 \leq \underline{K}_1$. Observation A1 implies that $\underline{K}_1 = K_{1a} \equiv \frac{1}{2}[3A - t]$ because $t < A$ in this case.

We first show that when $p_2 = c_2$ in this case, Firm 1 maximizes its profit by setting $p_1 = c_1 + 3A - K_1$, which ensures that all consumers buy its product.

[42] See Chakravorty and Sappington (2025).

[43] The proof of Observation A1 is straightforward. See Chakravorty and Sappington (2025) for details.

Lemmas A9 and A10 imply that if $p_2 = c_2$, then among all values of p_1 that ensure all consumers buy Firm 1's product (i.e., among all $p_1 \leq p_2 + G_1 - G_2 - K_1$), the unique value of p_1 that maximizes Firm 1's profit is:

$$p_1 = c_2 + G_1 - G_2 - K_1 = G_1 - c_1 - (G_2 - c_2) + c_1 - K_1 = c_1 + 3A - K_1 > 0. \quad (21)$$

The inequality in (21) holds because, by assumption:

$$3A - K_1 \geq 3A - \underline{K}_1 = 3A - \frac{1}{2}[3A - t] = \frac{3A}{2} + \frac{t}{2} > 0.$$

(21) implies that Firm 1's corresponding profit in the putative monopoly equilibrium is:

$$\pi_1 = p_1 - c_1 = 3A - K_1 > 0. \quad (22)$$

We now show that when $p_2 = c_2$, Firm 1 cannot increase its profit by setting $p_1 \in (p_2 + G_1 - G_2 - K_1, p_2 + G_1 - G_2 + K_2)$ or $p_1 \geq p_2 + G_1 - G_2 + K_2$.

From (5), when $p_2 = c_2$ and $p_1 = c_2 + G_1 - G_2 - K_1$:

$$\frac{\partial \pi_1}{\partial p_1} = -[p_1 - c_1] + t + G_1 - G_2 + p_2 - p_1 = t + 2K_1 - 3A \leq 0. \quad (23)$$

The inequality in (23) holds because $K_1 \leq \frac{1}{2}[3A - t]$, by assumption. (23) implies that $\frac{\partial^2 \pi_1}{\partial p_1^2} = -2 < 0$ when $p_1 \in (p_2 + G_1 - G_2 - K_1, p_2 + G_1 - G_2 + K_2)$. Therefore, (23) implies that when $p_2 = c_2$, Firm 1 cannot increase its profit by increasing p_1 from $c_1 + 3A - K_1$ to some $p_1 \in (p_2 + G_1 - G_2 - K_1, p_2 + G_1 - G_2 + K_2)$.

Lemmas A7 and A8 imply that if Firm 1 sets $p_1 > p_2 + G_1 - G_2 + K_2$, it will not sell any of its product, so its profit will be 0. Therefore, among all $p_1 \geq p_2 + G_1 - G_2 + K_2$, the price that maximizes Firm 1's profit is $p_1 = p_2 + G_1 - G_2 + K_2$. When $p_2 = c_2$, this price is $p_1 = c_2 + G_1 - G_2 + K_2$. Lemma A5 implies that when $p_1 = p_2 + G_1 - G_2 + K_2$, all consumers located in $[0, \frac{1}{2} - \frac{K_2}{2t}]$ buy Firm 1's product, whereas all consumers located in $(\frac{1}{2} - \frac{K_2}{2t}, 1]$ buy Firm 2's product. Therefore, Firm 1's profit is:

$$\tilde{\tilde{\pi}}_1 = [p_1 - c_1]\left[\frac{1}{2} - \frac{K_2}{2t}\right] = [3A + K_2]\left[\frac{1}{2} - \frac{K_2}{2t}\right]. \quad (24)$$

(22) and (24) imply that Firm 1 cannot increase its profit by setting $p_1 \geq p_2 + G_1 - G_2 + K_2$ because:

$$\pi_1 \geq \tilde{\tilde{\pi}}_1 \Leftrightarrow 3A - K_1 \geq [3A + K_2]\left[\frac{1}{2} - \frac{K_2}{2t}\right]. \quad (25)$$

The last inequality in (25) holds because $3A - K_1 \geq 3A - \underline{K}_1$, by assumption, and because:

$$3A - \frac{1}{2}[3A - t] \geq [3A + K_2]\left[\frac{1}{2} - \frac{K_2}{2t}\right] \Leftrightarrow 3A + t \geq [3A + K_2]\left[1 - \frac{K_2}{t}\right]. \quad (26)$$

The inequality in (26) holds because $K_2 \in [0, t)$.

In summary, we have established that when $K_1 \leq \underline{K}_1$ and $p_2 = c_2$, Firm 1 maximizes its profit by setting $p_1 = c_2 + G_1 - G_2 - K_1$, thereby ensuring that all consumers buy its product.

We now show that when Firm 1 sets $p_1 = c_2 + G_1 - G_2 - K_1$, Firm 2 cannot secure strictly more profit than it secures by setting $p_2 = c_2$. Lemmas A9 and A10 imply that when Firm 1 sets $p_1 = c_2 + G_1 - G_2 - K_1$, Firm 2 sells none of its product (so Firm 2 secures no profit) if it sets $p_2 = c_2$. Firm 2 continues to sell none of its product (so Firm 2 continues to secure no profit) if it sets $p_2 > c_2$. Firm 2 incurs negative profit if it sets $p_2 < c_2$. Therefore, Firm 2 cannot increase its profit by setting $p_2 \neq c_2$ when Firm 1 sets $p_1 = c_2 + G_1 - G_2 - K_1$.

Finally, Lemma A2 implies that all consumers located in the interval $(\frac{1}{2} + \frac{K_1}{2t}, 1]$ (and only these consumers) change the default setting on the product they purchase from Firm 1.

Case (ii). $t \geq A$ and $K_1 \leq \underline{K}_1$. Observation A1 implies that $\underline{K}_1 = K_{1b} \equiv 3A - \frac{1}{8t}[t + 3A]^2$ because $t \geq A$ in this case.

The proof of Case (i) implies that the identified monopoly equilibrium exists if $t \geq A$ and $K_1 \leq K_{1a}$. The remainder of the present proof establishes the corresponding existence when $t \geq A$ and $K_1 \in (K_{1a}, \underline{K}_1]$.

We first show that when $p_2 = c_2$ in this case, Firm 1 maximizes its profit by setting $p_1 = c_1 + 3A - K_1$, thereby ensuring that all consumers buy its product.

The proof that $p_1 = c_1 + 3A - K_1$ is the unique value of $p_1 \leq p_2 + G_1 - G_2 - K_1$ that maximizes Firm 1's profit (when $p_2 = c_2$) is analogous to the corresponding proof in Case (i). The inequality in (21) holds in the present case because, by assumption:

$$K_1 \leq \underline{K}_1 \Leftrightarrow 3A - K_1 \geq \frac{1}{8t}[t + 3A]^2 . \tag{27}$$

(22) and (27) imply that Firm 1's corresponding profit is:

$$\pi_1 = p_1 - c_1 = 3A - K_1 \geq \frac{1}{8t}[t + 3A]^2 > 0 . \tag{28}$$

We now show that when $p_2 = c_2$, Firm 1 cannot increase its profit by setting $p_1 \in (p_2 + G_1 - G_2 - K_1, p_2 + G_1 - G_2 + K_2)$ or $p_1 \geq p_2 + G_1 - G_2 + K_2$.

(7) implies that when $p_1 \in (p_2 + G_1 - G_2 - K_1, p_2 + G_1 - G_2 + K_2)$, the price that maximizes Firm 1's profit when $p_2 = c_2$ is:

$$p_1 = \frac{1}{2}[t + c_1 + G_1 - G_2 + p_2] = \frac{1}{2}[t + G_1 - G_2 + c_1 + c_2] . \tag{29}$$

(5) and (29) imply that Firm 1's corresponding profit is:

$$\pi_1^{'} = [p_1 - c_1]\frac{1}{2t}[t + G_1 - G_2 + p_2 - p_1] = \frac{1}{8t}[t + 3A]^2 . \tag{30}$$

(28) and (30) imply that Firm 1 cannot increase its profit by setting $p_1 \in (p_2 + G_1 - G_2 - K_1, p_2 + G_1 - G_2 + K_2)$ because:

$$\pi_1 \gtreqless \pi_1^{'} \Leftrightarrow 3A - K_1 \gtreqless \frac{1}{8t}[t + 3A]^2 \Leftrightarrow K_1 \lesseqgtr 3A - \frac{1}{8t}[t + 3A]^2 \equiv \underline{K}_1 . \tag{31}$$

The last inequality in (31) holds because, by assumption, $K_1 \leq \underline{K}_1$ in the present case.

The analysis in Case (i) implies that when $p_2 = c_2$, $p_1 = p_2 + G_1 - G_2 + K_2$ is the unique $p_1 \geq p_2 + G_1 - G_2 + K_2$ that maximizes Firm 1's profit. Furthermore, Firm 1's profit when it sets this price (and when $p_2 = c_2$) is $\widetilde{\widetilde{\pi}}_1$, as specified in (24). (22) and (24) imply Firm 1 cannot increase its profit by setting $p_1 \geq p_2 + G_1 - G_2 + K_2$ because:

$$\pi_1 \;\geq\; \tilde{\tilde{\pi}}_1 \;\Leftrightarrow\; 3A - K_1 \;\geq\; [\,3A + K_2\,]\left[\frac{1}{2} - \frac{K_2}{2t}\right]. \tag{32}$$

The last inequality in (32) holds because $3A - K_1 \geq \frac{1}{8t}[\,t + 3A\,]^2$ (since $K_1 \leq \underline{K}_1$, by assumption, in the present case) and because:

$$\begin{aligned} & \frac{1}{8t}[\,t + 3A\,]^2 \;\geq\; [\,3A + K_2\,]\left[\frac{1}{2} - \frac{K_2}{2t}\right] \\ \Leftrightarrow\quad & t^2 + 6At + 9A^2 \;\geq\; [\,12A + 4K_2\,][\,t - K_2\,] \\ \Leftrightarrow\quad & t^2 - 2[\,3A + 2K_2\,]\,t + [\,3A + 2K_2\,]^2 \;\geq\; 0 \quad\Leftrightarrow\quad [\,t - (\,3A + 2K_2\,)\,]^2 \;\geq\; 0\,. \end{aligned} \tag{33}$$

The last inequality in (33) always holds, so the first inequality in (33) holds under the specified conditions.

In summary, we have established that when $p_2 = c_2$ in the present case, Firm 1 maximizes its profit by setting $p_1 = c_2 + G_1 - G_2 - K_1$.

When Firm 1 sets $p_1 = c_2 + G_1 - G_2 - K_1$, Firm 2 maximizes its profit by setting $p_2 = c_2$, for the reasons explained in Case (i).

Finally, Lemma A2 implies that all consumers located in the interval $(\,\frac{1}{2} + \frac{K_1}{2t},\, 1\,]$ (and only these consumers) change the default setting on the product they purchase from Firm 1.

Case (iii). $K_1 > \underline{K}_1$. It remains to prove that the putative equilibrium identified in Case (i) and Case (ii) is not an equilibrium in Case (iii). $K_1 > K_{1a}$ and $K_1 > K_{1b}$ in the present case because $K_1 > \underline{K}_1 \equiv \max\{\,K_{1a},\, K_{1b}\,\}$. (22) establishes that Firm 1 secures profit $\pi_1 = 3A - K_1$ at the putative equilibrium identified in Case (i) and Case (ii). (29) and (30) establish that when Firm 2 sets $p_2 = c_2$, Firm 1 can secure profit $\pi_1^{'} = \frac{1}{8t}[\,t + 3A\,]^2$ by setting $p_1 = \frac{1}{2}[\,t + G_1 - G_2 + c_1 + c_2\,]$. (31) establishes that $\pi_1^{'} > \pi_1$ when $K_1 > \underline{K}_1$. Therefore, the putative equilibrium is not an equilibrium in Case (iii). ■

Lemma A18. *Suppose $K_1 \in (\,0,\, t\,)$ and $K_2 \in (\,0,\, t\,)$. Then the duopoly equilibrium identified in Lemma 3 (and Lemma 1) is the unique duopoly equilibrium.*

Proof. Let x_0 denote the location of the consumer who is indifferent between buying Firm 1's product and buying Firm 2's product in a duopoly equilibrium. (Lemma A17.) Lemma A14 implies that an equilibrium does not exist if $x_0 \in (\,0,\, \frac{1}{2} - \frac{K_2}{2t}\,]$. Lemmas A9 and A10 imply that all consumers buy Firm 1's product if $x_0 \in [\,\frac{1}{2} + \frac{K_1}{2t},\, 1\,)$, so a duopoly equilibrium does not exist in this case. The proof of Lemma 3 establishes that when $x_0 \in (\,\frac{1}{2} - \frac{K_2}{2t},\, \frac{1}{2} + \frac{K_1}{2t}\,)$, the unique duopoly equilibrium is the one characterized in Lemma 3 (and Lemma 1). ■

Observation A2.[44] *Suppose $A < t$. Then the conditions identified in Lemma 3 and the conditions identified in Lemma 4 are mutually exclusive because*

$$\overline{K}_1 \;\equiv\; \frac{1}{2\,t}\left[\,t^2 + 2\,A\,t - A^2\,\right] \;>\; 3\,A - \frac{1}{8\,t}\,\left[\,t + 3\,A\,\right]^2 \;\equiv\; \underline{K}_1. \tag{34}$$

Observation A3. *An equilibrium in which all consumers buy the product from Firm 2 does not exist when $A > 0$.*

Proof. In any equilibrium in which all consumers buy the product from Firm 2: (i) Firm 1's profit is 0 (because no consumers buy its product); and (ii) $p_2 \geq c_2$ (because Firm 2 must secure nonnegative profit). When $A > 0$, Firm 1 can secure strictly positive profit whenever Firm 2 sets $p_2 \geq c_2$. This is the case because Lemmas A1 and A2 imply that the consumer located at 0 strictly prefers to buy the product from Firm 1 rather than from Firm 2 if:

$$G_1 - p_1 \;\geq\; G_2 - p_2 - K_2\,. \tag{35}$$

Because $p_2 \geq c_2$, the inequality in (35) holds if:

$$G_1 - p_1 \;\geq\; G_2 - c_2 - K_2 \quad\Leftrightarrow\quad p_1 \;\leq\; c_1 + 3\,A + K_2\,. \tag{36}$$

Because $K_2 \geq 0$, the inequality in (36) (and thus the inequality in (35)) holds if $p_1 \leq c_1 + 3\,A$. Therefore, if Firm 1 sets $p_1 \in (\,c_1,\, c_1 + 3\,A\,)$, it can secure strictly positive profit by ensuring the patronage of consumers located close to 0. ■

Proof of Proposition 1. The proof employs the following four conclusions.

Conclusion 1.1. *A monopoly equilibrium in which all consumers purchase the product from Firm 1 does not exist if $K_1 > \underline{K}_1$.*

Proof. The Conclusion follows from Lemma 4. □

Conclusion 1.2. *No duopoly equilibrium exists if $A > t$.*

Proof. (9) and Lemma A18 imply that Firm 2's profit margin in a duopoly equilibrium is $p_2 - c_2 = t - A$. This profit margin is strictly negative when $A > t$. Therefore, a duopoly equilibrium does not exist when $A > t$. □

Conclusion 1.3. *No duopoly equilibrium exists if $K_1 < \overline{K}_1$.*

Proof. The Conclusion follows from Lemma 3 and Lemma A18. □

Conclusion 1.4. No equilibrium exists if $K_2 < \overline{K}_2$ and $K_1 > \underline{K}_1$.

Proof. Observation A3 establishes that when $A > 0$, no equilibrium exists in which all consumers buy Firm 2's product.

[44]The proof of Observation A2 is straightforward. See Chakravorty and Sappington (2025) for details.

Lemma 4 establishes that a monopoly equilibrium in which all consumers purchase Firm 1's product does not exist when $K_1 > \underline{K}_1$.

Lemma A18 and the proof of Lemma 3 establish that if a putative duopoly equilibrium exists, then $p_1 = c_1 + t + A$, $p_2 = c_2 + t - A$, and $\pi_2 = \frac{1}{2t}[t - A]^2$ in this putative equilibrium. The proof of Lemma 3 also establishes that when $p_1 = c_1 + t + A$, Firm 2 can secure profit that strictly exceeds $\frac{1}{2t}[t - A]^2$ by reducing p_2 sufficiently far below $c_2 + t - A$ to ensure that all consumers prefer to buy Firm 2's product than to buy Firm 1's product if $K_2 < \overline{K}_2$. Therefore, the putative equilibrium is not an equilibrium when $K_2 < \overline{K}_2$. □

The findings in Proposition 1(i) and Proposition 1(ii) follow from Observation A2 and Lemmas 3, 4, and A18. The finding in Proposition 1(iii)(a) follows from Observation A2 and Conclusions 1.1 and 1.2. The finding in Proposition 1(iii)(b) follows from Observation A2, and Conclusions 1.1 and 1.3. The finding in Proposition 1(iii)(c) follows from Observation A2 and Conclusion 1.4. ■

Proof of Lemma 5. Because no consumer changes the default horizontal characteristic of the product she purchases in the duopoly equilibrium identified in Lemma 3, consumer welfare in this equilibrium is:

$$\begin{aligned} W^{Cd} &= \int_0^{x_0} [G_1 - p_1 - t\,x]\,dx + \int_{x_0}^1 [G_2 - p_2 - t(1 - x)]\,dx \\ &= [G_1 - p_1]\,x_0 - \frac{t\,x_0^2}{2} + [G_2 - p_2][1 - x_0] - t[1 - x_0] + \frac{t}{2}[1 - x_0^2] \\ &= G_2 - p_2 + [G_1 - G_2 + p_2 - p_1]\,x_0 - \frac{t}{2}[1 - 2x_0 + 2x_0^2]\,. \end{aligned} \tag{37}$$

Lemma 3 further implies that at the identified equilibrium:

$$p_1 = c_1 + t + A \ \text{ and } \ p_2 = c_2 + t - A \ \Rightarrow \ p_2 - p_1 = c_2 - c_1 - 2A\,. \tag{38}$$

(4), (38), and Lemma 3 imply:

$$x_0 = \frac{1}{2t}[t + G_1 - G_2 + p_2 - p_1] = \frac{1}{2t}[t + A]\,. \tag{39}$$

(37) – (39) imply:

$$\begin{aligned} W^{Cd} &= G_2 - c_2 - t + A + [G_1 - G_2 + c_2 - c_1 - 2A]\,\frac{1}{2t}[t + A] - \frac{t}{2}[1 - 2x_0 + 2x_0^2] \\ &= G_2 - c_2 - \frac{5}{4}t + \frac{3}{2}A + \frac{A^2}{4t}\,. \end{aligned} \tag{40}$$

The (only) consumers who change the default horizontal characteristic of the product they purchase in the monopoly equilibrium identified in Lemma 2 are those located in $(\frac{1}{2}, 1]$. Therefore, because $K_1 = 0$, consumer welfare in this equilibrium is:

$$W_0^{Cm} \;=\; \int_0^{\frac{1}{2}} [\, G_1 - p_1 - t\, x\,]\, dx + \int_{\frac{1}{2}}^{1} [\, G_1 - p_1 - t\,(\,1-x\,)\,]\, dx \;=\; G_2 - c_2 - \frac{t}{4} \,. \;\blacksquare \qquad (41)$$

Proof of Proposition 2. Lemma 2 implies that when $K_1 = K_2 = 0$, Firm 1's equilibrium profit is:

$$\pi_{1a} \;=\; 3\,A. \qquad (42)$$

$A < t$ when $A < \frac{t}{2+\sqrt{3}}$. Therefore, the maintained assumptions ensure that the conditions in Lemma 3 are all satisfied. In the equilibrium identified in Lemma 3 (and Lemma 1), Firm 1's profit is:

$$\pi_{1b} \;=\; \frac{[\,t + A\,]^2}{2\,t} \,. \qquad (43)$$

(42) and (43) imply:

$$\pi_{1b} \;>\; \pi_{1a} \;\;\Leftrightarrow\;\; t^2 + 2\,A\,t + A^2 \;>\; 6\,A\,t \;\;\Leftrightarrow\;\; t^2 - 4\,A\,t + A^2 \;>\; 0\,. \qquad (44)$$

The ("t") roots of the quadratic equation associated with (44) are:

$$\frac{1}{2}\left[\,4\,A \pm \sqrt{16\,A^2 - 4\,A^2}\,\right] \;=\; \frac{1}{2}\left[\,4\,A \pm \sqrt{12\,A^2}\,\right] \;=\; A\left[\,2 \pm \sqrt{3}\,\right]. \qquad (45)$$

(44) and (45) imply that $\pi_{1b} > \pi_{1a}$ when $t > A\left[\,2 + \sqrt{3}\,\right] \Leftrightarrow A < \frac{t}{2+\sqrt{3}}$.

Lemma 2 implies that Firm 2's equilibrium profit is 0 when $K_1 = K_2 = 0$. Firm 2's profit in the duopoly equilibrium identified in Proposition 3 is $\frac{1}{2\,t}\,[\,t - A\,]^2 > 0$.

(40) and (41) imply:

$$W^{Cd} \gtreqless W_0^{Cm} \;\Leftrightarrow\; G_2 - c_2 - \frac{5\,t}{4} + \frac{3\,A}{2} + \frac{A^2}{4\,t} \gtreqless G_2 - c_2 - \frac{t}{4} \;\Leftrightarrow\; A^2 + 6\,t\,A - 4\,t^2 \gtreqless 0\,. \qquad (46)$$

The ("A") roots of the quadratic equation in (46) are:

$$\frac{1}{2}\left[\,-\,6\,t \pm \sqrt{36\,t^2 + 16\,t^2}\,\right] \;=\; \frac{1}{2}\left[\,-\,6\,t \pm \sqrt{52\,t^2}\,\right] \;=\; -\,3\,t \pm t\sqrt{13}\,. \qquad (47)$$

The smaller root is negative whereas the larger root is positive. Therefore, (46) and (47) imply that $W^{Cd} < W_0^{Cm}$ if $A \in (0, \left[\,\sqrt{13} - 3\,\right] t)$.

Observe that $\sqrt{13} - 3 \approx 0.606 > \frac{1}{2+\sqrt{3}} \approx 0.268$. Therefore, $A \in (0, \left[\,\sqrt{13} - 3\,\right] t)$ when $A < \frac{t}{2+\sqrt{3}}$. Consequently, $W^{Cd} < W_0^{Cm}$ when $A < \frac{t}{2+\sqrt{3}}$.

(12) and (13) imply that at the equilibrium identified in Lemma 3, industry profit is:

$$\frac{1}{2\,t}\,[\,t + A\,]^2 + \frac{1}{2\,t}\,[\,t - A\,]^2 \;=\; \frac{1}{2\,t}\left[\,2\,t^2 + 2\,A^2\,\right] \;=\; \frac{1}{t}\left[\,t^2 + A^2\,\right]. \qquad (48)$$

Let T^d denote total welfare in the duopoly equilibrium identified in Lemma 3. Also let T_0^m denote total welfare in the monopoly equilibrium identified in Lemma 2, where $K_1 = K_2 = 0$. (40) and (48) imply:

$$T^d \;=\; G_2 - c_2 - \frac{5}{4}\,t + \frac{3}{2}\,A + \frac{A^2}{4\,t} + \frac{1}{t}\left[\,t^2 + A^2\,\right] \;=\; G_2 - c_2 - \frac{t}{4} + \frac{3\,A}{2} + \frac{5\,A^2}{4\,t}\,. \qquad (49)$$

Lemma 2 implies that when $K_1 = K_2 = 0$, equilibrium industry profit is $\pi_1 + \pi_2 =$

$3\,A + 0 \;=\; 3\,A$. Therefore, (41) implies:

$$T_0^m \;=\; G_2 - c_2 - \frac{t}{4} + 3\,A\,. \tag{50}$$

(49) and (50) imply:

$$T_0^m \;>\; T^d \;\;\Leftrightarrow\;\; G_2 - c_2 - \frac{t}{4} + 3\,A \;>\; G_2 - c_2 - \frac{t}{4} + \frac{3\,A}{2} + \frac{5\,A^2}{4\,t} \;\;\Leftrightarrow\;\; A \;<\; \frac{6}{5}\,t\,.$$

The last inequality here holds because $A < t$, by assumption. ■

Proof of Proposition 3. Proof of (i). Observation A3 implies that no equilibrium exists in which all consumers buy Firm 2's product because $A > 0$, by assumption.

(9) and Lemma A18 imply that if $A > t$, then Firm 2's profit margin in a duopoly equilibrium ($p_2 - c_2 = t - A$) is negative. Therefore, a duopoly equilibrium does not exist when $A > t$.

(10) and Lemma A18 imply that if $A = t$, then in a putative duopoly equilibrium, the consumer who is indifferent between buying Firm 1's product and Firm 2's product is located at $x_0 = \frac{1}{2} + \frac{A}{2\,t} = \frac{1}{2} + \frac{A}{2\,A} = 1$. Therefore, no duopoly equilibrium exists if $A = t$.

Lemmas 2 and 4 establish that Firm 1's profit in any monopoly equilibrium in which all consumers buy Firm 1's product is $3\,A - K_1 \,\leq\, 3\,A$. Therefore, Firm 1's profit is highest in such an equilibrium when $K_1 = 0$.

Lemmas 1, 3, and A18 imply that Firm 1's profit in a duopoly equilibrium is $\frac{1}{2\,t}\,[\,t + A\,]^2$. Observe that:

$$\frac{1}{2\,t}\,[\,t + A\,]^2 \;>\; 3\,A \;\;\Leftrightarrow\;\; t^2 - 4\,A\,t + A^2 \;>\; 0\,. \tag{51}$$

(51) holds when $t < t_L$ or $t > t_H$, where:

$$t_L \;=\; \frac{1}{2}\left[\,4\,A - \sqrt{16\,A^2 - 4\,A^2}\,\right] \;=\; \left[\,2 - \sqrt{3}\,\right] A \quad \text{and} \quad t_H \;=\; \left[\,2 + \sqrt{3}\,\right] A\,. \tag{52}$$

(52) implies:

$$t \;<\; t_L \;\;\Leftrightarrow\;\; A \;>\; \frac{t}{2 - \sqrt{3}}\,; \quad t \;>\; t_H \;\;\Leftrightarrow\;\; A \;<\; \frac{t}{2 + \sqrt{3}}\,. \tag{53}$$

The inequalities in (53) do not hold when $A \in (\,\frac{t}{2+\sqrt{3}},\, t\,)$ (because $\frac{t}{2-\sqrt{3}\,]} \approx 3.732\,t \,>\, t$). Therefore, the analysis in (51) – (53) implies that if Firm 1 sets $K_1 > 0$ and a duopoly equilibrium ensues, Firm 1's profit does not exceed $3\,A$ when $A \in (\,\frac{t}{2+\sqrt{3}},\, t\,)$.

Finally, suppose a duopoly equilibrium exists when $A \in (\,\frac{t}{2+\sqrt{3}},\, t\,)$. Lemmas 1, 3, and A18 imply that $p_2 = c_2 + t - A$ and $\pi_1 = \frac{1}{2\,t}\,[\,t + A\,]^2$ in this putative equilibrium. The analysis in (51) – (53) implies that when $A \in (\,\frac{t}{2+\sqrt{3}},\, t\,)$, Firm 1 can secure strictly more profit by setting $K_1 = 0$ and reducing p_1 sufficiently to ensure that all consumers buy its product. Therefore, the putative duopoly equilibrium is not an equilibrium.

Proof of (ii). We first prove that the identified duopoly equilibria exist when $A < \frac{t}{2+\sqrt{3}}$, $K_1 \geq \overline{K}_1$ and $K_2 \geq \overline{K}_2$.

Observe that $A < t$ when $A < \frac{t}{2+\sqrt{3}}$ because $\frac{1}{2+\sqrt{3}} \approx 0.268 < 1$. Therefore, Lemma 3 establishes that a duopoly equilibrium exists under the specified conditions. Lemmas 3 and A18 imply that the profits of Firm 1 and Firm 2 in this equilibrium are $\pi_1^d = \frac{1}{2t}[t+A]^2 > 0$ and $\pi_2^d = \frac{1}{2t}[t-A]^2 > 0$, respectively. Observe that these profits do not vary with K_1 or K_2. Therefore, neither firm can increase its profit by unilaterally varying its default-switching cost if a duopoly equilibrium ensues.

Observation A3 implies that no equilibrium exists in which all consumers buy Firm 2's product because $A > 0$, by assumption.

The proof of Observation A2 implies that a monopoly equilibrium in which all consumers buy Firm 1's product does not exist when $A < t$ and $K_1 \geq \overline{K}_1$. Therefore, such an equilibrium does not exist under the specified conditions.

The analysis in (51) – (53) implies that a monopoly equilibrium in which all consumers buy Firm 1's product does not exist when $A < \frac{t}{2+\sqrt{3}}$ and $K_1 \leq \underline{K}_1$. This is the case because when $p_2 = c_2$, Firm 1's profit in a monopoly equilibrium is at most $3A$, which is strictly less than Firm 1's profit in the identified duopoly equilibrium $(\frac{1}{2t}[t+A]^2)$.

Now we prove that the identified monopoly equilibria can exist when $A < \frac{t}{2+\sqrt{3}}$, $K_1 = 0$, and $K_2 \in [0, \overline{K}_2)$. Lemmas 2 and 4 imply that in any monopoly equilibrium with the specified properties, Firm 1's profit is $\pi_1^m = 3A$ and Firm 2's profit is $\pi_2^m = 0$.

Lemma 2 implies that the identified monopoly equilibrium exists when $K_1 = 0$ and $K_2 = 0$. To determine when a monopoly equilibrium with $K_1 = 0$ and $K_2 \in (0, \overline{K}_2)$ exists, observe that $\frac{t}{3[3+2\sqrt{2}]} < \frac{t}{2+\sqrt{3}}$. Also observe that $A \leq t$ (so $\underline{K}_1 = K_{1b} \equiv 3A - \frac{1}{8t}[t+3A]^2$, from Observation A1) when $A < \frac{t}{2+\sqrt{3}}$. Furthermore:

$$K_{1b} \geq 0 \quad \Leftrightarrow \quad A \in \left[\frac{t}{3(3+2\sqrt{2})}, \frac{t}{3(3-2\sqrt{2})}\right]. \tag{54}$$

(54) holds because:

$$3A \geq \frac{1}{8t}[t+3A]^2 \quad \Leftrightarrow \quad [t+3A]^2 \leq 24At \quad \Leftrightarrow \quad t^2 - 18At + 9A^2 \leq 0. \tag{55}$$

The ("t") roots of the quadratic equation associated with (55) are:

$$\frac{1}{2}\left[18A \pm \sqrt{324A^2 - 36A^2}\right] = 9A \pm A\sqrt{72} = 3\left[3 \pm 2\sqrt{2}\right]A. \tag{56}$$

(55) and (56) imply that $3A \geq \frac{1}{8t}[t+3A]^2$ if and only if (54) holds. Consequently, Lemma 4 implies that the identified monopoly equilibria with $K_1 = 0$ and $K_2 \in (0, \overline{K}_2)$ exist when $A \in [\frac{t}{3(3+2\sqrt{2})}, \frac{t}{2+\sqrt{3}})$.

Lemma A18 and the proof of Lemma 3 establish that if Firm 1 increases K_1 above 0, a duopoly equilibrium does not ensue because $K_2 < \overline{K}_2$. Furthermore, the increase in K_1 would reduce Firm 1's profit (π_1) if a monopoly equilibrium in which all consumers buy Firm 1's product ensues (because $\pi_1 = 3A - K_1$ in any such equilibrium). Consequently, Firm 1 cannot secure a strict increase in profit by setting $K_1 > 0$ when $K_2 \in [0, \overline{K}_2)$.

If Firm 2 changes K_2, a duopoly equilibrium will not ensue because $K_1 = 0 < \overline{K}_1$.

(Recall Lemmas 3 and A18.) $\overline{K}_1 > 0$ when $A < \frac{t}{2+\sqrt{3}}$ because:

$$\overline{K}_1 \;=\; \frac{1}{2\,t}\left[\,t^2 + 2\,A\,t - A^2\,\right] \;>\; 0 \quad\Leftrightarrow\quad t^2 + 2\,A\,t + A^2 \;>\; 2\,A^2$$

$$\Leftrightarrow\quad t + A \;>\; \sqrt{2}\,A \quad\Leftrightarrow\quad A \;<\; \frac{t}{\sqrt{2}-1}\,. \tag{57}$$

The last inequality in (57) holds when $A < \frac{t}{2+\sqrt{3}}$ because $\frac{1}{\sqrt{2}-1} \approx 2.415 > 0.268 \approx \frac{1}{2+\sqrt{3}}$.

Furthermore, a change in K_2 will not change Firm 2's profit if a monopoly equilibrium in which all consumers buy Firm 1's product ensues. Therefore, when $K_1 = 0$ and $K_2 \in [\,0, \overline{K}_2\,)$, neither firm can strictly increase its profit above the level it secures in the monopoly equilibrium where all consumers buy Firm 1's product by changing its default-switching cost. ■

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